Dijet Production and $k_t$ Factorization: What Can We Learn from RHIC to EIC?

Bo-Wen Xiao

Department of Physics, Penn State University
AND
Lawrence Berkeley National Laboratory

  INT, workshop, Oct, 2010
The Redemption of the $k_t$ factorization.

- The Redemption of the $k_t$ factorization.
- The Salvation of the dijet production calculation.
Outline

Introduction to the $k_t$ factorization and its violation

Quark Distributions

Gluon Distributions
- DIS dijet
- $\gamma+$Jet in $pA$
- Gluon+Jet in $pA$

Conclusion

Backup
$K_t$ Factorization "expectation"

Consider the inclusive production of two high-transverse-momentum back-to-back particles in hadron-hadron collisions, i.e., in the process:

$$H_1 + H_2 \rightarrow H_3 + H_4 + X.$$ 

The standard $k_t$ factorization "expectation" is:

$$E_3 E_4 \frac{d\sigma}{d^3 p_3 d^3 p_4} = \sum \int d\hat{\sigma}_{i+j \rightarrow k+l} x f_{i/1} f_{j/2} d_3/k d_4/l + \cdots$$

- Convolution of $d\hat{\sigma}$ with $f(x, k_\perp)$ and $d$.
- **Factorization** ⇔ **Factorization formula** + Universality
- Only Drell-Yan process is proved for factorization in hadron-hadron collisions. [Bodwin; 85, 86], [Collins, Soper, Sterman; 85, 88].
- However, $k_t$ factorization is used to compute dijet production in $dAu$ collisions.
Breaking down of the $k_t$ factorization in di-hadron production

- [Bacchetta, Bomhof, Mulders and Pijlman; 04-06] Wilson lines approach
  Studies of Wilson-line operators show that the TMD parton distributions are not generally process-independent due to the complicated combination of initial and final state interactions. TMD PDFs admit process dependent Wilson lines.

- [Collins, Qiu; 07], [Collins; 07], [Vogelsang, Yuan; 07] and [Rogers, Mulders; 10]
  Scalar QED models and its generalization to QCD (Counterexample to Factorization)

- $O(g^2)$ calculation shows non-vanishing anomalous terms with respect to standard factorization.

- Remarks: $k_t$ factorization is violated in di-jet production.

- TMD parton distributions are non-universal.

- It seems worst nightmare becomes reality! For $pp$ and $AA$ collisions, no factorization formula at all for dijet production.
Why is the di-jet production process special?

Initial state interactions and/or final state interactions

▶ In Drell-Yan process, there are only initial state interactions.

\[
\int_{-\infty}^{+\infty} dk_+^g \frac{i}{-k_+^g - i\epsilon} A^+(k_g) = \int_{0}^{-\infty} d\zeta^- A^+(\zeta^-)
\]

Eikonal approximation \(\Rightarrow\) gauge links.

▶ In DIS, there are only final state interactions.

\[
\int_{-\infty}^{+\infty} dk_+^g \frac{i}{-k_+^g + i\epsilon} A^+(k_g) = \int_{0}^{+\infty} d\zeta^- A^+(\zeta^-)
\]

Eikonal approximation \(\Rightarrow\) gauge links.

▶ However, there are both initial state interactions and final state interactions in the di-jet process.
Existing calculations on dijet production

Let us first look back, and re-examine the existing calculations on dijet productions.

I. Quark+Gluon channel [Marquet, 07] and [Albacete, Marquet, 10]

- Prediction of saturation physics.
- All the framework is correct, but 4-point function is over simplified.
- The complete 4-point function [F. Dominguez, C. Marquet and B. Wu; 09.]
- Improvement [F. Dominguez, C. Marquet, BX and F. Yuan, in preparation.]

II. Gluon+Gluon channel [Tuchin, 09]

- Fit the RHIC data amazingly well.
- Not correct, since the starting formula is $K_t$ factorization formula.
- Should be small, since gluon distribution is very small at large $x$. 
Fear and Hope

- $k_t$ factorization in the "court of physics law", guilty as charged.

Trouble? or Disaster?

- Fear: loss of prediction power.
- Fear can hold you prisoner. Hope can set you free.

Hope: For $pA$ (dilute-dense system) collisions, there is an effective $k_t$ factorization.
No way out?

Need to use the effective TMD factorization.

