Probing the linear polarization of gluons in unpolarized hadrons at an EIC

Cristian Pisano

Università di Cagliari

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- Study of $e h \rightarrow e' Q \bar{Q} X$, $e h \rightarrow e' \text{jet jet} X$ in a generalized factorization scheme
- Azimuthal asymmetries attributed to TMD pdfs, in particular $h_1^g$
- Comparison with $h_1 h_2 \rightarrow Q \bar{Q} X$ and $h_1 h_2 \rightarrow \text{jet jet} X$
- Summary and conclusions

* In coll. with D. Boer (U. Groningen), S. Brodsky (SLAC), P. Mulders (VU Amsterdam)
We consider the process
\[ e(\ell) + h(P) \rightarrow e(\ell') + Q(K_1) + \bar{Q}(K_2) + X \]
where the \(Q\bar{Q}\) pair is almost back-to-back in the plane perpendicular to \(q\) and \(P\), \(q \equiv \ell - \ell'\): four-momentum of the exchanged virtual photon \(\gamma^*\).

Perpendicular vectors \(a_\perp\): components of the vector \(a\) orthogonal to both \(q\), \(P\).

To LO in pQCD the reaction is mediated by the 2→2 partonic hard scattering subprocess
\[ \gamma^*(q) + g(p) \rightarrow Q(K_1) + \bar{Q}(K_2) \]
**Theoretical framework**

- Collinear factorization: gluon density $f_1^g(x)$ describes the distribution of unpolarized gluons with momentum fraction $x$ inside an unpolarized hadron.

- Phenomenological assumption: instead of collinear factorization, we consider a generalized factorization scheme taking into account partonic transverse momenta.

- In this framework, gluons do not have to be unpolarized, even if the parent hadron itself is unpolarized!

- Cross sections and azimuthal asymmetries are generated by transverse momentum dependent pdfs (TMDs), for example $f_1^{q,g}(x) \rightarrow f_1^{q,g}(x, p_T^2)$. Information on such functions is encoded in the TMD correlators $\Phi_{q,g}(x, p_T)$. 
### Quark and antiquark correlators

- Parton correlators describe the hadron \( \rightarrow \) parton transitions and can be parameterized in terms of transverse momentum dependent (TMD) pdfs

\[
\Phi(p;P,S) = \frac{1}{2} \left\{ \begin{array}{l}
    f_1^q(x, p_T^2) \, P + i h_{1\perp}^q(x, p_T^2) \, \frac{(p_T, P)}{2M} \\
\end{array} \right\}
\]

- \( f_1^q(x, p_T^2) \) is the unpolarized quark distribution

- \( h_{1\perp}^q(x, p_T^2) \) is the \( T \)-odd, quark transverse spin distribution in an unpolarized hadron

- \( h_{1\perp}^q(x, p_T^2) = 0 \) in the absence of initial or final state interactions (ISI/FSI)

- The antiquark corr. \( \bar{\Phi}_q \) is obtained from \( \Phi_q \) by replacing \( f_1^q \rightarrow f_1^\bar{q} \) and \( h_{1\perp}^q \rightarrow h_{1\perp}^\bar{q} \)

D. Boer, P.J. Mulders, PRD 57 (1998) 5780
We introduce two light-like Sudakov vectors $n_+$ and $n_-$, satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$, and define the transverse projector $g_{\alpha\beta}^T \equiv g_{\alpha\beta} - n_+^{\{\alpha} n_-^{\beta\}}$

For an unpolarized hadron, at leading twist, omitting gauge links,

$$\Phi_{g}^{\alpha\beta}(x,p_T;P) \equiv \Gamma^{\alpha\beta} = \frac{1}{2x} \left\{ - g_{T}^{\alpha\beta} f_1^g(x,p_T^2) + \left( \frac{p_T^\alpha p_T^\beta}{M^2} + g_T^{\alpha\beta} \frac{p_T^2}{2M^2} \right) h_1^\bot g(x,p_T^2) \right\}$$

