SU(3) Breaking Effects in Axial-Current Matrix Elements

Martin J. Savage
University of Washington
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Flavor Matrix Elements

\[ \langle P | \bar{q} \gamma_\mu \gamma_5 q | P \rangle = \Delta q \bar{U}_P \gamma_\mu \gamma_5 U_P \]

\( j^{(1)}_{\mu,5} = \bar{q} \gamma_\mu \gamma_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} q \quad j^{(3)}_{\mu,5} = \bar{q} \gamma_\mu \gamma_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} q \quad j^{(8)}_{\mu,5} = \bar{q} \gamma_\mu \gamma_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix} q \]

(mixes with gluons)

3 matrix elements gives u,d,s contributions, \( \Delta u, \Delta d, \Delta s \)
Experimental Inputs: 1

\[ g_A = 1.2664(65) \]

\[ \langle P | j_{\mu,5}^{(3)} | P \rangle = \Delta u - \Delta d \]
Experimental Inputs: 2

Polarized Deep Inelastic Scattering

Analyticity and Completeness

\[ \langle P | \gamma_\mu \gamma_\nu \gamma_\alpha \rightarrow i \epsilon'^{\mu\nu\alpha\beta} \gamma_\beta \gamma_5 + ... \rangle \]

\[ \int_0^1 dx \ g_1^{(P)}(x, Q^2) = \frac{1}{2} \left( 1 - \frac{\alpha_s(Q)}{\pi} + .. \right) \langle P | \bar{q} \ Q_{EM}^2 \gamma_\mu \gamma_5 q | P \rangle \]
Experimental Inputs : 2(b)

Polarized Deep Inelastic Scattering

\[
\bar{q}Q_{\text{EM}}^2 \gamma_\mu \gamma_5 q = \frac{3}{18} j_{\mu,5}^{(3)} + \frac{1}{18} j_{\mu,5}^{(8)} + \frac{2}{9} j_{\mu,5}^{(1)}
\]

0.120(16) : \( Q^2 = 10 \text{ GeV}^2 \) [SMC]

0.133(10) : \( Q^2 = 3 \text{ GeV}^2 \) [E143]

0.118(08) : \( Q^2 = 5 \text{ GeV}^2 \) [E155]

0.120(09) : \( Q^2 = 2.5 \text{ GeV}^2 \) [Hermes]
Experimental Inputs : 3

\[ \langle P | j^{(8)}_{\mu,5} | P \rangle \]

Semi-leptonic Hyperon Decays via approximate SU(3) Flavor Symmetry

\[ B^a_b = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^+}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^+}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix} \]

In limit of EXACT SU(3) symmetry : \( m_s = m_u = m_d \)

\[ \langle B' | \bar{q}O^{(a)} \gamma_\mu \gamma_5 q | B \rangle = D \langle B' | \text{Tr} \left[ \bar{B} \gamma_\mu \gamma_5 \{O^{(a)}, B\} \right] | B \rangle + F \langle B' | \text{Tr} \left[ \bar{B} \gamma_\mu \gamma_5 \left[ O^{(a)}, B \right] \right] | B \rangle \]

Flavor-conserving matrix elements related to flavor-changing matrix elements : 2 independent measurements required
Experimental Inputs : 3(b)

\[ \langle B' | \bar{q} \gamma^{(a)} \gamma_{5} \gamma \bar{B} | B \rangle = D \langle B' | \text{Tr} \left[ \bar{B} \gamma^{a} \gamma_{5} \{ \mathcal{O}^{(a)}, B \} \right] | B \rangle + F \langle B' | \text{Tr} \left[ \bar{B} \gamma^{a} \gamma_{5} \right] \mathcal{O}^{(a)} | B \rangle \]

\[ \langle P | j^{(8)}_{\mu} | P \rangle = 3F - D \]

How to constrain \( F \) and \( D \) from data that exists in the presence of SU(3)-violation - found at 25%-level ??