The effective $k_t$ factorization for $pA$ (dilute-dense system) collisions

$$\frac{d\sigma^{pA\rightarrow qfX}}{d^2P_\perp d^2q_\perp dy_1 dy_2} = x_p q(x_p, \mu^2) x_f(x, q^2_\perp) \frac{1}{\pi} \frac{d\hat{\sigma}}{d\hat{t}}.$$

Remarks:

- **Penalty**: $K_t$ dependent Parton distributions $x_f(x, q^2_\perp)$ are not universal.
- $x_p q(x_p, \mu^2)$ is the Feynman parton distribution of the dilute projectile.
- Thanks to the nuclear enhancement, soft gluon exchange from the dilute proton can be neglected.
Breaking down of the $k_t$ factorization at small-$x$

[Brodsky, et al, 02](DIS)
[Collins, Qiu; 07], [Collins; 07], [Vogelsang, Yuan; 07](di-jet)

We employ the same scalar QED model, and study the TMD parton distributions at small-$x$ in di-jet production process in pA collisions.

- This model allows us to calculate the TMD quark distributions up to all order in a few of different processes exactly in the small-$x$ limit.
- The goal is to compute the TMD quark distribution in di-jet production and compare it with those calculated in DIS and Drell-Yan process.
- Establish an **effective generalized** TMD factorization in pA collision.
Comparison of the TMD pdfs among di-jet production, DIS and Drell-Yan

The total amplitude ⇒ the TMD pdf for the di-jet process \[\text{Xiao, Yuan, 10}\]

\[
\tilde{q}_{\text{di-jet}}(x, q_\perp) = \frac{xP^2}{8\pi^4} \int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp \cdot (R_\perp - R'_\perp)}
\]
\[
\times V(r_\perp) V(r'_\perp) e^{-igg_2(G(R_\perp) - G(R'_\perp))}
\]
\[
\times \left\{ 1 - e^{igg_1[G(R_\perp + r_\perp) - G(R_\perp)]} \right\}
\]
\[
\times \left\{ 1 - e^{-igg_1[G(R'_\perp + r'_\perp) - G(R'_\perp)]} \right\} .
\]

TMD pdf in DIS and DY \[\text{Brodsky, et al; 02}, \text{Belitsky, Ji, Yuan; 03}, \text{Peigne; 02}\]

\[
\tilde{q}_{\text{DIS, DY}}(x, q_\perp) = \frac{xP^2}{8\pi^4} \int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp \cdot (R_\perp - R'_\perp)}
\]
\[
\times V(r_\perp) V(r'_\perp)
\]
\[
\times \left\{ 1 - e^{igg_1[G(R_\perp + r_\perp) - G(R_\perp)]} \right\}
\]
\[
\times \left\{ 1 - e^{-igg_1[G(R'_\perp + r'_\perp) - G(R'_\perp)]} \right\} .
\]

with \(r'_\perp = R_\perp + r_\perp - R'_\perp\).
Non-Universality

Relating the quark distributions

\[ x\tilde{q}^{\text{di-jet}}(x, q_\perp) = \int d^2 l_\perp x\tilde{q}^{\text{DIS}}(x, l_\perp) F(q_\perp - l_\perp, Q_s), \]

with \( F(k_\perp, Q_s) \) being the normalized unintegrated gluon distribution

\[ F(k_\perp, Q_s) = \int \frac{d^2 r_\perp}{(2\pi)^2} e^{-ik_\perp \cdot r_\perp} \frac{\text{Tr}\langle U(R_\perp) U^\dagger(R_\perp + r_\perp)\rangle_\rho}{N_c} \approx \frac{1}{\pi Q_s^2} \exp\left(-\frac{k_\perp^2}{Q_s^2}\right). \]

Now let us compare it with [Bomhof et al; 06]

\[ q^{\text{di-jet}}(x, q_\perp) = \frac{1}{2} \int \frac{d^2 \xi_\perp d\xi^-}{(2\pi)^2} e^{-ix^+ \xi^- - iq_\perp \cdot \xi_\perp} \langle P|\bar{\Psi}(\xi)[G]\Psi(0)|P\rangle, \]

with \( G = \frac{N_c^2 + 1}{N_c^2 - 1} \frac{\text{Tr}(U[\Box])}{N_c} U^{[+] -} - \frac{2}{N_c^2 - 1} U^{[-]}. \) For DIS, \( G = U^{[+]}. \) In large \( N_c \) limit,

\[ \text{Tr}\langle U(R_\perp) U^\dagger(R_\perp + r_\perp)\rangle_\rho \Leftrightarrow \frac{\text{Tr}\left(U[\Box]\right)}{N_c}. \]
Non-Universality

Transverse Momentum Dependent (TMD) quark distributions:

- Comparison of quark distributions $\frac{4\pi^4}{N_c} \frac{d\tilde{q}_{\text{tot}}(x, q_{\perp})}{d^2 R_{\perp}}$ as functions of $\frac{q_{\perp}^2}{Q_s^2}$ in DIS (or Drell-Yan) and di-hadron production. The solid curve stands for the quark distribution in DIS and Drell-Yan process, and the dash curve represents the distribution involved in di-hadron production.