- $f_1^g(x,p_T^2)$ represents the usual unpolarized gluon distribution
- $h_1^\bot g(x,p_T^2)$ is the $T$-even distribution of linearly pol. gluons in an unp. hadron


- $h_1^\bot g$ is a helicity-flip distribution, and a second rank tensor in $p_T$ ($p_T$-even)
- $h_1^\bot g(x,p_T^2) \neq 0$ in the absence of ISI or FSI, but, as any TMD, it will receive contributions from ISI/FSI \(\rightarrow\) non-universal?
The function $h_{1} \perp q$: phenomenology

- $h_{1} \perp q$ recently extracted from analysis of the azimuthal asymmetry
  \[ A \sim \cos 2\phi \ h_{1} \perp q \otimes H_{1} \perp : \text{Collins ff} \]

measured by the HERMES and COMPASS collaborations in unpolarized SIDIS


- Azimuthal asymmetry appearing in the Drell-Yan process:
  \[ \nu_{DY} \sim \cos 2\phi \ h_{1} \perp q \otimes h_{1} \perp \bar{q} \]

D. Boer, PRD 60 (1999) 014012
Photon-jet production in hadronic collisions

- In principle, $h_1^\perp q$ can be extracted also from the process

$$h_1(P_1) + h_2(P_2) \rightarrow \gamma(K\gamma) + \text{jet}(K_j) + X$$

where the photon-jet pair in the final state is almost back-to-back in the plane perpendicular to the beam axis

D. Boer, P. Mulders, CP, PLB 660 (2008) 360

- To LO in pQCD the reaction is mediated by the $2 \rightarrow 2$ partonic subprocesses

$$q\bar{q} \rightarrow \gamma g$$

$$qg \rightarrow \gamma q$$

- $h_1^\perp g$ turns out to yield a power-suppressed contribution to the cross section
Azimuthal asymmetry in photon-jet production

- For $|q_T| \ll |K_{\gamma\perp}| \approx |K_{j\perp}|$, with $q_T = |q_T|(\cos \phi_T, \sin \phi_T) \equiv K_{\gamma\perp} + K_{j\perp}$, and $K_{\perp} = |K_{\perp}|(\cos \phi_{\perp}, \sin \phi_{\perp}) \equiv (K_{\gamma\perp} - K_{j\perp})/2$

\[
A(z, x_1, x_2, q_T^2) = \nu_{DY}(x_1, x_2, q_T^2) R(z, x_1, x_2, q_T^2)
\]

$x_{1,2}$: partonic lightcone momentum fractions, and in term of rapidities:

\[
z = \frac{1}{e^{\eta_{\gamma} - \eta_j} + 1}
\]

- $\nu_{DY}$ contains the $\cos 2\phi \equiv \cos 2(\phi_T - \phi_{\perp})$ dependence and it is probed at the scale $|K_{\perp}| (\neq Q)$

- In $p \bar{p}$ DY $\nu_{DY} \sim 30\%$ or higher for $|q_T|$ of a few GeV and $Q$ of $\mathcal{O}(1 - 10)$ GeV


- We need an estimate of $R$, which only depends on the T-even pdfs $f_{1^{q,g}}(x, p_T^2)$

- Assumption: $f_{1^{q,g}}(x, p_T^2) = f_{1^{q,g}}(x) T(p_T^2)$, with $T(p_T^2)$ being a generic function, taken to be the same for all partons $\implies R \approx R(z, x_1, x_2)$
The proportionality factor $R$

- In $p\bar{p}$, using that the antiquark contribution in $\bar{p}$ equals the quark contribution in $p$:

$$R(z, x_1, x_2) = \frac{2N^2z(1-z)\sum_q e_q^2 f_1^q(x_1)f_1^q(x_2)}{D(z, x_1, x_2)}$$

$$D(z, x_1, x_2) = \sum_q e_q^2\left\{ N(1-z)(1+z^2)f_1^q(x_1)f_1^q(x_2) \\
+ Nz(1+(1-z)^2)f_1^q(x_2)f_1^q(x_1) \\
+ 2(N^2-1)(z^2+(1-z)^2)f_1^q(x_1)f_1^q(x_2) \right\}$$

