Current Understanding of TMDs and Future Perspectives

Alexei Prokudin

Jefferson Lab
Thomas Jefferson National Accelerator Facility

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Spin structure of spin-1/2 nucleon is described by 8 TMDs. Each of them depend on two independent variables $x$ and $p_\perp$.

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Kotzinian 1995; Mulders, Tangerman 1995; Boer and Mulders 1997; Bacchetta et al 2007

T-odd TMD FFs survive due to Final State Interactions.

Plot courtesy of B. Musch
Quark-quark Correlator and TMDs

\[ \Phi_{ij}(p, P, S) = \int \frac{d^4 \xi}{(2\pi)^4} e^{ik\cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^{-}) \psi_i(\xi) | P, S \rangle \]

Mulders, Tangerman 95; Goeke, Metz, Schlegel 05, Bacchetta et al 07

Twist-2 decomposition (= leading terms in \( P^+ = x p^+ \) expansion) contains 8 functions:

\[ \Phi[\gamma^+](x, p_T, S) = f_1(x, p_T^2) - \frac{\epsilon_{ij} p_{T i} S_{T j}}{M} f_{1T}^\perp(x, p_T^2) \]

\[ \Phi[\gamma^+ \gamma_5](x, p_T, S) = S_L g_{1L}(x, p_T^2) - \frac{p_{T i} S_{T i}}{M} g_{1T}^\perp(x, p_T^2) \]

\[ \Phi[i\sigma^i \gamma_5](x, p_T, S) = S_T^i h_1 + S_L \frac{p_T^i}{M} h_{1L}^\perp - \frac{p_T^i p_T^j}{M^2} - \frac{1}{2} p_T^2 g_{ij} \]

Alexei Prokudin, INT, Seattle 2010
Fragmentation of a quarks into spin-1/2 hadron is described by 8 TMD FFs. Each of them depend on two independent variables $z$ and $k_\perp$.

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Mulders, Tangerman 1995; Meissner, Metz, Pitonyak 2010
Quark-quark Correlator and TMD FFs

\[ \Delta_{ij}(p, P_h, S_h) = \frac{1}{4z} \sum_X \int \frac{d^4 \xi}{(2\pi)^4} e^{ip \cdot \xi} \langle 0 | \mathcal{W} \psi_j(0) | P_h, S_h; X \rangle \langle P_h, S_h; X | \bar{\psi}_i(\xi) \mathcal{W}^\dagger | 0 \rangle \]

Mulders, Tangerman 95; Meissner, Metz, Pitonyak 2010

Twist-2 decomposition (= leading terms in \( P_h^- = k^- / z \) expansion) contains 8 functions:

\[ \Delta^{[\gamma^\perp]}(z, k_T, S_h) = D_1(z, z^2 k_T^2) + \frac{\epsilon_{ij} k_{Ti} S_{hTj}}{M_h} D_{1T}^\perp(z, z^2 k_T^2) \]

\[ \Delta^{[\gamma^\perp \gamma_5]}(z, k_T, S_h) = S_h L G_{1L}(z, z^2 k_T^2) - \frac{k_{Ti} S_{hTi}}{M} G_{1T}^\perp(z, z^2 k_T^2) \]

\[ \Phi^{[i \sigma^i - \gamma_5]}(z, k_T, S_h) = S_{hT}^i H_1 + S_{hL} \frac{k_T^i}{M_h} H_{1L}^\perp - \frac{k_T^i k_T^j - 1/2 k_T^2 g_T^{ij}}{M_h^2} S_{hTj} H_{1T}^\perp - \frac{\epsilon_{ij} k_T^j}{M_h} H_1^\perp \]
Polarised Semi Inclusive Deep Inelastic Scattering

Asymmetry in $\gamma^* p$ cm frame of $\ell p^\uparrow \rightarrow \ell' h X$

TMD functions can be studied in asymmetries

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{\frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow)}$$

Unpolarised electron beam, Transversely polarised proton. Azimuthal dependence on $\phi_h$ and $\phi_S$ singles out different combinations.

Contributions at leading twist

$$d\sigma^\uparrow - d\sigma^\downarrow \propto f_{1T}^\perp \otimes D_1 \sin(\phi_h - \phi_S) +$$

$\{\text{Sivers effect}\}$

$$+ h_1 \otimes H_1^\perp \sin(\phi_h + \phi_S) +$$

$\{\text{Collins effect}\}$

$$+ \ldots$$

$$\frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow) \propto f_1 \otimes D_1 \equiv F_{UU}$$

Kotzinian 1995; Mulders, Tangerman 1995; Boer and Mulders 1997; Bacchetta et al 2007
Methods to study TMDs: Spin Asymmetries

Structure functions that appear in Spin Asymmetries are convolutions of TMDs, experimental data are usually integrated in all variables apart from one: $x$, $z$ or $P_{h\perp}$.

$$C[w f D] = \int d^2p_T d^2K_T d^2l_T \delta^{(2)}(zp_T + K_T + l_T - P_{h\perp})$$
$$\cdot w(p_T, K_T) f(x, p_T^2) D(z, K_T^2) U(l_T^2)$$

Some model is needed to disentangle $k_T$ dependence of TMDs $f$ and $D$.

