HIGH-PRECISION $\ell N$ COLLISIONS:
BASICS OF RADIATIVE CORRECTIONS

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EIC Workshop
Week 6, Oct. 21, 2010

H. Spiesberger
The goal:

High-precision measurements of the 'nucleon structure'

- measure form factors, structure functions, (generalized) parton distribution functions, ...

- at low $Q^2$ elastic and quasi-elastic scattering
  - form factors, polarizabilities, ...

- at high $Q^2$ deep inelastic scattering
  - parton distribution functions, GPDs, GDAs, ...

The interesting physics is encoded in FFs, PDFs, ...
test the dynamics of the strong interaction

QCD precision physics — the main topic of this workshop

Lepton scattering: only via electromagnetic and weak interaction

- well-controlled and separable perturbative treatment
Measure FFs, PDFs, etc by comparing data with theoretical predictions:

\[ \sigma_{\text{exp}} = \sigma_{\text{theory}}[F_n(x, Q^2, \ldots)] \]

High precision requires knowledge of higher-order corrections

\[ \sigma_{\text{theory}} = \sigma^{(0)} + \alpha_{\text{em}} \sigma^{(1)} + \ldots \]

- Emission of real photons
  experimentally often not distinguished from non-radiative processes:
  soft photons, collinear photons
  ➔ "radiative corrections"

- Virtual corrections: loop diagrams
  needed to cancel infrared divergences (Bloch-Nordsieck)

- Electroweak effects
  \( Z-, W\)-boson exchange (\( O(G_F) \))
  and higher-order electroweak corrections (\( O(\alpha G_F) \))
Radiative corrections have to be ‘removed’ to uncover the interesting physics and radiative corrections often considered the uninteresting part but:

- radiation from the nucleon: DVCS deeply virtual Compton scattering, $\gamma$-PDF, is part of the 1-photon radiative corrections
- $2\gamma$ exchange (and $\gamma Z$ exchange) is part of the 1-loop photonic corrections: box diagrams corresponding infrared divergences cancel with the interference between photon radiation from the lepton and from the nucleon
- $Z$-exchange gives rise to $P$- and $C$-violating interactions, charge and polarization asymmetries

$\rightarrow$ week 7
Classification of $O(\alpha)$ corrections

- Radiation from the lepton model independent (universal)
- Vacuum polarization (boson self energy) universal
- Radiation from the hadronic initial/final state parton model: radiation from quarks to be considered as a part of the nucleon structure
- Interference of leptonic and hadronic radiation $2\gamma$ exchange new structure
- Purely weak corrections

Note: for NC-scattering straightforward separation
Rule: respect gauge invariance
IR divergences: need to combine real and virtual radiation
Leptonic radiation

Feynman diagrams for leptonic radiation at $O(\alpha)$ (NC) for $q\bar{q}$ scattering:

The radiative leptonic tensor $S_{\mu \nu}(l, l', k)$ is:
- gauge invariant
- infrared finite
- universal

(includes Born + loops: $\delta^{(4)}(k^\mu)$)

Radiative Corrections

H. Spiesberger (Mainz)
Leptonic radiation

Observed cross section: convolution of true cross section $\otimes$ radiator function

$$d\sigma^{\text{obs}}(p, q) = \int \frac{d^3k}{2k^0} R(l, l', k)d\sigma^{\text{true}}(p, q - k)$$

or, for the structure functions:

$$F_n^{\text{obs}}(x, Q^2) = \int d\tilde{x}d\tilde{Q}^2 R_n(x, Q^2; \tilde{x}, \tilde{Q}^2)F_n^{\text{true}}(\tilde{x}, \tilde{Q}^2)$$

Can be extended to include higher-order effects: multi-photon emission, soft-photon exponentiation, $e^+e^-$-pair creation, $R_n$ known analytically to second order, $O(\alpha^2)$

In turn: determination of the true $F_n =$ unfolding, may be ill-defined

Difficult to treat radiative and detector effects separately (acceptance cuts, efficiencies, ...)

H. Spiesberger (Mainz)
**Leptonic radiation**

\[ F_n^{\text{obs}}(x, Q^2) = \int d\tilde{x} d\tilde{Q}^2 R_n(x, Q^2; \tilde{x}, \tilde{Q}^2) F_n^{\text{true}}(\tilde{x}, \tilde{Q}^2) \]

Note: shifted kinematics, e.g.,

\[ Q^2 = -(l - l')^2 \rightarrow \tilde{Q}^2 = -(l - l' - k)^2 \]

- expect strong dependence on experimental prescriptions for measuring kinematic variables
- **leptonic variables**: measure \( E \) and \( \theta \) of scattered lepton \( \rightarrow x \) and \( Q^2 \)
- **hadronic variables**: measure \( E, \theta \) from hadronic final state \( \rightarrow \tilde{x} \) and \( \tilde{Q}^2 \)
- **mixed variables**: combine information from leptonic and hadronic final state

- need full Monte-Carlo modelling

\( \rightarrow \) No numerical results for an EIC in this talk
with partial fractioning, write: \[ R(l, l', k) = \frac{l}{k \cdot l} + \frac{F}{k \cdot l'} + \ldots \]