$N$: number of colors

- $R$ increases as $x_1, x_2$ increase, due to the small contribution, in the denominator, of the gluon distribution $f_1^g(x)$ in the valence region
Results for $R = R(z)$ at the Tevatron

- $x_1, x_2, |K_{\gamma \perp}|$ are fixed at values typical of the Tevatron experiment in the central rapidity region, where $\eta_j \approx \eta_\gamma \approx 0$, $x_1 \approx x_2$ and $z \approx 0.5$

O. Atramentov, DØ Coll., talk at DIS07
A. Kumar, DØ Coll., JPCS 110 (2008) 022025

- unp. pdfs: GRV98 LO at the scale $\mu^2 = K_{\gamma \perp}^2$, only light quarks included

$R \approx 10\% - 50\% \implies A \approx 5\% - 15\%$ is possible in the central region. This could allow a study of $h_{1q}^\perp$ in $p \bar{p} \rightarrow \gamma \text{jet } X$ at the Tevatron
The function $h_1^\perp g$: phenomenology

- So far, no experimental studies of the function $h_1^\perp g$ have been performed.

- $h_1^\perp g$ contributes to the so-called dijet imbalance in hadronic collisions, commonly used to extract the average partonic intrinsic transverse momentum. Complication!

  It is likely too hard to measure the azimuthal asymmetry for $p p \rightarrow \text{jet jet } X$,

  $$\mathcal{A} \sim \cos 4\phi \ h_1^\perp g \otimes h_1^\perp g \quad \phi = \phi_T - \phi_\perp$$

  $\phi_T (\phi_\perp)$: azim. angle of the difference (sum) of the trans. momenta of the jets

  D. Boer, P. Mulders, CP, PRD 80 (2009) 094017

- One would like to obtain an extraction of $h_1^\perp g$ in a simpler manner, for example from

  $$\mathcal{A} \sim \cos 2\phi \ h_1^\perp g$$

- In $e p$ instead of $p p$ collisions: only one TMD is involved. Construction of the transverse momenta of the jets or heavy quarks would be essential.
Calculation of the cross section

- At high energies the electroproduction cross section factorizes in a leptonic tensor $L$, a soft parton correlator $\Phi$ for the incoming hadron and a hard part $H$

TMD master formula:

$$
\frac{d\sigma}{d^3\ell'} = \frac{1}{2s} \frac{d^3K_1}{(2\pi)^3 2E_1} \frac{d^3K_2}{(2\pi)^3 2E_2} \int dx \frac{d^2p_T}{2} (2\pi)^4 \delta^4(q+p-K_1-K_2) \\
\times \sum_{a,b,c} \frac{1}{Q^4} L(\ell, q) \otimes \Phi_a(x, p_T) \otimes |H_{\gamma^* a \to b c}(q, p, K_1, K_2)|^2
$$

- Leptonic tensor:

$$
L^{\mu\nu}(\ell, q) = -g^{\mu\nu} Q^2 + 2 (\ell^{\mu} \ell^{\nu} + \ell^{\nu} \ell^{\mu})
$$

- $|H_{\gamma^* a \to b c}|^2 = |H_{\gamma^* g \to Q \bar{Q}}(q, p, K_1, K_2)|^2$ is obtained from the diagrams for $\gamma^* g \to Q \bar{Q}$:
Kinematics

- Lightcone decomposition of the four-momenta in terms of two light-like Sudakov vectors $n_+$ and $n_-$, satisfying $n_+^2 = n_-^2 = 0$ and $n_+ \cdot n_- = 1$:

$$P = n_+ + \frac{M^2}{2} n_- \approx n_+, \quad q = -x_B n_+ + \frac{Q^2}{2x_B} n_- \approx -x_B P + (P \cdot q) n_-$$