*Probably neural networks can help.*

Multidimensional binning in $x$, $z$ and $P_{h\perp}$ at JLab and EIC can help to resolve the problem.

Work is under investigation.
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Methods to study TMDs: Weighted Spin Asymmetries

$P_{h\perp}$ weighted Spin Asymmetries help to resolve the convolution into a simple product of moments of TMDS Mulders 95, Boer 97, Kotzinian 97

$v(P_{h\perp}) \propto P_{h\perp}^n$

$$C[v(P_{h\perp})w f D] = \int_0^\infty d^2P_{h\perp} v(P_{h\perp})C[v(P_{h\perp})w f D]$$

$$= f^{(k)}(x) D^{(m)}(z) U^{(0)} + ...$$

Moments are defined as

$$f^{(k)}(x) = \int d^2p_T \left( \frac{p_T^2}{2M^2} \right)^k f(x, p_T^2)$$

$$D^{(m)}(z) = \int d^2K_T \left( \frac{K_T^2}{2z^2 M^2} \right)^m D(z, K_T^2).$$

However, it implies integration over the whole range in $P_{h\perp}$ and higher orders in $\alpha_s$ involve complicated multiparton correlations that are not present in TMD expansion (Vogelsang, Yuan, 09) that leads to possible contamination of this simple picture.
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Methods to study TMDs: Bessel Weighted Spin Asymmetries

We need a weight that regularize high $P_{h\perp}$. This can be achieved by Bessel function weighting – product of Fourier transform of TMDs at some certain transverse distance $\mathcal{R}$ appear Boer, Gamberg, Musch, AP 2010 in preparation $P_{h\perp}^n \rightarrow J_n(P_{h\perp} \mathcal{R})$ (idea of Berni Musch)

$$\hat{C}[J_n(P_{h\perp} \mathcal{R}) w f D] = \int_0^\infty d^2P_{h\perp} J_n(P_{h\perp} \mathcal{R}) C[w f D]$$

$$= \tilde{f}^{(k)}(x, z^2 \mathcal{R}^2) \tilde{D}^{(m)}(z, \mathcal{R}^2) \tilde{U}^{(0)}(\mathcal{R}^2)$$

Fourier transformed TMDs are defined as

$$\tilde{f}^{a}(x, \mathcal{R}_T^2) = \int d^2p_T e^{ip_T \cdot \mathcal{R}_T} f^{a}(x, \mathcal{R}_T^2)$$

$$\tilde{f}^{a(n)}(x, \mathcal{R}_T^2) = \left(\frac{2}{M^2}\right)^n (\partial \mathcal{R}_T^2)^n \tilde{f}^{a}(x, \mathcal{R}_T^2)$$

Two methods are equivalent in $\mathcal{R} \rightarrow 0$ limit.

$P_{h\perp}$ resolution should be studied in case of EIC and other experiments.
Methods to study TMDs: Bessel Weighted Spin Asymmetries

We need a weight that regularize high \( P_h\perp \). This can be achieved by Bessel function weighting – product of Fourier transform of TMDs at some certain transverse distance \( R \) appear Boer, Gamberg, Musch, AP 2010 in preparation

\[ P_h\perp \to J_n(P_h\perp R) \] (idea of Berni Musch)

\[
\hat{C}[J_n(P_h\perp R) w f D] = \int_0^\infty d^2 P_h\perp J_n(P_h\perp R) C[w f D]
\]

\[ = \tilde{f}^{(k)}(x, z^2 R^2) \tilde{D}^{(m)}(z, R^2) \tilde{U}^{(0)}(R^2) \]

Fourier transformed TMDs are defined as

\[
\tilde{f}^a(x, R_T^2) \equiv \int d^2 p_T e^{i p_T \cdot R_T} f^a(x, R_T^2)
\]

\[
\tilde{f}^a(n)(x, R_T^2) \equiv \left( \frac{2}{M^2} \right)^n (\partial R_T^2)^n \tilde{f}^a(x, R_T^2)
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Flavour decomposition of TMDs

We can achieve flavour decomposition by using different targets $p(uud)$, $n(ddu)$, deuterium and different hadron produced $\pi (u\bar{d}, \bar{u}d)$, $K (u\bar{s}, \bar{u}s)$, $\rho, \eta, p, D, ...$

Note that we need information on fragmentation for those hadrons. Possibly $x$, $z$, $P_{h\perp}$ binning will help to study simultaneously distribution and fragmentation.

Target examples

Proton target

$e_u^2 f_{u/p} D_{u/\pi^+} + e_d^2 f_{d/p} D_{d/\pi^+} + ...$

$u$ quark “dominance”. Up to now is the case in all TMD studies, $u$ quark TMDs are much better constrained.

Deuterium target

$e_u^2 (f_{d/p} + f_{u/p}) D_{u/\pi^+} + e_d^2 (f_{u/p} + f_{d/p}) D_{d/\pi^+}$

d quark is also probed.

Hadron examples

Proton target $\pi^-$ production

$e_u^2 f_{u/p} D_{u/\pi^-} + e_d^2 f_{d/p} D_{d/\pi^-} + e_u^2 f_{\bar{u}/p} D_{\bar{u}/\pi^-} + e_d^2 f_{\bar{d}/p} D_{\bar{d}/\pi^-}$. $D_{d/\pi^-}$ and $D_{\bar{u}/\pi^-}$ are favoured fragmentation functions.