- Non-Universality (as a result of initial and final state interactions) and $k_t$ factorization violation in CGC. However, they are calculable.

- Integrated quark distributions are universal.
A Tale of Two Gluon Distributions

In small-x physics, two gluon distributions are widely used:

I. **Weizsäcker Williams** gluon distribution (MV model):

\[
xG^{(1)} = \frac{S_{\perp}}{\pi^2 \alpha_s} \frac{N_c^2 - 1}{N_c} \leq \frac{N_c}{2\pi^2 \alpha_s}
\]

\[
\times \int \frac{d^2 r_{\perp}}{(2\pi)^2} \frac{e^{-ik_{\perp} \cdot r_{\perp}}}{r_{\perp}^2} \left(1 - e^{-\frac{r_{\perp}^2 Q_s^2}{2}}\right)
\]

II. **Color Dipole** gluon distributions:

\[
xG^{(2)} = \frac{S_{\perp}}{\pi^2 \alpha_s} \frac{N_c}{N_c} \leq \frac{N_c}{2\pi^2 \alpha_s}
\]

\[
\times \int \frac{d^2 r_{\perp}}{(2\pi)^2} e^{-ik_{\perp} \cdot r_{\perp}} \nabla_{r_{\perp}}^2 N(r_{\perp})
\]

Remarks:

- The WW gluon distribution simply counts the number of gluons.
- The Color Dipole gluon distribution often appears in calculations. \(N(r_{\perp})\) is the color dipole amplitude. It is now in fundamental representation. The adjoint representation form is similar and also widely used.
- Does this mean that gluon distributions are non-universal? Answer: Yes and No!
- These two distributions are used in \(R_{pA}\) calculation. [Kharzeeve, Kovchegov, Tuchin; 03].
A Tale of Two Gluon Distributions

In small-x physics, two gluon distributions are widely used:

I. Weizsäcker Williams gluon distribution (MV model):

\[
\frac{d^2 \rho}{(2\pi)^2} \frac{N_c^2}{N_c} \left( 1 - e^{-\frac{r^2 Q_s^2}{2}} \right)
\]

II. Color Dipole gluon distributions:

\[
\frac{d^2 \rho}{(2\pi)^2} e^{-i k \cdot r} \nabla r^2 N(r)
\]
A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, to be published]

I. Weizsäcker Williams gluon distribution (MV model):

\[ xG^{(1)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3} P^+ e^{i x P^+ \xi^- - ik_\perp \cdot \xi_\perp} Tr<P|F^{+i}(\xi^-, \xi_\perp)U^+[+]|F^{+i}(0)U^+[+]|P>. \]

II. Color Dipole gluon distributions:

\[ xG^{(2)} = 2 \int \frac{d\xi^- d\xi_\perp}{(2\pi)^3} P^+ e^{i x P^+ \xi^- - ik_\perp \cdot \xi_\perp} Tr<P|F^{+i}(\xi^-, \xi_\perp)U^-[-]|F^{+i}(0)U^+[+]|P>. \]

Remarks:

- The WW gluon distribution is the conventional gluon distributions. In light-cone gauge, it is the gluon density. (Only final state interactions.)
- The dipole gluon distribution has no such interpretation. (Initial and final state interactions.)
- Both definitions are gauge invariant.
- Same after integrating over \(q_\perp\).
- Same perturbative tail.
A Tale of Two Gluon Distributions

In terms of operators, we find these two gluon distributions can be defined as follows: [F. Dominguez, C. Marquet, BX and F. Yuan, to be published]

I. Weizsäcker Williams gluon distribution (MV model):

\[
xG^{(1)} = 2 \int \frac{d\xi^- d\xi_{\perp}}{(2\pi)^3} e^{ixP^+ \xi^- - ik_{\perp} \cdot \xi_{\perp}} \text{Tr}\langle P\mathcal{F}_+^{+i}(\xi^-, \xi_{\perp})\mathcal{U}^{[+]\dagger}\mathcal{F}_+^{+i}(0)\mathcal{U}^{[+]}|P\rangle.
\]