The common procedure : \( \langle P | j^{(8)}_{\mu} | P \rangle \sim 0.585(25) \)

1. \( g_{A} = D+F \) is well known - use this.
2. Use semi-leptonic hyperon decays to fix independent combination.

Problems :
1. \( g_{A} = D+F + \text{SU(3)}\)-violating (20% ?)
2. \( \chi^{2}/\text{dof} > 3 \) - clearly not adequate theoretical description of data
1. Fit F and D to data including theoretical uncertainty of \( \pm 20\% \) in chi-squared: 

\[
D = 0.79(10), \quad F = 0.47(07)
\]

\[
D + F = 1.26(08), \quad 3F - D = 0.65(21)
\]

Our “best” estimate of matrix elements in the SU(3) limit

Uncertainty = 10 \times (commonly used) !!!!!!!

2. Estimate SU(3)-violation with chiral loops

\[
D = 0.64(06), \quad F = 0.34(04), \quad C = 1.37(05), \quad H = -2.7(5)
\]

Our “best” estimate of couplings to be used at 1-loop in chiral perturbation theory
SU(3)-Violation [Chiral PT]

\[ \langle P | j_{\mu,5}^{(8)} | P \rangle \neq 3F - D \]

\[ \langle P | j_{\mu,5}^{(8)} | P \rangle = 0.65(21) \text{(tree level)} , \ 0.45(20) \text{(1 loop)} \]
\[ \langle P | j_{\mu,5}^{(1)} | P \rangle = 0.10(11) \text{(tree level)} , \ 0.16(10) \text{(1 loop)} \]

\[ \Delta u = 0.77(04) \text{ tree , } \ 0.76(04) \text{ loop} \]
\[ \Delta d = -0.49(04) \text{ tree , } \ -0.51(04) \text{ loop} \]
\[ \Delta s = -0.18(09) \text{ tree , } \ -0.10(09) \text{ loop} \]
SU(3)-Violation [Large Nc]

Constraints from Large-Nc limit of QCD

e.g., the baryon masses (simpler than the axial matrix elements)

\[ M_B = N_C \left[ M_0 + \alpha \left( \frac{J^2}{N_C} \right)^2 + \beta \left( \frac{J^2}{N_C} \right)^4 + \ldots \right] \]

\[ \langle P | j^{(8)}_{\mu, 5} | P \rangle = 0.40(06)(08) \]
Lattice Contributions

\[ \langle B|j_{\mu,5}^{(3)}|B \rangle \]

0.35 < \( g_{\Sigma\Sigma} \) < 0.55 (\( \chi \)PT) and 0.30 < \( g_{\Sigma\Sigma} \) < 0.36 (large \( N_C \))
\[ g_{\Sigma\Sigma} = 0.450(21)(27) \text{ (lattice QCD, 2007)} \]

0.18 < \( -g_{\Xi\Xi} \) < 0.36 (\( \chi \)PT) and 0.26 < \( -g_{\Xi\Xi} \) < 0.30 (large \( N_C \))
\[ g_{\Xi\Xi} = -0.277(15)(19) \text{ (lattice QCD, 2007)} \]

\( m_q \)-dependence differs from expectation! - convergence?

Lattice calculations will reduce uncertainties in matrix elements. Either by direct calculation or by better constraints on large-\( N \) and chiral fits - to be done.
Other Neutral Currents

\[ j_{\mu,5}^{(Z^0)} = \overline{q} \gamma_\mu \gamma_5 \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} q = j_{\mu,5}^{(3)} - j_{\mu,5}^{(s)} \]

Direct determination of third matrix element

Electroweak-DIS may be another way
Conclusions

★ SU(3)-violation is generally under-estimated in axial matrix elements

★ Little improvement in the last 10 years
   ★ New analysis should be done - lattice QCD results

★ Lattice QCD is the only way to reliably improve hyperon decay input

★ A third (neutral-current) matrix element (?)
The END