Proton target $K^-$ production

$e_u^2 f_{u/p} D_{u/K^-} + e_d^2 f_{d/p} D_{d/K^-} + e_u^2 f_{\bar{u}/p} D_{\bar{u}/K^-} + e_d^2 f_{\bar{d}/p} D_{\bar{d}/K^-} + ...$ $D_{\bar{u}/K^-}$ is favoured fragmentation function.
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**Target examples**

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$$e_u^2 (f_{d/p} + f_{u/p}) D_{u/\pi^+} + e_d^2 (f_{u/p} + f_{d/p}) D_{d/\pi^+}$$

$d$ quark is also probed.

**Hadron examples**

Proton target $\pi^-$ production

$$e_u^2 f_{u/p} D_{u/\pi^-} + e_d^2 f_{d/p} D_{d/\pi^-} + e_u^2 f_{\bar{u}/p} D_{\bar{u}/\pi^-} + e_d^2 f_{\bar{d}/p} D_{\bar{d}/\pi^-}.$$  

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**Target examples**

**Proton target**

\[
e^{2}_u f_{u/p} D_{u/\pi} + e^{2}_d f_{d/p} D_{d/\pi} + ... \\
\]

\( u \) quark “dominance”. Up to now is the case in all TMD studies, \( u \) quark TMDs are much better constrained.

**Deuterium target**

\[
e^{2}_u (f_{d/p} + f_{u/p}) D_{u/\pi} + e^{2}_d (f_{u/p} + f_{d/p}) D_{d/\pi} + \\
\]

\( d \) quark is also probed.

**Hadron examples**

**Proton target \( \pi^- \) production**

\[
e^{2}_u f_{u/p} D_{u/\pi} + e^{2}_d f_{d/p} D_{d/\pi} + e^{2}_u f_{\bar{u}/p} D_{\bar{u}/\pi} + e^{2}_d f_{\bar{d}/p} D_{\bar{d}/\pi}. \\
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**Hadron examples**

**Proton target** \( \pi^- \) production

\( e_u^2 f_u/p D_{u/\pi^-} + e_d^2 f_d/p D_{d/\pi^-} + e_u^2 f_{\bar{u}}/p D_{\bar{u}/\pi^-} + e_d^2 f_{\bar{d}}/p D_{\bar{d}/\pi^-} \).

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$d$ quark is also probed.

**Hadron examples**

*Proton target $\pi^-$ production*$ \quad e_u^2 f_{u/p} D_u/\pi - + e_d^2 f_{d/p} D_d/\pi - + e_u^2 f_{\bar{u}/p} D_{\bar{u}}/\pi - + e_d^2 f_{\bar{d}/p} D_{\bar{d}}/\pi -$. $D_d/\pi-$ and $D_{\bar{u}}/\pi-$ are favoured fragmentation functions.

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Alexei Prokudin, INT, Seattle 2010
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**Hadron examples**

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$$e_u^2(f_{d/p} + f_{u/p})D_{u/\pi^+} + e_d^2(f_{u/p} + f_{d/p})D_{d/\pi^+}$$

d quark is also probed.

**Hadron examples**

**Proton target** $\pi^-$ production

$$e_u^2f_{u/p}D_{u/\pi^-} + e_d^2f_{d/p}D_{d/\pi^-} + e_u^2f_{\bar{u}/p}D_{\bar{u}/\pi^-} + e_d^2f_{\bar{d}/p}D_{\bar{d}/\pi^-}.$$  

$D_{d/\pi^-}$ and $D_{\bar{u}/\pi^-}$ are favoured fragmentation functions.

**Proton target** $K^-$ production

$$e_u^2f_{u/p}D_{u/K^-} + e_d^2f_{d/p}D_{d/K^-} + e_u^2f_{\bar{u}/p}D_{\bar{u}/K^-} + e_d^2f_{\bar{d}/p}D_{\bar{d}/K^-} + ....$$

$D_{\bar{u}/K^-}$ is favoured fragmentation function.
Typical ingredients used in TMD phenomenology

We start from TMD factorization Ji, Ma, Yuan 05

\[ F_{UU} = \int d^2p_T d^2K_T d^2l_T \delta^{(2)}(z p_T + K_T + l_T - P_{h\perp}) f_1(x, p_T^2) D_1(z, K_T^2) U(l_T) \]

Soft factor \( U(l_T) \) is not included

Gaussian parametrization of TMDs

\[ f_1(x, p_T) = f_1(x) \frac{1}{\pi \langle p_T^2 \rangle} e^{-p_T^2/\langle p_T^2 \rangle}, \quad \int d^2p_T f_1(x, p_T) \equiv f_1(x) \]

\[ D_1(z, K_T) = f_1(x) \frac{1}{\pi \langle K_T^2 \rangle} e^{-K_T^2/\langle K_T^2 \rangle}, \quad \int d^2K_T D_1(x, K_T) \equiv D_1(x) \]

Justified by lattice QCD studies Musch et al, 2008 and phenomenology Metz, Teckentrup, Schweitzer, 2010

TMDs are parametrized in such a way that positivity constraints Bacchetta, Boglione, Henneman, Mulders, 2000 are taken into account. Usually \( \propto x^\alpha (1 - x)^\beta \).