$\implies P, q$ determine the lightcone directions. All the momenta expanded w.r.t. $P$ and $n$

$$n \equiv n_- = \frac{1}{P \cdot q} \left(q + x_B P\right)$$

Utilized DIS variables and relations:

$$s \equiv (\ell + P)^2$$
$$Q^2 = -q^2$$
$$x_B \equiv \frac{Q^2}{2P \cdot q}$$
$$y \equiv \frac{P \cdot q}{P \cdot \ell} = \frac{Q^2}{sx_B}$$
$$W^2 \equiv (q + P)^2 = Q^2 \frac{1-x_B}{x_B} = (1-x_B)ys$$
Decomposition of the momenta

- The leptonic momenta define a plane transverse w.r.t. $q$ and $P$, identified by $\hat{\ell}_\perp$

\[
\ell = \frac{1-y}{y} x_B P + \frac{1}{y} \frac{Q^2}{2x_B} n + \frac{\sqrt{1-y}}{y} Q \hat{\ell}_\perp
\]

\[
\ell' = \frac{1}{y} x_B P + \frac{1-y}{y} \frac{Q^2}{2x_B} n + \frac{\sqrt{1-y}}{y} Q \hat{\ell}_\perp
\]

- Partonic (gluon) momentum

\[p = x P + p_T + (p \cdot P - x M^2) n \approx x P + p_T, \quad x = p \cdot n\]

- Heavy quark and antiquark momenta

\[K_1 = z_1 (P \cdot q) n + \frac{M_Q^2 + K_{1\perp}^2}{2z_1 P \cdot q} P + K_{1\perp}\]

\[K_2 = z_2 (P \cdot q) n + \frac{M_Q^2 + K_{2\perp}^2}{2z_2 P \cdot q} P + K_{2\perp}, \quad K_i^2 = M_Q^2, \quad K_{i\perp} = -K_{i\perp}^2\]
**Kinematics**

- **Approximation:** since $Q$ and $\bar{Q}$ are produced almost back-to-back in azimuth, if we define $K_\perp \equiv (K_{1\perp} - K_{2\perp})/2$, $q_T \equiv K_{1\perp} + K_{2\perp}$, then $|q_T| \ll |K_\perp|$ and

  \[
  K_{1\perp} \approx K_\perp, \quad K_{2\perp} \approx -K_\perp
  \]

  \[
  M^2_{i\perp} \approx M^2_\perp = M^2_Q + K^2_\perp
  \]

- In the partonic subprocess $q + p = K_1 + K_2$, from which one gets, using the previous decompositions of momenta, the useful relations

  \[
  x = x_B + \frac{M^2_\perp}{y z_1 z_2 s}
  \]

  \[
  z_1 + z_2 = 1
  \]

  \[
  p_T = q_T
  \]

- Denoting with $y_1, y_2$ the rapidities of $Q$ and $\bar{Q}$ resp. in the photon-hadron cms:

  \[
  z \equiv z_2 = \frac{1}{e^{y_1 - y_2} + 1} = -\frac{\hat{t} - M^2_Q}{\hat{s} + Q^2}, \quad 1 - z = z_1 = \frac{1}{e^{y_2 - y_1} + 1} = -\frac{\hat{u} - M^2_Q}{\hat{s} + Q^2}
  \]

  $\hat{s}, \hat{t}, \hat{u}$: Mandelstam variables for the subprocess $\gamma^*(q) + g(p) \rightarrow Q(K_1) + \bar{Q}(K_2)$
Azimuthal asymmetry

- For $|q_T| \ll |K_T|$, with $q_T = |q_T|(\cos \phi_T, \sin \phi_T)$, $K_\perp = |K_\perp|(\cos \phi_\perp, \sin \phi_\perp)$

\[
\frac{d\sigma}{dy_1 \, dy \, dx_B \, d^2q_T \, d^2K_\perp} \sim \frac{\alpha^2 \alpha_s}{s M_\perp^2} \left[ A + \frac{q_T^2}{M^2} B \cos 2(\phi_T - \phi_\perp) \right]
\]

- Angular independent part of the cross section (summed over flavors)

\[
A = \sum_Q e_Q^2 A^{eg \rightarrow eQ\bar{Q}} \left( y, z, \frac{Q^2}{M_\perp^2}, \frac{M_Q^2}{M_\perp^2} \right) f_1^g(x, q_T^2)
\]

