A Study of Gluonic Pole Matrix Elements

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- Correlators in high energy scattering processes
- Distribution and fragmentation
- Gauge invariance
- Single spin asymmetries and time reversal invariance
- Transverse moments: gluonic pole contributions
- Spectral analysis of gluonic pole matrix elements in spectator framework
- Model Independent Analysis
- Conclusion

In collaboration with L. P. Gamberg (Penn State-Berks), P. J. Mulders (VU Amsterdam)
Hard partonic subprocess contribution
parton($p_1$)+parton($p_2$) → parton($k_1$)+parton ($k_2$)

To the process at the level of amplitude
Hadron($P_1$)+Hadron($P_2$) → Hadron($K_1$)+Hadron($K_2$)+X

Subprocess is hard implies $P_1 \cdot P_2 \sim P_1 \cdot K_1 \sim Q^2$
Correlators:

The quark distribution correlator can be written as,

\[ \Phi_{ij}(k, P) = \frac{1}{(2\pi)^4} \int d^4\xi e^{ik \cdot \xi} \langle P | \bar{\psi}_i(0) \psi_j(\xi) | P \rangle \]

And the fragmentation correlator,

\[ \Delta_{ij}(k, P) = \frac{1}{(2\pi)^4} \sum_X \int d^4\xi e^{ik \cdot \xi} \langle 0 | \psi_i(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle \]
Correlators:

Inside the correlator: all momenta of hadrons and quarks and gluons are soft →
\( p^2 \sim p \cdot P \sim P^2 = M^2 \ll Q^2 \sim s \)

- Integrate the correlator over \( p \cdot P \sim \sigma \)

\[ \Phi_{ij}(x, p_T) = \int d(p \cdot P) \Phi_{ij}(p, P) \]

- Integrating over \( p_T \) one gets collinear correlators

- Collinear correlators appear in processes where only the hard scale (\( \sim Q^2 \)) is measured, like DIS; transverse momentum dependent (TMD) correlators appear when hadronic scale variables are measured (like correlations or transverse momentum in jets, off-collinear configurations)
Distribution and Fragmentation Functions

Cross section can be expressed in terms of the distribution and fragmentation functions:

\[ \Phi(x, p_T) = \int \frac{d(\xi \cdot P)d^2\xi T}{(2\pi)^3} e^{ip \cdot \xi} \langle P \mid \bar{\psi}(0)\psi(\xi) \mid P \rangle_{\xi \cdot n=0} \]

\[ p^\mu = xP^\mu + p_T^\mu + (p \cdot P - xM^2)n^\mu \]

\[ \Delta(x, k_T) = \sum_X \int \frac{d(\xi \cdot K)d^2\xi T}{(2\pi)^3} e^{ik \cdot \xi} \langle 0 \mid \psi(\xi) \mid K, X \rangle \langle K, X \mid \bar{\psi}(0) \mid 0 \rangle_{\xi \cdot n=0} \]

\[ k^\mu = \frac{1}{z} K^\mu + k_T^\mu + (k \cdot K - z^{-1}M_h^2)n^\mu \]

The correlators are defined on the light-front \( \xi \cdot n = 0 \).

These are transverse momentum dependent (TMD) distribution and fragmentation correlators.
Integrating the distribution and fragmentation functions over $p_T$, we get the lightcone (collinear) correlators

$$
\Phi(x) = \int d^2 p_T \Phi(x, p_T) \int d(\xi \cdot P) \frac{e^{i p \cdot \xi}}{2\pi} \langle P \mid \bar{\psi}(0) \psi(\xi) \mid P \rangle \xi \cdot n=0,\xi_T=0
$$

Integrating over $x$ we get a completely local object

$$
\Phi = \int dx \Phi(x) = \langle P \mid \bar{\psi}(0) \psi(0) \mid P \rangle
$$
Transverse Momentum Dependent Parametrization

Unpolarized quarks

\[ \Phi[\gamma^+](x, p_T) = f_1(x, p_T^2) + \frac{(p_T \times S_T)}{M} f_{1T}^\perp(x, p_T^2) \]