Evolution in \( Q^2 \) is taken for moments of TMDs and usually is supposed to be the same as for collinear distributions. Kang, Qiu, Zhou, Yuan evolution to be implemented.
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Soft factor \( U(l_T) \) is not included

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\[ f_1(x, p_T) = f_1(x) \frac{1}{\pi \langle p_{\perp}^2 \rangle} e^{-p_T^2 / \langle p_{\perp}^2 \rangle}, \quad \int d^2 p_T f_1(x, p_T) = f_1(x) \]

\[ D_1(z, K_T) = f_1(x) \frac{1}{\pi \langle K_{\perp}^2 \rangle} e^{-K_T^2 / \langle K_{\perp}^2 \rangle}, \quad \int d^2 K_T D_1(x, K_T) = D_1(x) \]

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Gaussian parametrization of TMDs

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Soft factor \( U(l_T) \) is not included

Gaussian parametrization of TMDs

\[ f_1(x, p_T) = f_1(x) \frac{1}{\pi \langle p^2 \rangle} e^{-p_T^2 / \langle p^2 \rangle}, \quad \int d^2 p_T f_1(x, p_T) \equiv f_1(x) \]

\[ D_1(z, K_T) = f_1(x) \frac{1}{\pi \langle K^2 \rangle} e^{-K_T^2 / \langle K^2 \rangle}, \quad \int d^2 K_T D_1(x, K_T) \equiv D_1(x) \]

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Error estimates of TMDs

Part of $x$ coverage of an EIC will be away from the region covered by existing experiments $0.006 < x < 0.3$. Extrapolation of known TMDs are needed for predictions

- Error estimates are usually done with $\Delta \chi^2 \neq 1$, typically $\Delta \chi^2 \approx 3\% \chi_{total}$. This accounts partly to the theoretical error of parametrization $(x^\alpha (1 - x)^\beta)$

- Going directly to positivity bounds is not a good idea. It is **unreasonable** to assume that TMD will suddenly saturate bounds at low $x$.

- *In order to estimate improvement of TMD extraction pseudodata from EIC generators will be fitted.* Work in progress
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\[ f_1 \]
TMD ($\sigma_0$) and collinear QCD ($\sigma_1$) cross sections are included:

$$\sigma = \sigma_0 + K \alpha_s \sigma_1$$

$$\sigma_0 \propto f_1(x, P_T^2) \otimes D_1(z, K_T^2) = f_1(x) D_1(z) \frac{1}{\pi \langle P_{h\perp}^2 \rangle} e^{-P_{h\perp}^2/\langle P_{h\perp}^2 \rangle},$$

where $\langle P_{h\perp}^2 \rangle = z^2 \langle p_{\perp}^2 \rangle + \langle K_{\perp}^2 \rangle$
From low to high $P_T$ Anselmino et al 2007

\[ \frac{1}{\sigma_{\text{DIS}}} \frac{d\sigma}{dP_T} (\text{GeV/c})^{-1} \]

\begin{align*}
0.1 < z < 0.2 & \quad W^2 < 90 \text{ (GeV}^2) \\
0.2 < z < 0.4 & \quad W^2 > 350 \text{ (GeV}^2) \\
0.4 < z < 1 & \quad \text{not shown}
\end{align*}

\[ 10^{-3} \quad 10^{-2} \quad 10^{-1} \quad 10 \quad 100 \]
\[ P_T (\text{GeV/c}) \]

\[ 0 \quad 1 \quad 2 \quad 3 \quad 4 \]

TMD ($\sigma_0$) and collinear QCD ($\sigma_1$) cross sections are included:

\[ \sigma = \sigma_0 + K \alpha_s \sigma_1 \]

The data are from EMC $Q^2 > 5 \text{ GeV}^2$ and ZEUS $Q^2 > 10 \text{ GeV}^2$.


The result of extraction Anselmino et al 2007 $\langle p_{\perp}^2 \rangle = 0.25 \text{ GeV}^2$ $\langle K_{\perp}^2 \rangle = 0.2 \text{ GeV}^2$
TMD ($\sigma_0$) and collinear QCD ($\sigma_1$) cross sections are included:

$$\sigma = \sigma_0 + K \alpha_s \sigma_1$$

The cross sections $\sigma_0$ and $\sigma_1$ are summed out.
No resummation is done, $\sigma_1$ is cut at some value of $P_T \sim 1$ GeV.
The result of extraction Anselmino et al 2007 $\langle p^2_{\perp} \rangle = 0.25$ GeV$^2$ $\langle K^2_{\perp} \rangle = 0.2$ GeV$^2$
TMD ($\sigma_0$) and collinear QCD ($\sigma_1$) cross sections are included:

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The cross sections $\sigma_0$ and $\sigma_1$ are summed out.

No resummation is done, $\sigma_1$ is cut at some value of $P_T \sim 1$ GeV.

