Neutrino mass and weak lensing

• The Good
• The Bad
• The Ugly
• A Proposal
Gravitational Potentials Decay if Neutrinos have Mass

- Massive neutrinos contribute to the energy density
- This fraction of the matter does not clump on small scales
- Destroys a delicate balance between expansion and gravitational instability
- Produces relatively large changes in the power spectrum of matter

\[ f_v \equiv \frac{\rho_v}{\rho_{\text{matter}}} = 0.1 \left( \sum \frac{m_v}{1\text{eV}} \right) \]
Growth of Structure: Gravitational Instability

Define overdensity:

\[ \delta(\vec{x}, t) \equiv \frac{\rho(\vec{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)} \]

Fundamental equation governing overdensity in a matter-dominated universe when scales are within horizon:

\[ \ddot{\delta} + 2H \dot{\delta} - 4\pi G \bar{\rho}_m \delta = 0 \]
Growth of Structure: Gravitational Instability

\[ \ddot{\delta} + 2H \dot{\delta} - 4\pi G \bar{\rho}_m \delta = 0 \]

**Example 1:** No expansion (H=0, energy density constant)

\[ \delta \propto e^{\pm t \sqrt{4\pi G \bar{\rho}_m}} \]

- Two modes: growing and decaying
- Growing mode is exponential (the more matter there is, the stronger is the gravitational force)
Gravitational Instability in an Expanding Universe

\[ \ddot{\delta} + 2H\dot{\delta} - 4\pi G \bar{\rho}_m \delta = 0 \]

**Example 2:** Matter density equal to the critical density in an expanding universe.

The coefficient of the 3rd term is then \( 3H^2/2 \), and \( H=2/(3t) \)

\[ \ddot{\delta} + \frac{4}{3t} \dot{\delta} - \frac{2}{3t^2} \delta = 0 \]
Gravitational Instability in an Expanding Universe

\[ \ddot{\delta} + \frac{4}{3t} \dot{\delta} - \frac{2}{3t^2} \delta = 0 \]

Insert solution of the form: \( \delta \sim t^p \)

\[ p = \begin{cases} 
2/3 \\
-1 
\end{cases} \]

Growing mode: \( \delta \sim a \). Dilution due to expansion counters attraction due to overdensity. Result: power law growth instead of exponential growth.
Gravitational Potential

Poisson Equation:
\[ \nabla^2 \Phi = 4\pi G \rho \delta \]

In Fourier space, this becomes:
\[ -\frac{k^2}{a^2} \Phi \propto \frac{\delta}{a^3} \]

So the gravitational potential remains constant! Delicate balance between attraction due to gravitational instability and dilution due to expansion.

Only holds if all the energy is in non-relativistic matter. Dark energy and massive neutrinos lead to potential decay.
Massive Neutrino Suppress Growth on Small Scales

A fraction $f_\nu$ of the total density does not participate in collapse on scales smaller than the free-streaming scale

$$k_{fs}^{-1} \approx \frac{vt}{a} \approx \frac{(T/m)H^{-1}}{a}$$
The Power Spectrum Quantifies this Suppression

\[ \left\langle \tilde{\delta}(\vec{k})\tilde{\delta}(\vec{k}') \right\rangle = (2\pi)^3 \delta^3(\vec{k} + \vec{k}') P(k) \]

Even for a small neutrino mass, get large impact on structure: power spectrum is excellent probe of neutrino mass
# Overview of Cosmological Probes of Neutrino Mass

<table>
<thead>
<tr>
<th>Technique</th>
<th>Underlying Physics</th>
<th>Linear?</th>
<th>Un-biased?</th>
<th>3D?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lensing of Galaxies</td>
<td>Decaying $\Phi$</td>
<td>No</td>
<td>Yes</td>
<td>No+</td>
</tr>
<tr>
<td>Lensing of CMB</td>
<td>Decaying $\Phi$</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Galaxy Clustering</td>
<td>Decaying $\Phi$</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Lyman alpha forest</td>
<td>Decaying $\Phi$</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>CMB</td>
<td>Acoustic Oscillations</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Red/No is Bad
Gravitational Lensing

The geodesic equation

\[ \frac{d^2 x^i}{d\lambda^2} = -\Gamma^i_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} \]

in a perturbed Friedman-Robertson-Walker metric reduces to

\[ \frac{d^2}{d\chi^2} (\chi \theta^i) = 2\Phi, i \]

Distance from us

Angular position
Integrate to find the deflection

\[ \theta^i_S = \theta^i + \int d\chi \, W(\chi_s, \chi) \Phi, i \]

with kernel

\[ W(\chi_s, \chi) \equiv \begin{cases} 
2 \left( 1 - \frac{\chi}{\chi_s} \right) & \chi < \chi_s \\
0 & \text{otherwise} \end{cases} \]

Define the distortion tensor

\[ \psi_{ij}(\vec{\theta}) \equiv \frac{\partial \delta \theta_i}{\partial \theta_j} = \int d\chi W(\chi_s, \chi) \chi \Phi_{ij}(\chi, \vec{\theta}) \]
Distortion Tensor is Measurable!