- The term $B$ contains the information about the linear polarization of gluons

\[
B = \sum_Q e_Q^2 B^{eg \rightarrow eQ\bar{Q}} \left( y, z, \frac{Q^2}{M_\perp^2}, \frac{M_Q^2}{M_\perp^2} \right) h_1^g(x, q_T^2)
\]

and can be singled out by weighting the cross section with $\cos 2(\phi_T - \phi_\perp)$
Electroproduction of two jets

- The cross section for the process

\[ e(\ell) + h(P) \rightarrow e(\ell') + \text{jet}(K_1) + \text{jet}(K_2) + X \]

where the two jets are almost back-to-back in the plane perpendicular to \( q \) and \( P \), can be calculated in the same way, taking \( M_Q = 0 \)

\[ \frac{d\sigma}{d\eta_1 \, dy \, dx_B \, d^2q_T \, d^2K_\perp} \sim \frac{\alpha^2\alpha_s}{sK_\perp^2} \left[ A + \frac{q_T^2}{M^2} B \cos 2(\phi_T - \phi_\perp) \right] \]

- \( y_i \rightarrow \eta_i = -\ln (\tan(\frac{1}{2}\theta_i)) \), \( \theta_i \) being the polar angles of the final partons in the virtual photon-hadron cms frame

- To LO in pQCD the reaction is mediated by the \( 2 \to 2 \) partonic hard scattering subprocesses \( \gamma^* \, q \rightarrow q \, g \) and \( \gamma^* \, g \rightarrow q \, \bar{q} \):

\[
A = \sum_{q, \bar{q}} e_q^2 A^{eq \rightarrow eqg} \left( x_B, y, z, \frac{Q^2}{K_\perp^2} \right) f_1^q(x, q_T^2) \\
+ \sum_{q} e_q^2 A^{eg \rightarrow eq\bar{q}} \left( y, z, \frac{Q^2}{K_\perp^2} \right) f_1^q(x, q_T^2)
\]
The cos 2φ angular distribution

- The expression for the $B$ term relative to $e h \rightarrow e \text{jet jet } X$ can be obtained from the one for $e h \rightarrow e Q \bar{Q} X$ with $M_Q = 0$. Namely,

$$B^{eh \rightarrow e \text{jet jet } X}(y, z, \frac{Q^2}{K^2_\perp}, q_T^2) = \sum_q e_q^2 B^{eg \rightarrow eq\bar{q}}(y, z, \frac{Q^2}{K^2_\perp}) h^\perp_1 g(x, q_T^2)$$

- Explicitly:

$$B^{eg \rightarrow eq\bar{q}}(y, z, \frac{Q^2}{K^2_\perp}) = \frac{1}{2} \frac{z(1-z)}{D^3} a(y) \left[ \left( -1 + \frac{1}{2} z(1-z) b(y) \right) \frac{Q^2}{K^2_\perp} \right]$$

$$D \equiv D \left( z, \frac{Q^2}{K^2_\perp} \right) = 1 + z(1-z) \frac{Q^2}{K^2_\perp}$$

$$a(y) = 2 - y(2 - y), \quad b(y) = \frac{6 - y(6 - y)}{2 - y(2 - y)}$$
Comments on the results

• From the expression of the term $B$ for $e p \rightarrow e' \text{jet jet} \ X$ one can see that the $\cos 2\phi$ coplanar correlation due to $h_{\perp}^g$ is not power suppressed if $Q^2 \sim K^2_{\perp}$

• Depending on which of the two scales is larger, the asymmetry can become $Q^2 / K^2_{\perp}$ or $K^2_{\perp} / Q^2$ suppressed if the difference between them becomes too large

• If instead of jets, a heavy quark pair is produced, then the situation is identical, except that for bottom quarks there is a third large scale that may enter in the ratios

• Jet and heavy quark production in SIDIS are equally interesting processes: they produce a $\cos 2\phi$ asymmetry that could be measured at an EIC and at the LHeC

• What about hadronic collisions at RHIC, Tevatron, LHC?
**Hadroproduction of heavy quarks**