The result of extraction Anselmino et al 2007 $\langle p_{\perp}^2 \rangle = 0.25$ GeV$^2$ $\langle K_{\perp}^2 \rangle = 0.2$ GeV$^2$
FUTURE PROSPECTIVES

- Gluon TMD $g(x, k_T)$
- From large to low $x$, from quarks to sea quarks and gluons
- Scale dependence and energy studies $f_1(x, k_T)$ and its width $\langle p_{\perp}^2 \rangle (Q^2, s)$
- Flavour dependence studies of width $\langle p_{\perp}^2 \rangle_{u,d,\bar{u},\bar{d}}$
  Gluon and quark distributions may have different widths (phenomenology and models).
- Gaussian vs non gaussian shape studies
- $P_{h,\perp}$ range of EIC will allow us to study interplay of TMD and collinear factorizations
- Etc
SIVERS FUNCTION

$\text{fit}$
Sivers function

Sivers function Sivers 1990

\[ f_{q/P\uparrow}(x, p_\perp, S) = f_1(x, p_\perp^2) - \frac{S \cdot (\hat{P} \times p_\perp)}{M} f_{1T}(x, p_\perp^2) \]

Sivers function for quarks and gluons is a candidate for **GOLDEN EXPERIMENT** at **EIC**

- This function gives access to 3D imaging
- It is spin-orbit correlation
- Physics of gauge links is represented
- Access to Orbital Angular Momentum
Physics of gauge links: process dependence

Sivers function [Sivers 1990] can be measured in both SIDIS and DY processes.

\[ f_{q/P\uparrow}(x, k_\perp, S) = f_1(x, k^2) - \frac{S \cdot (\hat{P} \times k_\perp)}{M} f_{1T}(x, k_\perp) \]

Drell Yan \( A^\uparrow B \rightarrow l^+ l^- X \)

\[ A^{\sin(\phi_\gamma - \phi_S)}_{UT} \sim f_{1T}^{DY}(x, k_\perp) \otimes f_{\bar{q}/B}(x, p_\perp) \]

SIDIS \( \ell P^\uparrow \rightarrow \ell' h X \)

\[ A^{\sin(\phi_H - \phi_S)}_{UT} \sim f_{1T}^{SIDIS}(x, k_\perp) \otimes D_{h/q}(z, p_\perp) \]
Modified universality

Sivers function is process dependent. Collins 2002

\[ f_{1T}^{DY} = -f_{1T}^{SIDIS} \]

Let’s consider a simple model of Final State Interactions as in Brodsky, Hwang, Schmidt 2002,
proton = quark$^+$ + antiquark$^-$

Experimental test of this relation is fundamental for our understanding of the origin of the correlation between parton angular momentum and the spin of the proton and the gauge link formalism itself.

Experimental DY data are not available, experiments are planned.
EXPERIMENTAL DATA.

**HERMES**

\( ep \to e\pi X, \ p_{lab} = 27.57 \ \text{GeV}. \)

**COMPASS**

\( \mu D \to \mu\pi X, \ p_{lab} = 160 \ \text{GeV}. \)

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**Anselmino et al 2010 in preparation**

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\( l p^\uparrow \to l\pi^+ X \simeq -f_{1T}^{\perp u} \otimes D_{u/\pi^+} > 0 \) thus \( f_{1T}^{\perp u} < 0 \)

\( lD^\uparrow \to l\pi^+ X \simeq (f_{1T}^{\perp u} + f_{1T}^{\perp d}) \otimes D_{u/\pi^+} \simeq 0 \) thus \( f_{1T}^{\perp u} \sim -f_{1T}^{\perp d} \) in accordance with large \( N_C \) predictions Pobylitsa hep-ph/0301236
**EXPERIMENTAL DATA.**

**HERMES**

\[ ep \rightarrow e\pi X, \quad p_{lab} = 27.57 \text{ GeV}. \]

Anselmino et al 2010 in preparation

Arnold et al 2008

\[ lp^{\uparrow} \rightarrow l\pi^{+} X \simeq -f_{1T}^{u} \otimes D_{u/\pi^{+}} > 0 \text{ thus } f_{1T}^{u} < 0 \]

\[ lD^{\uparrow} \rightarrow l\pi^{+} X \simeq (f_{1T}^{u} + f_{1T}^{d}) \otimes D_{u/\pi^{+}} \simeq 0 \text{ thus } f_{1T}^{u} \sim -f_{1T}^{d} \text{ in accordance with large } N_{C} \text{ predictions Pobylitsa hep-ph/0301236} \]
Sivers functions

\[ \Delta^N f_q^{(1)}(x) \equiv \int d^2 k_\perp \frac{k_\perp}{4m_p} \Delta^N f_q/p^\uparrow(x, k_\perp) = -f_{1T}^{(1)q}(x). \]

Anselmino et al 2010 in preparation. Extraction from the newest data.

- Sivers functions for \( u, d \) and \( s \) sea quarks are extracted from HERMES and COMPASS data.

- \( \Delta^N f_u > 0, \Delta^N f_d < 0 \), hints on nonzero sea quark Sivers functions.

- Sea quark functions may grow at low \( x \).
  \[ f_{1T}^{\perp \text{sea}} \propto x^0 f_1^{\text{sea}} \]
How do we “see” sea quarks in the data?