- initial state radiation, \( k \cdot l \) small for \( \chi(e_{\text{in}}, \gamma) \to 0 \)
- final state radiation, \( k \cdot l' \) small for \( \chi(e_{\text{out}}, \gamma) \to 0 \)

narrow peaks, width \( \simeq \sqrt{m_e/E_e} \): collinear or mass singularities

upon angular integration: large logarithm \( \propto \frac{\alpha}{\pi} \log \frac{Q^2}{m_e^2} \simeq 10\% \)

Note: \( E_{\gamma,\text{max}}^2 \propto Q^2 \frac{1 - x}{x} \)

→ large corrections at large \( Q^2 \) and at small \( x \)
→ Radiation suppressed at small \( Q^2 \) and at large \( x \),
  large negative corrections from uncancelled virtual contributions
e.g., for initial-state radiation:
assume \( k^\mu = (1 - z)l^\mu \)

\[ R_{\text{ISR}} = \frac{\alpha}{2\pi} \frac{1 + z^2}{1 - z} \log \frac{Q^2}{m_e^2} \]

\((+\delta(1 - z) \text{ from loops } \rightarrow +\text{-distribution } 1/(1 - z)_+)\)

\[ d\sigma_{\text{ISR}} = \int \frac{dz}{z} R_{\text{ISR}}(z) d\sigma_{\text{Born}}(l^\mu \rightarrow zl^\mu) \]

(similar for final-state radiation)

Can be extended to include multi-photon emission:

\[ R_{\text{ISR}}^{(2)}(z) = \int_z^1 \frac{dz'}{z'} R_{\text{ISR}}^{(1)}(z') R_{\text{ISR}}^{(1)}(z/z') + \ldots \]

Solution of evolution equations like DGLAP

Known at \( O(\alpha^2) \) (complete) and partially at \( O(\alpha^3) \)
Corrections due to soft photons are universal

sum of real and virtual contributions: \( \delta^{\text{IR}} \) (finite and gauge invariant)

\[
1 + \delta^{\text{tot}} = 1 + \delta^{\text{IR}} + \delta^{\text{fin}} \to \exp(\delta^{\text{IR}})(1 + \delta^{\text{fin}})
\]

\( \delta^{\text{IR}} \) contains \( \log(E^\gamma_{\text{max}}) \) and \( L_e = \log(m_e^2/Q^2) \):

\[
1 + \frac{\alpha}{2\pi}(L_e - 1) \ln \frac{E^\gamma_{\text{max}}}{E_e} + \ldots \to \left( \frac{E^\gamma_{\text{max}}}{E_e} \right)^{\frac{\alpha}{2\pi}(L_e-1)} (1 + \ldots)
\]

(in the \( \gamma^* p \) cms: \( E^\gamma_{\text{max}} = \frac{1}{2} \sqrt{y(1-x)S} \), i.e. important at low \( y \) and large \( x \))

Yennie, Frautschi, Suura, 1961
Radiative tail

Radiation of (hard) photons $\rightarrow$ shifted kinematic variables:

$$Q^2 = -(l - l')^2 \rightarrow \tilde{Q}^2 = -(l - l' - k)^2$$

and

$$x = \frac{Q^2}{2P \cdot (l - l')} \rightarrow \tilde{x} = \frac{\tilde{Q}^2}{2P \cdot (l - l' - k)}$$

Radiator function is folded with

$$d\sigma(\tilde{x}, \tilde{Q}^2) \propto \frac{1}{\tilde{Q}^2}$$

$\rightarrow$ correction factor $d\sigma_{O(\alpha)}(x, Q^2)/d\sigma_{\text{Born}}(x, Q^2)$ enhanced by $Q^2/\tilde{Q}^2$

Note: $\tilde{Q}^2 \ll Q^2$ possible: $\tilde{Q}_{\text{min}}^2 = \frac{x^2}{1-x} M_N^2$

$\rightarrow$ radiative tail, Compton peak

back to photoproduction

$\rightarrow \gamma$-PDF
**AN EXAMPLE FOR HERA KINEMATICS**

\[ \delta_{RC}(A) = \frac{d\sigma_{O(\alpha)}(A)}{d\sigma_{\text{Born}}(A)} - 1 \] for nuclei with \( A = 2Z \) and \( \delta(A) \): corrections to the ratio \( d\sigma_{O(\alpha)}(A)/d\sigma_{O(\alpha)}(D) \)

**Contribution from inelastic tail:**

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**Graphs:**

- Full lines: leptonic variables without cuts (inelastic tail only)
- Dashed lines: with cuts on \( E - p_z \) and \( p_{T,\text{had}} \)

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**HERA workshop 1991**
Contribution from **elastic and quasi-elastic tails** (scattering off the nucleus or off individual nucleons)

![Graph showing the contribution from elastic and quasi-elastic tails.](image)

Full lines: inelastic contribution  
dashed lines: fully inclusive corrections, incl. elastic and quasi-elastic tails