II. Color Dipole gluon distributions:

\[
xG^{(2)} = 2 \int \frac{d\xi^- d\xi_{\perp}}{(2\pi)^3} e^{ixP^+ \xi^- - ik_{\perp} \cdot \xi_{\perp}} \text{Tr}\langle P\mathcal{F}_+^{+i}(\xi^-, \xi_{\perp})\mathcal{U}^{[-]\dagger}\mathcal{F}_+^{+i}(0)\mathcal{U}^{[+]}|P\rangle.
\]

Questions:

- Can we distinguish these two gluon distributions? Yes, We Can.
- How to measure \( xG^{(1)} \) directly? DIS dijet.
- How to measure \( xG^{(2)} \) directly? Direct γ+Jet in pA collisions. Maybe single-inclusive particle production in pA (Subtle).
- What happens in gluon+jet production in pA collisions? It’s complicated!
**DIS dijet**

The dijet production in DIS.

\[
\begin{align*}
\text{TMD factorization approach:} & &
\frac{d\sigma}{d\mathcal{P}. S.} & = \delta(x - x') G^{(1)}(x, q) \cdot H_{T}^{g \rightarrow q \bar{q}}, \\
\end{align*}
\]

with \( d\mathcal{P}. S. = dy_1 dy_2 d^2 P \cdot d^2 q \).

**CGC approach:**

\[
\begin{align*}
\frac{d\sigma}{d\mathcal{P}. S.} & \propto N_c \alpha_{em} e_q^2 \int \frac{d^2 x}{(2\pi)^2} \frac{d^2 x'}{(2\pi)^2} \frac{d^2 b}{(2\pi)^2} \frac{d^2 b'}{(2\pi)^2} e^{-i k_1 \cdot (x-x')} \\
& \times e^{-i k_2 \cdot (b-b')} \sum \psi^*(x - b) \psi_T(x' - b') [1 \\
& + S^{(4)}(x, b; b', x') - S^{(2)}(x, b) - S^{(2)}(b', x')] ,
\end{align*}
\]

Two independent calculations agree perfectly in the correlation limit (Large \( P_\perp \) and Small \( q_\perp \)).
DIS dijet

The dijet production in DIS.

TMD factorization approach:

\[
\frac{d\sigma_{\gamma^* A \rightarrow q\bar{q} + X}}{dP \cdot S} = \delta(x_{\gamma^*} - 1) x_g G_{(1)}(x_g, q_\perp) H_{\gamma^* g \rightarrow q\bar{q}},
\]

Remarks:

- Dijet in DIS is the **perfect physical** process which can measure **Weizsäcker Williams** gluon distributions.

- **Golden measurement** for the **Weizsäcker Williams** gluon distributions of nuclei at small-x. The cross section is directly related to the WW gluon distribution.

- **EIC** will provide us a **perfect machine** to study the strong gluon fields in nuclei.

- Here comes the **redemption of \(k_t\)** factorization and the salvation of dijet production. **Not disaster! Opportunity!**
Dijet Production and $k_t$ Factorization: What Can We Learn from RHIC to EIC?

Bo-Wen Xiao

Introduction to the $k_t$ factorization and its violation

Quark Distributions

Gluon Distributions

DIS dijet

$\gamma + \text{Jet in } pA$

Gluon+Jet in $pA$

Conclusion

Backup

DIS dijet correlation

Azimuthal angle correlation of dijet in DIS

Remarks:

- $k_{1\perp} = 5.5\text{Gev}, k_{2\perp} = 5.0\text{Gev}$ and $Q_s^2 = 1, 1.5, 3\text{GeV}$;
- Only away side peak is plotted due to the correlation limit.
- Suppression of away side peak at higher energy.
EIC predictions
Azimuthal angle correlation of dijet in DIS

Remarks:

- $\sqrt{s} = 100\text{Gev}$, $z_1 = z_2 \sim 0.3$ and $Q_{sA}^2 = 1.3A^{1/3}Q_{sp}^2$;
- Only away side peak is plotted due to the correlation limit.
- Additional fragmentation contribution.
**γ+Jet in pA collisions**

The direct photon + jet production in pA collisions.