\( \pi^+ (ud) \quad \pi^- (\bar{u}d) \)

HERMES no sea contributions

\( \Delta^N f_{\bar{u}/p}^\uparrow = 0 \)
How do we “see” sea quarks in the data?

\[ \pi^+(u\bar{d}) \quad \pi^-(\bar{u}d) \]

HERMES with sea contributions

HERMES with sea contributions

\[ \Delta^N f_{\bar{u}/p} > 0 \]
How do we “see” sea quarks in the data?

$K^+(u\bar{s})$

$K^-(\bar{u}s)$

HERMES no sea contributions

$\Delta^N f_{\bar{u}/p^\uparrow} = 0$
How do we “see” sea quarks in the data?

$K^+(u\bar{s})$  

$K^-(\bar{u}s)$

HERMES with sea contributions

HERMES with sea contributions

$\Delta^N f_{\bar{u}/p^\uparrow} > 0$
How do we “see” sea quarks in the data?

\[ \pi^+ (u\bar{d}) \quad \text{COMPASS no sea contributions} \]

\[ \pi^- (\bar{u}d) \quad \text{COMPASS no sea contributions} \]

\[ \Delta^N f_{\bar{u}/p}^+ + \Delta^N f_{d/p}^+ = 0 \]
How do we “see” sea quarks in the data?

\[ \pi^+ (u \bar{d}) \]

\[ \pi^- (\bar{u}d) \]

COMPASS with sea contributions

\[ \Delta_N f_{\bar{u}/p} + \Delta_N f_{\bar{d}/p} < 0 \]

\[ \Rightarrow \Delta_N f_{\bar{d}/p} < 0 \]
How do we “see” sea quarks in the data?

$K^+(u\bar{s})$  

$K^-(\bar{u}s)$

COMPASS no sea contributions

$\Delta^N f_{\bar{u}}/p^{\uparrow} + \Delta^N f_{\bar{d}}/p^{\uparrow} = 0$
How do we “see” sea quarks in the data?

$K^+(u\bar{s})$

$K^-(\bar{u}s)$

COMPASS with sea contributions

\[
\Delta_N f_{\bar{u}/p^\uparrow} + \Delta_N f_{\bar{d}/p^\uparrow} < 0 \\
\Rightarrow \Delta_N f_{\bar{d}/p^\uparrow} < 0
\]
Sivers function comparison with models

There is a number of model calculations of Sivers function:

Pasquini and Yuan 2010

Pasquini and Yuan arXiv:1001.5398

Alessandro Bacchetta et al 2010

Bacchetta et al arXiv:1003.1328

• Scale dependence of asymmetries $A_{UT}(Q^2)$
• Gluon and sea quark Sivers functions $f_{1T}^{\perp \bar{u}, \bar{d}, s, \bar{s}}$. $D$ meson production will help.
• Low to high $P_{h\perp}$ and Sivers function versus Qiu-Sterman matrix elements $f_{1T}^{\perp(1)} \propto T_F(x, x)$.
• Interplay of TMD and collinear factorization
• $Q^2$ evolution of Sivers function
• Different hadron production $\pi^\pm$, $K^\pm$, $D$ etc
• Weighted asymmetries, etc
Transversity cannot be studied in DIS as QED and QCD interactions conserve helicity up to corrections $O(m_q/E)$.

Transversity can be measured if coupled with another chiral-odd function. This can be done in Semi Inclusive DIS (SIDIS), quark fragments into unpolarised hadron. It couples to so called Collins Fragmentation function that describes how a polarised quark fragments into unpolarised hadron.

Golden channel to study transversity is proton - antiproton double spin asymmetry at GSI $A_N \propto h_{q/P}(x)h_{\bar{q}/\bar{P}}(x)$. 

Alexei Prokudin, INT, Seattle 2010
How to measure transversity? SIDIS and $e^+e^-$ annihilation

$lN \rightarrow l'H_1X$

$e^+e^- \rightarrow H_1H_2X$


$$A_{UT}^{\sin(\phi_H+\phi_S)} \propto \sum_q h^q_1(x, p_T) \otimes H_1^\perp q(z, K_T)$$


$$A^{\sin(\phi_1+\phi_2)} \propto \sum_{q, \bar{q}} H_1^\perp q(z_1, K_{T1}) \otimes H_1^\perp \bar{q}(z_2, K_{T2})$$
Description of BELLE data $e^+e^-$

$\cos(2\varphi_0)$ hadron method

$\cos(\varphi_1 + \varphi_2)$ thrust axis method

$e^+e^- \rightarrow \pi\pi X$, $s \equiv Q^2 = 100$ GeV$^2$


The extraction is done for Favoured and Unfavored $H_1^\perp q$.

Note that 16 points from $e^+e^-$ are apparently not enough:

Bins in $z$ are too wide to allow detailed extraction beyond $H_1^\perp q_{\text{fav/unfav}}$ model

No information on $Q_T$ dependence thus no hope to estimate Sudakov suppression for the data

BABAR data is wanted!
Collins fragmentation function

compared (left) to Ref. [1] (dashed line), Ref. [2] (dotted line)


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Collins fragmentation function

\[ \Delta^N D_{fav} = \Delta^N D_{u/\pi^+} > 0 \]
\[ \Delta^N D_{unf_{fav}} = \Delta^N D_{u/\pi^-} < 0 \]
\[ |\Delta^N D_{fav}| \approx |\Delta^N D_{unf_{fav}}| \]
Experimental data