- The cross section for the process

\[ h_1(P_1) + h_2(P_2) \rightarrow Q(K_1) + \bar{Q}(K_2) + X , \]

is calculated in the hadronic cms frame, similar to hadroproduction of two jets

D. Boer, P. Mulders, CP, PRD 80 (2009) 094017

- For \(|q_T| \ll |K_T|\), with \(q_T = |q_T| \cos \phi_T, \sin \phi_T\), \(K_\perp = |K_\perp| \cos \phi_\perp, \sin \phi_\perp\)

\[
\frac{d\sigma}{dy_1 dy_2 d^2 K_{1\perp} d^2 K_{2\perp}} = \frac{\alpha_s^2}{s M_2^2} \left[ A + B \, q_T^2 \cos(\phi_T - \phi_\perp) + C \, q_T^4 \cos(4\phi_T - 4\phi_\perp) \right]
\]

- The terms \(A, B, C\) depend on \(q_T^2, z, M_Q^2 / M_\perp^2\) and on the lightcone momentum fractions of the two incoming partons (carrying momenta \(p_1, p_2\))

\[
x_1 = \frac{1}{\sqrt{s}} \left( M_{1\perp} \, e^{y_1} + M_{2\perp} \, e^{y_2} \right), \quad x_2 = \frac{1}{\sqrt{s}} \left( M_{1\perp} \, e^{-y_1} + M_{2\perp} \, e^{-y_2} \right)
\]

\[
q_T \equiv K_{1\perp} + K_{2\perp} = p_{1T} + p_{2T}
\]

- \(A, B, C\) contain convolutions of the various pdfs and are calculated at LO in pQCD
The $\cos 2\phi$ angular distribution

- In the process $h_1 h_2 \rightarrow Q \bar{Q} X$ the role of $Q^2$ is taken on by $M_Q^2$

\[
B = \mathcal{B}^{q\bar{q} \rightarrow Q\bar{Q}} \left(z, x_1, x_2, \frac{M_Q^2}{M_\perp^2}, q_T^2\right) + \frac{M_Q^2}{M_\perp^2} \mathcal{B}^{g\bar{g} \rightarrow Q\bar{Q}} \left(z, x_1, x_2, \frac{M_Q^2}{M_\perp^2}, q_T^2\right)
\]

\[
\mathcal{B}^{q\bar{q} \rightarrow Q\bar{Q}} \sim h_1^\perp q \otimes h_1^\perp \bar{q}
\]

\[
\mathcal{B}^{g\bar{g} \rightarrow Q\bar{Q}} \sim f_1^g \otimes h_1^\perp g + q_T^2 \frac{M_Q^2}{M_\perp^2} h_1^\perp g \otimes h_1^\perp g
\]

- For charm quarks the $\cos 2\phi$ asymmetry becomes essentially higher twist, assuming $M_c^2 \ll K_\perp^2$

- For bottom quarks, the contribution coming from $f_1^g \otimes h_1^\perp g$ can be unsuppressed if $M_b^2 \sim K_\perp^2$, the corresponding asymmetry $\sim M_Q^2/M_\perp^2$

- $h_1^\perp g \otimes h_1^\perp g$: double suppression, assuming $h_1^\perp g \ll f_1^g$
The cos 4φ angular distribution

- The cos 4(φ_T − φ⊥) angular distribution of the Q ¯Q pair is related exclusively to the presence of linearly polarized gluons in unpolarized hadrons

\[ C = C^{gg \rightarrow Q \bar{Q}} = C^{HQ}(z) \left[ C^g_1(x_1, x_2, q^2_T) + \frac{M^4_Q}{M^4_T} C^g_2(x_1, x_2, q^2_T) \right] \]

- Explicitly, denoting with N the number of colors,

\[ C^{HQ}(z) = -\frac{N}{N^2 - 1} \frac{z(1 - z)}{4} \left( z^2 + (1 - z^2) - \frac{1}{N^2} \right) \]