---

**TMD factorization approach:**

\[
\frac{d\sigma^{(pA\rightarrow\gamma q+X)}}{d\mathcal{P}\cdot\mathcal{S}} = \sum_f x_1 q(x_1, \mu^2) x_g G^{(2)}(x_g, q_\perp) H_{qg\rightarrow\gamma q}.
\]

Remarks:

- Independent CGC calculation gives the identical result in the correlation limit.
- Direct measurement of the **Color Dipole** gluon distribution.
- The RHIC and future LHC experiments shall provide us some information on this.
γ+Jet correlation

Azimuthal angle correlation of γ+Jet:

Remarks:

- \( k_1^\perp = 5.5\text{ GeV}, k_2^\perp = 5.0\text{ GeV} \) and \( Q_s^2 = 1, 1.5, 3\text{ GeV} \);
- Only away side peak is plotted due to the correlation limit. We should be able to plot the correlation from 0 to \( 2\pi \). Need more work.
- Suppression of away side peak at higher energy.
- Double peak structure on the away side comes from the fact that \( xG^{(2)} \propto q_\perp^2 \) in the small \( q_\perp \) limit.
Recent RHIC data on di-hadron correlation in $dA$ collisions

- There is no sign of suppression in the $p + p$ and $d + Au$ peripheral data.
- The suppression of the away side jet in $d + Au$ central collisions is due to the multiple interactions between jets and dense nuclear matter (CGC).
- We employ two approaches to calculate this process and find perfect agreement between each others. One is based on color dipole formalism (similar to Marquet’s framework), the other is based on the effective TMD factorization.
Gluon+Jet in $pA$ collisions

Gluon+Jet in $pA$ collisions is the **dominant** channel for dijet production. For quark+gluon channel in the **TMD approach**, we have the following hard cross sections:

![Diagrams](image)

**Hard cross sections and Color factors:**

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\frac{4(\hat{t}^2 - \hat{s}u)^2}{\hat{t}^2 \hat{s}u}$</td>
<td>$\frac{2(\hat{u}^2 + \hat{t}^2)}{\hat{s}u}$</td>
<td>$\frac{2(\hat{t}^2 - \hat{s}u)(\hat{u} - \hat{t})}{\hat{s}tu}$</td>
<td>$\frac{2(\hat{s}^2 + \hat{t}^2)}{\hat{s}u}$</td>
<td>$\frac{2(\hat{t}^2 - \hat{s}u)(\hat{s} - \hat{t})}{\hat{s}tu}$</td>
<td>$\frac{2\hat{t}^2}{\hat{s}u}$</td>
</tr>
<tr>
<td></td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4}$</td>
<td>$\frac{1}{4N_c^2}$</td>
</tr>
</tbody>
</table>
Gluon+Jet in $pA$ collisions

For quark+gluon channel in the TMD approach, different graph corresponds to different gluon distributions:[Bomhof, Mulders and Pijlman; 06]

\[
\Phi_g^{(1)} = \left\langle \text{Tr} \left[ F(\xi) \left\{ \frac{1}{2} \text{Tr} \left[ \frac{\mathcal{U}[\square]}{N_c} \right] \mathcal{U}^{[+]\dagger} + \frac{1}{2} \mathcal{U}^{[-]\dagger} \right] \mathcal{F}(0) \mathcal{U}^{[+]} \right] \rightangle,
\]

\[
\Phi_g^{(2)} = \left\langle \text{Tr} \left[ F(\xi) \left\{ \frac{N_c^2}{N_c^2 - 1} \text{Tr} \left[ \frac{\mathcal{U}[\square]}{N_c} \right] \mathcal{U}^{[+]\dagger} - \frac{1}{N_c^2 - 1} \mathcal{U}^{[-]\dagger} \right] \mathcal{F}(0) \mathcal{U}^{[+]} \right] \rightangle,
\]

\[
\Phi_g^{(3)} = \left\langle \text{Tr} \left[ F(\xi) \frac{\text{Tr} \left[ \frac{\mathcal{U}[\square]}{N_c} \right]}{N_c} \mathcal{U}^{[+]\dagger} \mathcal{F}(0) \mathcal{U}^{[+]\dagger} \right] \rightangle,
\]

\[
\Phi_g^{(4),(5),(6)} = \left\langle \text{Tr} \left[ F(\xi) \mathcal{U}^{[-]\dagger} \mathcal{F}(0) \mathcal{U}^{[+]\dagger} \right] \rightangle.
\]
Gluon+quark jets correlation

At the end of the day, the cross section reads:

\[
\frac{d^3\sigma_{pA\rightarrow qgX}}{d^2p_\perp d^2q_\perp dy_1dy_2} = \sum_f x_p q_f(x_p, \mu^2) \frac{g^4}{32\pi^2} \frac{1}{p_\perp^4} \left[ 1 + (1 - z)^2 \right] (1 - z) \left[ (1 - z)^2 xG^{(2)} + xG^{(3)} \right].
\]