\[ \psi_{ij} \equiv \begin{pmatrix} -\kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & -\kappa + \gamma_1 \end{pmatrix} \]

\( \kappa \) is the projected density, a measure of the convergence of light rays.
\( \gamma_1 \) are the two components of shear.
Galaxy ellipticities are probes of cosmic shear

- Mean shear is zero
- RMS shear is 0.01
- Galaxy has intrinsic ellipticity (randomly oriented on the sky) ~0.3
- To get S/N~1, need ~1000 galaxies
We can compute the 2-point functions of cosmic shear

\[
C_l = \frac{l^4}{4} \int_0^\infty d\chi \frac{g^2(\chi)}{\chi^2} P_\Phi(l/\chi)
\]

Curves: Dodelson, Shapiro, & White 2005

Points: N Body Simulations
Lensing Improves Current Constraints

Ichiki, Takada, & Takahashi 2009
Projections for Future Constraints

Abazajian & Dodelson 2003

De Bernardis et al. 2009

Scott Dodelson (INT Workshop)
The Bad

- Degeneracies

- Nonlinearities

- Baryons
Degeneracies

\[ \Delta C_l / C_l \]

\[ m_2 = 0.05 \text{ eV} \]
Degeneracies

![Graph showing the change in $\Delta C/L$ with $l$ for $m_\nu = 0.05$ eV]
Degeneracies

![Graph showing degeneracies with labels: 1% Change in $\sigma$, $m_\nu = 0.05$ eV, $\Delta \sigma_1 = 0.01$.]
Nonlinearities

Dimensionless measure of nonlinearity

\[ \Delta^2(k) \equiv \frac{k^3 P(k)}{2\pi^2} \]
Nonlinearities

Abazajian et al. 2005

Saito, Takada, & Taruya 2008

Brandbyge & Hannestad 2009

Shoji & Komatsu 2009

Wong 2008

Lesgourgues et al. 2009

2/8/2010 Scott Dodelson (INT Workshop)
Baryons affect the gravitational potential on small scales

Rudd, Zentner & Kravtsov 2007
The Ugly

Use shapes of stars (point-like) to correct for PSF

Stripe 82 Gang: Soares-Santos, Simet, Dodelson, Lin, Kubo, Annis et al.
Use stars to model PSF across the field. These plots show difference between actual stellar ellipticities and model ellipticities.

Stripe 82 Gang: Soares-Santos, Simet, Dodelson, Lin, Kubo, Annis et al.
Proposal: Cosmological Neutrino Challenge

• Mock Sky using Input Cosmology
  – 3D Density (Linear, Non-Linear, Baryons, Bias, ...)
  – Shear Maps (Ray Tracing)
  – Lyman alpha maps (HPM, Hydro, ...)
  – Lensed CMB Maps (including tSZ, kSZ, Reionization, ...)

• Blind Analysis
  – Apply Planck Priors
  – Extract Neutrino Mass

We will not be believed (nor will we believe) until we do this
Back Up Slides
Learn more form lensing if redshifts of background galaxies are known
Tomography

\[ \phi \text{ (growth)} \]

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\[ m_{\text{tot}} = 0.2 \text{eV} \]

Hu 2002
Abazajian et al. 2005
Order of Magnitude Estimate

\[ \psi_{ij} = -2 \int_0^{\chi_s} d\chi \chi \frac{\chi_s - \chi}{\chi_s} \frac{\partial^2 \Phi}{\partial x^i \partial x^j} \]

First guess:

\[ \psi_{rms} \sim \left( \frac{r_H}{\lambda_{max}} \right)^2 \Phi_{rms} \]

Only Fourier modes contribute which do not vary along line of sight

\[ \psi_{rms} \sim \left( \frac{r_H}{\lambda_{max}} \right)^{3/2} \Phi_{rms} \sim 1\% \]
Review of First Order Calculation

- \( \psi \) is shear; \( \chi \) comoving distance; \( x_\perp \) perpendicular deflection; \( p \) is direction vector
- Solve 1st order geodesic eqn for \( f \) in terms of first order gravitational potential \( \phi \)
- To be consistent, evaluate \( \phi \) at undeflected \( x \)

\[
\psi_{ij}(\vec{\theta}, \chi_s) = \frac{1}{\chi_s} \frac{\partial x_i^j}{\partial \theta_j}. 
\]

\[
x^{(1)i}_\perp(\theta) = \int_0^{\chi_s} d\chi (\chi_s - \chi) f^{(1)i}_\perp(\chi, \vec{\theta}). 
\]

\[
f^{(1)i}_\perp(\chi, \vec{\theta}) = -\Gamma^{(1)i}_{\alpha\beta} p^{(0)\alpha} p^{(0)\beta} = -2 \frac{\partial \Phi^{(1)}(\vec{x}^{(0)}(\theta, \chi))}{\partial x^i}. 
\]

\[
\psi^{(1)}_{ij}(\vec{\theta}, \chi_s) = -2 \int_0^{\chi_s} d\chi \chi \frac{\chi_s - \chi}{\chi_s} \frac{\partial^2 \Phi^{(1)}(\vec{x}^{(0)}(\theta, \chi))}{\partial x^i \partial x^j}. 
\]