- Convolutions, \( \hat{h} = q_T/|q_T| \),

\[ C^{gg}_i \equiv \frac{1}{M^2_1 M^2_2} \int d^2 p_1T \ d^2 p_2T \ \delta^2(p_1T + p_2T - q_T) \mathcal{H}_i h_{1g}^i(x_1, p^2_{1T}) h_{1g}^j(x_2, p^2_{2T}) \]

\[ q^4_T \mathcal{H}_1 = 2 \left[ 2 (\hat{h} \cdot p_{1T})(\hat{h} \cdot p_{2T}) - (p_{1T} \cdot p_{2T}) \right]^2 - p^2_{1T} p^2_{2T} \]

\[ q^4_T \mathcal{H}_2 = 2 \left( 2(\hat{h} \cdot p_{1T})^2 - p^2_{1T} \right) \left( 2(\hat{h} \cdot p_{2T})^2 - p^2_{2T} \right) - 2 (p_{1T} \cdot p_{2T})^2 + p^2_{1T} p^2_{2T} \]

- Not higher twist, but cos 4φ asymm. more difficult to measure than the cos 2φ one!
Hadroproduction of dijets

- In the hadronic cms frame, for $|q_T| \ll |K_T|$, the cross section for the process
  
  $$h_1(P_1)+h_2(P_2) \to \text{jet}(K_1)+\text{jet}(K_2)+X,$$
  
  has the following structure, analogous to the heavy quark production case,

  $$\frac{d\sigma}{d\eta_1 d\eta_2 d^2K_{1\perp} d^2K_{2\perp}} = \frac{\alpha_s^2}{sM^2} \left[ A + B q_T^2 \cos(\phi_T-\phi_\perp) + C q_T^4 \cos 4(\phi_T-\phi_\perp) \right]$$

  D. Boer, P. Mulders, CP, PRD 80 (2009) 094017

- The term $B$ receives contributions from $qq \to qq$, $q\bar{q} \to q\bar{q}$, $q\bar{q} \to q'\bar{q}'$, $q\bar{q} \to gg \to \text{no cos 2}\phi$ asymmetry related to $h_1^{\perp g}$

- The $\cos 4\phi$ asymmetry arises from the convolution $C^{gg} \sim h_1^{\perp g} \otimes h_1^{\perp g}$:

  $$C = C^{gg\to q\bar{q}} + C^{gg\to gg} = C^{JJ}(z) C^{gg}(x_1, x_2, q_T^2)$$

  $$C^{JJ}(z) = \frac{N}{N^2-1} z(1-z) \left[ N(1-z(1-z)) - \frac{1}{4} \left(z^2 + (1-z)^2 - \frac{1}{N^2} \right) \right]$$
In order to project out the $C$ term of the cross section

$$\left\langle \cos 4(\phi_T - \phi_\perp) \right\rangle \propto \int \frac{d\phi_T}{2\pi} \cos 4(\phi_T - \phi_\perp) \frac{d\sigma^{h_1 h_2 \rightarrow Q \bar{Q} X}}{dy_1 dy_2 d^2 K_\perp d^2 q_T} = \frac{1}{2} \frac{\alpha_s^2}{s M_\perp^2} q_T^4 C$$

Integrating over the length of $q_T$ gives with possible inclusion of additional weighting with powers of $q_T^2$:

$$\pi \int dq_T^2 \left( \frac{q_T^2}{M_1 M_2} \right)^M \left\langle \cos 4(\phi_T - \phi_\perp) \right\rangle = \int d^2 q_T \left( \frac{q_T^2}{M_1 M_2} \right)^M \cos 4(\phi_T - \phi_\perp) d\sigma$$

for $M = 2$ one gets the de-convoluted results

$$\pi \int dq_T^2 \left( \frac{q_T^2}{M_1 M_2} \right)^2 q_T^4 C_i^{gg} \propto h_1^{\perp g(2)}(x_1) h_1^{\perp g(2)}(x_2)$$

$$h_1^{\perp g(n)}(x) = \int d^2 p_T \left( \frac{p_T^2}{2M^2} \right)^n h_1^{\perp g}(x, p_T^2)$$