**HERMES** \( A_{UT}^{\sin(\phi_h + \phi_S)} \)

**COMPASS** \( A_{UT}^{\sin(\phi_h + \phi_S + \pi)} \)

\[ \text{ep}\rightarrow e\pi X, \ p_{lab} = 27.57 \ \text{GeV.} \]

\[ \mu D\rightarrow \mu\pi X, \ p_{lab} = 160 \ \text{GeV} \]


Description of the data

Predictions for COMPASS operating on PROTON target

COMPASS $A_{UT}^{\sin(\phi_h+\phi_S+\pi)}$

Comparison with preliminary COMPASS data arXiv:0808.0086

Anselmino et al 2009
Transversity vs. helicity

1. Solid red line – transversity distribution
   \[ \Delta_T q(x) \]
   this analysis at \( Q^2 = 2.4 \text{ GeV}^2 \).

2. Solid blue line – Soffer bound
   \[ |\Delta_T q(x)| < \frac{q(x) + \Delta q(x)}{2} \]

3. Dashed line – helicity distribution
   \[ \Delta q(x) \]
   \( \text{GRSV98LO} \)}
Transversity

- This is the extraction of \textit{transversity} from existing experimental data. Anselmino et al. 2009
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- $|\Delta_T q(x)| < |\Delta q(x)|$
- EIC will allow study of $Q^2$ dependence and together with JLab @ 12 GeV provides wider region of $x$ for tensor charge extraction.
Transversity, comparison with models

The extraction is close to most models. Possible caveats: $Q^2$ value of SIDIS data $\sim 2 \text{ GeV}^2$, BELLE data $\sim 100 \text{ GeV}^2$. Sudakov suppression should be estimated. Cross check with IFF extraction? Evolution of Collins FF is supposed to be similar to transversity talk by Zhongbo Kang

0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. Bacchetta, Conti, Radici, (08)
Tensor charges

\[ \delta_T q = \int_0^1 dx (h_{1q} - h_{1\bar{q}}) = \int_0^1 dx h_{1q} \]
\[ \delta_T u = 0.54^{+0.09}_{-0.22}, \delta_T d = -0.23^{+0.09}_{-0.16} \text{ at } Q^2 = 0.8 \text{ GeV}^2 \]

1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB 659, 214 (2008), \( Q^2 = 0.4 \text{ GeV}^2 \)

2. CQSM:
   \( Q^2 = 0.3 \text{ GeV}^2 \)

3. Lattice QCD:
   M. Gockeler et al.,
   \( Q^2 = 4 \text{ GeV}^2 \)

4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, \( Q^2 \sim 1 \text{ GeV}^2 \)

5. Constituent quark model:
   B. Pasquini, M. Pincetti, and S. Boffi,
   PRD72(2005)094029 and PRD76(2007)034020,
   \( Q^2 \sim 0.8 \text{ GeV}^2 \)

6. Spin-flavour SU(6) symmetry
   L. Gamberg, G. Goldstein,
   Phys.Rev.Lett.87:242001,2001 \( Q^2 \sim 1 \text{ GeV}^2 \)
Transversity is one of the three fundamental collinear PDFs.
Scale dependence is known up to NLO, study of $A_{UT}(Q^2)$ is important.
Tensor charge is not measured with a good precision.
Sea quark transversity $h_{1\bar{u}, \bar{d}, s, \bar{s}}$.
Weighted asymmetries.
Extraction of transversity with dihadron IFF.
talk by Marco Radici.
BOER-MULDERS FUNCTION
Boer-Mulders effect

Boer-Mulders function

\[
f_{q^+/p}(x, k_\perp) = \frac{1}{2} \left[ f_{q/p}(x, k_\perp) - h_1^{\perp q}(x, k_\perp) \frac{s \cdot (\hat{P} \times k_\perp)}{M} \right]
\]

Sivers function

\[
f_{q/p}^+ (x, k_\perp) = f_{q/p}(x, k_\perp) - f_{1T}^{\perp q}(x, k_\perp) \frac{S_T \cdot (\hat{P} \times k_\perp)}{M}
\]

Both functions measure correlation of \((\hat{P} \times k_\perp)\) and \(S_T\) or s.

Matthias Burkardt (Burkardt 2005) conjecture: \(h_1^{\perp q}(x, k_\perp) \sim f_{1T}^{\perp q}(x, k_\perp)\), \(h_1^{\perp u,d}(x, k_\perp) < 0\).

Expected values: \(h_1^{\perp u}/f_{1T}^{\perp u} \sim 1.8, \ h_1^{\perp d}/f_{1T}^{\perp d} \sim -1\)

Data are available from HERMES and COMPASS. The best fit is (Barone, AP, Melis 2010): \(h_1^{\perp u}/f_{1T}^{\perp u} = 2.1 \pm 0.1, \ h_1^{\perp d}/f_{1T}^{\perp d} = -1.1 \pm 0.001\) A good accordance with expectations.
Boer-Mulders data

**COMPASS** $A^{\cos 2\phi_h}_{UU}$

**HERMES** $A^{\cos 2\phi_h}_{UU}$

- **COMPASS SIDIS UNPOLARISED**
  - $X - \pi \to l_D = 17.3276$ (GeV) s
  - $0.1 < y < 0.90 < x < 10.2 < z < 0.85$