Azimuthal angle correlation of gluon+quark jets:

Remarks:

- \( k_{1\perp} = 5.5\text{Gev}, k_{2\perp} = 5.0\text{Gev} \) and \( Q_s^2 = 1, 1.5, 3\text{GeV} \);
- Suppression of away side peak at higher energy.
Including the \( gg \to gg \) and \( gg \to q\bar{q} \) channels, a lengthy calculation gives

\[
\frac{d\sigma^{(pA\to\text{Dijet}+X)}}{dP.S.} = \sum_q x_1 q(x_1, \mu^2) \frac{\alpha_s^2}{\hat{s}^2} \left[ F^{(1)}_{gg} H^{(1)}_{gg} + F^{(2)}_{gg} H^{(2)}_{gg} \right] \\
+ x_1 g(x_1, \mu^2) \frac{\alpha_s^2}{\hat{s}^2} \left[ F^{(1)}_{gg} \left( H^{(1)}_{gg\to q\bar{q}} + H^{(1)}_{gg\to gg} \right) \right. \\
\left. + F^{(2)}_{gg} \left( H^{(2)}_{gg\to q\bar{q}} + H^{(2)}_{gg\to gg} \right) + F^{(3)}_{gg} H^{(3)}_{gg\to gg} \right],
\]

with the various gluon distributions defined as

\[
F^{(1)}_{gg} = xG^{(2)}(x, q_\perp), \quad F^{(2)}_{gg} = \int xG^{(1)} \otimes F, \\
F^{(1)}_{gg} = \int xG^{(2)} \otimes F, \quad F^{(2)}_{gg} = \int \frac{q_1 \perp \cdot q_2 \perp}{q_1 \perp^2} xG^{(2)} \otimes F, \\
F^{(3)}_{gg} = \int xG^{(1)}(q_1) \otimes F \otimes F,
\]

where \( F = \int \frac{d^2r_\perp}{(2\pi)^2} e^{-iq_\perp \cdot r_\perp} \frac{1}{N_c} \langle \text{Tr} U(r_\perp) U^\dagger(0) \rangle_{x_g} \).
Illustration of gluon distributions

The various gluon distributions:

\[
\mathcal{F}^{(1)}_{qg} = xG^{(2)}(x, q_\perp), \quad \mathcal{F}^{(2)}_{qg} = \int xG^{(1)} \otimes F,
\]

\[
\mathcal{F}^{(1)}_{gg} = \int xG^{(2)} \otimes F, \quad \mathcal{F}^{(2)}_{gg} = \int \frac{q_{1\perp} \cdot q_{2\perp}}{q_{1\perp}^2} xG^{(2)} \otimes F,
\]

\[
\mathcal{F}^{(3)}_{gg} = \int xG^{(1)}(q_1) \otimes F \otimes F,
\]
Compare to STAR data

Azimuthal angle correlation of dijet in PP collisions:

Remarks:
- \( \eta_1 = \eta_2 \sim 3.1 \) and \( Q_s^2 \sim (3 \times 10^{-4})^{0.28}\) GeV²;
- GBW model but without small-\( x \) evolution.
- Only away side peak is plotted due to the correlation limit.
- Additional fragmentation contribution.
Compare to STAR data

Azimuthal angle correlation of dijet in minimum bias $dAu$ collisions:

![Graph showing dijet correlation](image)

Remarks:
- $\eta_1 = \eta_2 \sim 3.2$ and $Q_{sA}^2 = 0.8A^{1/3}Q_{sp}^2$;
- GBW model but without small-$x$ evolution.
- Only away side peak is plotted due to the correlation limit.
- Additional fragmentation contribution.
Compare to STAR data

Azimuthal angle correlation of dijet in central $dAu$ collisions:

Remarks:
- $\eta_1 = \eta_2 \sim 3.1$ and $Q_{sA}^2 = 1.3A^{1/3}Q_{sp}^2$;
- GBW model but without small-$x$ evolution.
- Only away side peak is plotted due to the correlation limit.
- Additional fragmentation contribution.
Conclusion and Remarks

- The effect of the $k_t$ factorization violation is calculable and resummmable. This eventually helps us to reach an effective factorization for the collisions between a dilute projectile and a dense target.

- DIS dijet provides direct information of the WW gluon distributions. Perfect for testing CGC, and ideal measurement for EIC.

- Parton distributions are not universal, but they can be built from universal building blocks.