Similarly, $\left\langle \cos 2(\phi_T - \phi_\perp) \right\rangle \propto q_T^2 B$, and weighting with powers of $q_T^2$, for $M = 1$ one obtains factorized results

$$\sim h_1^{\perp q(1)}(x_1) h_1^{\perp \bar{q}(1)}(x_2), \quad f_1^{g}(x_1) h_1^{\perp g(2)}(x_2)$$

For $M = 0$, the above expressions for $B$ and $C$ do not de-convolute.
Gaussian Ansatz for the transverse momentum shape of $h_{1}^{±g}$

- In order to evaluate the integral without weights ($M = 0$) one can employ a Gaussian model for $h_{1}^{±g}$ with a factorized $x$ and $p_T$ dependence

$$h_{1}^{±g}(x, p_T^2) = \frac{R_h^2}{\pi} h_{1}^{±g}(x) e^{-R_h^2 p_T^2}$$

$$h_{1}^{±g}(n)(x) = \frac{n!}{(2 M^2 R_h^2)^n} h_{1}^{±g}(x)$$

$R_h$ is a size parameter related to the average partonic $p_T^2$ by the relation $R_h^2 = 1/\langle p_T^2 \rangle$. For incoming (anti)protons, $R_p = R_{\bar{p}} \equiv R$, so one has

$$\int d^2 q_T q_T^4 C_i^{gg} = \frac{1}{M_1^2 M_2^2} \int d^2 q_T d^2 p_{1T} d^2 p_{2T} \delta^2(p_{1T} + p_{2T} - q_T) \mathcal{H}_i$$

$$\times \frac{R^4}{\pi^2} h_{1}^{±g}(x_1) h_{1}^{±g}(x_2) e^{-R^2(p_{1T}^2 + p_{2T}^2)}$$

- Using the $p_{2T}$ integration to eliminate the $\delta$-function and shifting the integration variable $p_{1T} \rightarrow p'_{1T} = p_{1T} - \frac{1}{2} q_T$, one arrives at

$$\int d^2 q_T q_T^4 C_i^{gg} \propto \frac{1}{M_1^2 M_2^2} \frac{1}{R^4} h_{1}^{±g}(x_1) h_{1}^{±g}(x_2)$$
**Gaussian Ansatz for the transverse momentum shape of $h_1^⊥g$**

- **Final result for the unweighted average for $pp → Q\bar{Q}X$:**

  \[
  \pi \int dq_T^2 \langle \cos 4(\phi_T - \phi_⊥) \rangle = \frac{\alpha_s^2}{s M^2_⊥} C^{HQ}(z) \left( 1 + 2 \frac{M^4_Q}{M^4_⊥} \right) h_1^⊥g^{(1)}(x_1) h_1^⊥g^{(1)}(x_2)
  \]

  \[
  C^{HQ}(z) = - \frac{N}{N^2 - 1} \frac{z(1 - z)}{4} \left( z^2 + (1 - z^2) - \frac{1}{N^2} \right)
  \]

- **Same result holds for $pp → jet jet X$, but with $M_Q = 0$ and**

  \[
  C^{JJ}(z) = \frac{N}{N^2 - 1} z(1 - z) \left[ N (1 - z(1 - z)) - \frac{1}{4} \left( z^2 + (1 - z^2) - \frac{1}{N^2} \right) \right]
  \]
Summary and conclusions

- We have calculated the cross section for heavy quark and dijet production in $e h$ and $h h$ collisions within a generalized factorization scheme, taking into account the transverse momentum of the partons in the initial hadron(s).


- Contribution from the TMD pdf $h_{1}^{\perp} g$ to $\cos 2\phi$ and $\cos 4\phi$ azimuthal asymmetries

- $\cos 2\phi$ asymmetries could be measured both in $ep$ (EIC, LHeC) and in $pp$ or $p\bar{p}$ collisions (RHIC, LHC, Tevatron) probing the distribution of linearly polarized gluons inside nucleons

- From a theoretical viewpoint these asymmetries are among the simplest TMD observables since the number of partonic subprocesses is in each case limited to just one, this avoids having to consider complicated linear combinations of ISI/FSI