- **HERMES UNPOLARISED**
  - $X + \pi \to l_P = 7.25374$ (GeV) s
  - $0.3 < y < 0.850.023 < x < 10.2 < z < 1$

Barone, Melis, AP PRD81, 2010

$$F^{\cos 2\phi_S}_{UU} = h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$$

Twist-2 contribution (the dashed line) is comparable to higher twist (the dotted line) contribution at low $Q^2$. EIC at high $Q^2$ allows to measure $h_1^\perp \otimes H_1^\perp$ without higher twists.
Barone, Melis, AP PRD81, 2010

\[ F_{UU}^{\cos 2\phi_S} = h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1 \]

Twist-2 contribution (the dashed line) is comparable to higher twist (the dotted line) contribution at low \( Q^2 \). EIC at high \( Q^2 \) allows to measure \( h_1^\perp \otimes H_1^\perp \) without higher twists.
Boer-Mulders data

COMPASS $A_{UU}^{\cos^2 \phi_h}$

Boer-Mulders functions

Barone, Melis, AP PRD81, 2010

$$F_{UU}^{\cos 2\phi_S} = h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1$$

Twist-2 contribution (the dashed line) is comparable to higher twist (the dotted line) contribution at low $Q^2$. EIC at high $Q^2$ allows to measure $h_1^\perp \otimes H_1^\perp$ without higher twists.
Boer-Mulders functions

\[ F_{UU}^{\cos 2\phi_S} = h_1^\perp \otimes H_1^\perp + \frac{1}{Q^2} f_1 \otimes D_1 \]

Twist-2 contribution (the dashed line) is comparable to higher twist (the dotted line) contribution at low \( Q^2 \). EIC at high \( Q^2 \) allows to measure \( h_1^\perp \otimes H_1^\perp \) without higher twists.

Barone, Melis, AP PRD81, 2010

Alexei Prokudin, INT, Seattle 2010
Boer-Mulders function comparison with models

There is a number of model calculations of Boer-Mulders function:
- Light-cone quark model by Barbara Pasquini and Feng Yuan 2010.
- Diquark model by Alessandro Bacchetta et al 2010, Leonard Gamberg, Gary Goldstein, and Marc Schlegel 2008 etc.

Pasquini and Yuan 2010

Alessandro Bacchetta et al 2010

Pasquini and Yuan arXiv:1001.5398

Bacchetta et al arXiv:1003.1328

Reasonable agreement of the extracted Boer-Mulders functions by Enzo Barone, Stefano Melis, AP et al 2010 and Lu and Schmidt et al 2009 and model calculations.
- Scale dependence of asymmetries $A_{UU}^{\cos(2\phi_h)}(Q^2)$ will allow us to distinguish twist-2 from twist-3 contribution
- Sea quark Boer-Mulders functions $h_{1}^{\perp \bar{u}, \bar{d}, s, \bar{s}}$
- Low to high $P_T$ and Boer-Mulders function versus Qiu-Sterman matrix elements $h_{1}^{\perp (1)} \propto T^\sigma_F(x, x)$
- Different hadron production $\pi^\pm, K^\pm$ etc
OTHER TMD DISTRIBUTIONS

• Non trivial width of $g_1$ with respect to $f_1$

• Wandzura-Wilcek relations for $g_{1T}^\perp$ and $h_{1L}^\perp$. Some models predict $g_{1T}^{\perp(1)} = -h_{1L}^{\perp(1)}$. Numerically feasible in WW approximation.

• $h_{1T}^\perp$ is quadrupole deformation in $k_T$ thus more involved structure. Some models predict $h_{1T}^\perp = h_1 - g_1$, connection to $L^q$.

• P. S. Once we measure azimuthal modulations we can measure all TMDs. Each of them represent different physics. NO STAMP COLLECTION.

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THE GOAL: THREE DIMENTIONAL PICTURE OF THE PROTON

The proton moves along $-Z$ direction (into the screen) and $S_T$ is along $Y$.

This is the three dimensional view of the proton as “seen” by the virtual photon.

Red color – more quarks. Blue color – less quarks. Distributions of quarks are not symmetrical and shifted due to final state interactions.

$x = 0.2$
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The proton moves along $-Z$ direction (into the screen) and $S_T$ is along $Y$.

Sivers functions for $u$, $d$ and sea quarks are extracted from HERMES and COMPASS data. Red color – more quarks. Blue Color – less quarks. Sivers functions is a left – right asymmetry of quark distribution.

$x = 0.01$

More information on sea quarks. Future Electron Ion Collider and JLab will contribute.
CONCLUSIONS

- EIC will be a powerful tool to study parton dynamics and TMDs.
  - High $Q^2$ range will allow to study twist-2 functions and higher twist content of the nucleon.
  - Range of $P_T$ will allow to study intermediate region where both TMD and collinear factorizations are applicable.
  - $Q^2$ range at some fixed $x$ will provide information on $Q^2$ behavior of the asymmetries and $Q^2$ evolution of TMDs.
  - Range of $P_T$ will allow for measurements of weighted asymmetries at EIC, so that moments of TMDs could be extracted from the data.
  - Full flavor and spin decomposition of TMDs can be attempted at EIC.
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THANK YOU!