<table>
<thead>
<tr>
<th></th>
<th>Inclusive</th>
<th>Single Inc</th>
<th>DIS dijet</th>
<th>$\gamma$+jet</th>
<th>g+jet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$xG^{(1)}$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>$xG^{(2)}$, F</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
<td>$\times$</td>
<td>$\checkmark$</td>
<td>$\checkmark$</td>
</tr>
</tbody>
</table>

$\times \Rightarrow$ Do Not Appear. $\checkmark \Rightarrow$ Appear.

- Two fundamental gluon distributions. Other gluon distributions are just different combinations and convolutions of these two.
Zihuatanejo: Life is Good.
The beach didn’t look quite like that in ’The Shawshank Redemption’...

But it is nice as well.

- The effective $k_t$ factorization does not look quite like the expected $k_t$ factorization!
- But it is nice too...
Color Glass Condensate

**Color** (partons are colored objects) **Glass** (Analogy to random spin glass) **Condensate** (High gluon density)

In QCD, the McLerran-Venugopalan Model describes high density gluon distribution in a relativistic large nucleus by solving the classical Yang-Mills equation:

\[ [D_\mu, F_{\mu\nu}] = g J^\nu \]

with the color source

\[ J^\nu = \delta^\nu_\nu + \rho_a(x^-, x_\perp) T^a. \]
Color Glass Condensate

Classical Yang-Mills equation:

\[ [D_\mu, F^{\mu\nu}] = g J^\nu \quad \text{COV gauge} \Rightarrow - \nabla_\perp^2 A^+ = g \rho, \]

The Wilson line

\[ V(x_\perp) = T \exp \left[ -ig^2 \int dz^- d^2z_\perp G(x_\perp - z_\perp) \rho(z^-, z_\perp) \right] \]

In addition, MV model assumes that the density of color charges follows a Gaussian distribution

\[ W[\rho] = \exp \left[ - \int dz^- d^2z_\perp \frac{\rho_a(z^-, z_\perp) \rho_a(z^-, z_\perp)}{2\mu^2(z^-)} \right]. \]

With such a weight, average of two color sources is

\[ \langle \rho_a \rho_b \rangle = \int D[\rho] W[\rho] \rho_a(x^-, x_\perp) \rho_b(y^-, y_\perp) = \mu^2(x^-) \delta_{ab} \delta(x^- - y^-) \delta(x_\perp - y_\perp). \]

MV model is often used as an initial condition at some moderate \( x_0 \). The evolution equation (JIMWLK)

\[ \partial_Y W_Y[\rho] = \mathcal{H} W_Y \quad \text{with} \quad Y = \ln(1/x). \]
The Quark Distribution for a large nucleus in DIS

- TMD pdf in DIS [Brodsky, et al; 02], [Belitsky, Ji, Yuan; 03]

\[ \tilde{q}_{\text{DIS}}(x, q_\perp) = \frac{x P^2 + 2}{8 \pi^4} \int dp^- p^- \int d^2 R \, d^2 R' \, d^2 r_\perp e^{i q_\perp \cdot (R_\perp - R'_\perp)} V(r_\perp) V(r'_\perp) \times \left\{ 1 - e^{i g_1 [G(R_\perp + r_\perp) - G(R_\perp) \}} \right\} \left\{ 1 - e^{-i g_1 [G(R'_\perp + r'_\perp) - G(R'_\perp)]} \right\} . \]

\[ \frac{d x \tilde{q}(x, q_\perp)}{d^2 R_\perp} = \frac{N_c}{16 \pi^6} \int dy d^2 r_\perp d^2 r'_\perp e^{-i q_\perp \cdot (r_\perp - r'_\perp)} \nabla_{r_\perp} K_0 \left( \sqrt{y} r_\perp \right) \cdot \nabla_{r'_\perp} K_0 \left( \sqrt{y} r'_\perp \right) \times \left\{ 1 + \exp \left[ -Q_s^2 (r_\perp - r'_\perp)^2 / 4 \right] - \exp \left[ -Q_s^2 r_\perp^2 / 4 \right] - \exp \left[ -Q_s r'_\perp^2 / 4 \right] \right\} , \]

- Use fermionic quark.