→ **cut on mass of hadronic final state:**  
\[ W_A = \frac{Q^2(1 - x)}{x} + M_A^2 \]
(deep) inelastic $ep$:

factorized into PDFs $\otimes 2\gamma$-box

for $eq$ scattering

Need interference of radiation from the lepton and the hadron to obtain IR-finite result

elastic $ep$:

- assume dominance of a few intermediate states: $p +$ resonances
- assume factorization into GPDs $\otimes$ partonic scattering

Dedicated precision measurements to determine $2\gamma$ contributions: lepton charge asymmetry ($Re$) and lepton polarization asymmetry ($Im$)
Vacuum polarization

Self energy diagrams of the exchanged boson (γ and Z)

\[ \propto \log \frac{Q^2}{m_f^2} \rightarrow O(10\%) \quad \text{small, } O(1\%) \]

Photon self energy = vacuum polarization, absorbed in the running fine structure constant:

\[ \alpha \rightarrow \alpha(Q^2) = \frac{\alpha}{1 - \Pi_{\gamma}(Q^2)} \]

Z-boson self energy: a small correction if written in terms of:

\[ \frac{\alpha}{s_W^2 c_W^2} \rightarrow \frac{M_Z^2 G_\mu \sqrt{2}}{\pi} \frac{1 - \Delta r}{1 - \Pi_Z(Q^2)} \]

(with \( s_W^2, c_W^2 \): sin and cos of the weak mixing angle; \( G_\mu \) the muon decay constant; \( \Delta r \) one-loop corrections to the muon decay: renormalization)
at large $Q^2$: DIS, parton model emission of photons like emission of gluons

infrared divergences (soft photons / gluons) cancel with loops, collinear emission gives rise to corrections $\frac{\alpha}{2\pi} \log m^2_q$, but quark masses are ill-defined

$\Rightarrow$ factorize and absorb collinear divergences into parton distribution functions

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m^2_q)) q_f(x)$$

$$d\sigma = \sum_f d\hat{\sigma}_f (1 + \delta_f(Q^2; m^2_q)) q_f(x) = \sum_f d\hat{\sigma}_f \hat{q}_f(x, Q^2)$$

renormalized parton distribution functions

$$\hat{q}_f(x, Q^2) = (1 + \delta_f(Q^2; m^2_q)) q_f(x)$$

$\Rightarrow$ modified scaling violations

well-known in QCD, $\overline{\text{MS}}$ factorization
different charges of $u$- and $d$-quarks $\rightarrow$ isospin-violating effect

implemented in MRST2004

relevant for precision predictions, e.g. $W$ production at the LHC

HS; Roth, Weinzierl, PLB590
Different point of view: not as part of radiative corrections but with observed photon

→ DVCS: deeply virtual Compton scattering

→ direct photons . . .
Classical analytical approach: Mo, Tsai 
often used in ’private’ implementations of experimental collaborations

Full Monte-Carlo approach:
**HERACLES**: complete electroweak corrections at $O(\alpha)$ (parton model) for 
NC and CC scattering at HERA, including polarization

Full event generation:
**DJANGO**: universal leptonic corrections at $O(\alpha)$, interface to QCD-based 
event generation of jets, parton showers, hadronic final state, includes models for low $Q^2$ behaviour: elastic tail, **SOPHIA** for low-mass hadronic final states

Specialized: **VANDERHAEGHEN ET AL.** QED $O(\alpha)$ corrections to virtual Compton scattering

... and many more tools: **TERAD** by Bardin et al., 
**HEKTOR, KRONOS, FRANEQ, RADGEN**...
More dedicated efforts needed to include:

- IR/soft photon exponentiation and radiator functions at $O(\alpha^2)$ → multi-photon emission

- Radiation from quarks:
  - Subtraction and modified parton showers including $q \rightarrow q + \gamma$
  - (mixed QED+QCD corrections
  - Lepton-hadron interference and 2-photon exchange)
Electroweak effects

$Z$-boson exchange
Charge and polarization asymmetries

\[ A_{\pm} = \frac{\sigma(e_{L}^{\pm}) - \sigma(e_{R}^{\pm})}{\sigma(e_{L}^{\pm}) + \sigma(e_{R}^{\pm})}, \quad B_{\pm} = \frac{\sigma(e_{L}^{\pm}) - \sigma(e_{R}^{\mp})}{\sigma(e_{L}^{\pm}) + \sigma(e_{R}^{\mp})}, \quad C_{L,R} = \frac{\sigma(e_{L,R}^{-}) - \sigma(e_{L,R}^{+})}{\sigma(e_{L,R}^{-}) + \sigma(e_{L,R}^{+})} \]
Z-boson exchange
Polarization asymmetry

\[ A_\pm = \frac{\sigma(e_L^\pm) - \sigma(e_R^\pm)}{\sigma(e_L^\pm) + \sigma(e_R^\pm)}, \]

→ precision measurement of $\sin^2 \theta_W$ → week 7
• High precision needs careful treatment of radiative corrections
• Closely related to experimental conditions
• Photon radiation provides access to interesting physics: direct photons, DVCS, $\gamma$-PDF, $2\gamma$-box
• Impact on possible precision for electroweak effects: $\sin^2 \theta_W$