- Perform a replacement as follows:

\[ e^{-i g_1 [G(x_\perp)]} \implies U(x_\perp) = T \exp \left[ -i g_1 \int dz^- d^2 z_\perp G(x_\perp - z_\perp) \rho_a (z^-, z_\perp) t^a \right] . \]

Note that \[ U(x_\perp) \implies e^{-i g_1 [G(x_\perp)]} \]

\[ \rho_a(z^-, z_\perp) t^a = \delta(z_\perp) \delta(z^-) \]

- Average the distribution over the gaussian distribution \[ W[\rho] . \]

- Agrees with [Mueller, 99]. Drell-Yan also agree with [Gelis, Jalilian-Marian, 02]
The Quark Distribution for a large nucleus in di-jet production

▶ TMD pdf in di-jet production [Work in progress]

\[
\tilde{q}_{\text{di-jet}}(x, q_\perp) = \frac{x^{P+2}}{8\pi^4} \int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp \cdot (R_\perp - R'_\perp)}
\]

\[\times V(r_\perp) V(r'_\perp) e^{-i g_2 (G(R_\perp) - G(R'_\perp))}\]

\[\times \left\{ 1 - e^{ig_1 [G(R_\perp + r_\perp) - G(R_\perp)]} \right\} \left\{ 1 - e^{-ig_1 [G(R'_\perp + r'_\perp) - G(R'_\perp)]} \right\}
\]

\[
\downarrow
\]

\[
\frac{dx\tilde{q}(x, q_\perp)}{d^2R_\perp} = \frac{N_c}{16\pi^6} \int dy^2 r_\perp d^2r'_\perp e^{-i q_\perp \cdot (r_\perp - r'_\perp)} \nabla r_\perp K_0(\sqrt{y}r_\perp) \cdot \nabla r'_\perp K_0(\sqrt{y}r'_\perp)
\]

\[\times \left\{ \exp \left[ -Q_s^2 (r_\perp - r'_\perp)^2 / 4 \right] + \exp \left[ -Q_s^2 (r_\perp - r'_\perp)^2 / 2 \right] \right\}
\]

\[\left\{ -\exp \left[ -Q_s^2 \left( (r_\perp - r'_\perp)^2 + r^2_\perp \right) / 4 \right] - \exp \left[ -Q_s^2 \left( (r_\perp - r'_\perp)^2 + r'^2_\perp \right) / 4 \right] \right\}
\]

▶ Large $N_c$ approximation.

▶ Two point functions $\langle U(x_\perp) U^\dagger(x'_\perp) \rangle$ (DIS)

and Four point functions $\langle U(x_\perp) U^\dagger(x'_\perp) U(y_\perp) U^\dagger(y'_\perp) \rangle$. 

---

The Quark Distribution for a large nucleus in di-jet production

- TMD pdf in di-jet production [Work in progress]

\[
\tilde{q}_{\text{di-jet}}(x, q_\perp) = \frac{x^{P+2}}{8\pi^4} \int dp^- p^- \int d^2R_\perp d^2R'_\perp d^2r_\perp e^{iq_\perp \cdot (R_\perp - R'_\perp)}
\]

\[\times V(r_\perp) V(r'_\perp) e^{-i g_2 (G(R_\perp) - G(R'_\perp))}\]

\[\times \left\{ 1 - e^{ig_1 [G(R_\perp + r_\perp) - G(R_\perp)]} \right\} \left\{ 1 - e^{-ig_1 [G(R'_\perp + r'_\perp) - G(R'_\perp)]} \right\}
\]

\[
\downarrow
\]

\[
\frac{dx\tilde{q}(x, q_\perp)}{d^2R_\perp} = \frac{N_c}{16\pi^6} \int dy^2 r_\perp d^2r'_\perp e^{-i q_\perp \cdot (r_\perp - r'_\perp)} \nabla r_\perp K_0(\sqrt{y}r_\perp) \cdot \nabla r'_\perp K_0(\sqrt{y}r'_\perp)
\]

\[\times \left\{ \exp \left[ -Q_s^2 (r_\perp - r'_\perp)^2 / 4 \right] + \exp \left[ -Q_s^2 (r_\perp - r'_\perp)^2 / 2 \right] \right\}
\]

\[\left\{ -\exp \left[ -Q_s^2 \left( (r_\perp - r'_\perp)^2 + r^2_\perp \right) / 4 \right] - \exp \left[ -Q_s^2 \left( (r_\perp - r'_\perp)^2 + r'^2_\perp \right) / 4 \right] \right\}
\]

▶ Large $N_c$ approximation.

▶ Two point functions $\langle U(x_\perp) U^\dagger(x'_\perp) \rangle$ (DIS)

and Four point functions $\langle U(x_\perp) U^\dagger(x'_\perp) U(y_\perp) U^\dagger(y'_\perp) \rangle$. 

---