Longitudinally polarized quarks

\[ \Phi[\gamma^+\gamma_5](x, p_T) = S_L g_{1L}(x, p_T^2) + \frac{(p_T \cdot S_T)}{M} g_{1T}(x, p_T^2) \]

Transversely polarized quark

\[ \Phi[i\sigma^i\gamma_5](x, p_T) = S_T^i h_1(x, p_T^2) + S_L \frac{p_T^i}{M} h_{1L}^\perp(x, p_T^2) \]
\[ + \frac{p_T^i(p_T \cdot S_T) - \frac{1}{2} p_T^2 S_T^i}{M} h_{1T}^\perp(x, p_T^2) + \frac{\epsilon_{ij} p_T^j}{M} h_1^\perp(x, p_T^2) \]

- \( f_{1T}^\perp(x, p_T^2) \) and \( h_1^\perp(x, p_T^2) \) : Time reversal odd
Gauge Invariance of Quark Correlators

Presence of gauge link needed for color gauge invariance of the correlators
Collinear correlator: bilocality on the light cone

\[ \Phi_{ij}^q(x, n) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ip \cdot \xi} \langle P \mid \bar{\psi}_j(0)U^{(n)}_{[0, \xi]} \psi_i(\xi) \mid P \rangle_{\xi \cdot n = \xi_T = 0} \]

TMD correlator: bilocality on the light front

\[ \Phi_{ij}^q(x, p_T, n, C) = \int \frac{d(\xi \cdot P)d^2 \xi_T}{(2\pi)^3} e^{ip \cdot \xi} \langle P \mid \bar{\psi}_j(0)U^{(C)}_{[0, \xi]} \psi_i(\xi) \mid P \rangle_{\xi \cdot n = 0} \]

Gauge link \( U^{(C)}_{[0, \xi]} = \mathcal{P} e^{\frac{ig}{P} \int_0^\xi ds^\mu A^a_\mu T_a} \)

\( C \) is the integration path with endpoints 0 and \( \xi \); \( \mathcal{P} \) denotes a path ordering.

Gauge invariant quark correlator not only contains the quark fields but a varying number of gluon fields.
Gauge Links

- Diagrams with additional gluons coupled to the hard function are not power suppressed if these gluons are polarized collinear to the parent (or daughter) hadron, sometimes even transversely polarized.

- These contributions are resummed together to get the color gauge invariant correlator coupling to the tree level hard function.

- These gauge links or Wilson Lines are fixed by hard partonic interactions $\rightarrow$ process dependence.

- In $\Phi(x)$, parton fields are separated in the light cone direction $\xi \cdot P$ : Wilson line in the lightcone $n$ direction formed by gluons polarized along the momentum of the hadron:

$$U^{(n)}_{[\xi,\xi+\eta]} = \mathcal{P} \exp[-ig \int_0^{\frac{n \cdot P}{n \cdot p}} d\lambda n \cdot A^a(\xi + \lambda n) T^a]$$

Efremov and Radyushkin (1981)
**Gauge Links**

- In $\Phi(x, p_T)$, quark fields are separated also in the transverse direction → both collinear and transversely polarized gluons have to be taken into account to get the full gauge link.

- Transverse gauge link due to transversely polarized gluons at lightcone $\pm \infty$

$$U^T_{[\xi, \xi + \eta]} = \mathcal{P} e^{-ig \int_{\xi T}^{\xi T + \eta T} d\xi T A_T^a(\xi) T^a}$$

- These transverse gluon contributions are not suppressed


- These formalized model calculations of Brodsky, Huang, Schmidt (2002) that Sivers effect is nonzero → arises when the gauge link is included; without a violation of time-reversal invariance
Single Spin Asymmetries and $T$-invariance

- Single transverse spin asymmetry

$$A_N = \frac{\sigma(+S_T) - \sigma(-S_T)}{\sigma(+S_T) + \sigma(-S_T)}$$

Azimuthal imbalance caused by transverse spin effects can give rise to finite transverse spin asymmetries

- Nonvanishing single-spin asymmetries requires a large complex phase difference between the helicity flip and non-flip amplitudes. Several mechanisms have been proposed to generate this phase difference, both in the initial state as well as in the final state

  \[ \text{Kane, Pumplin, Repko (1978)} \]

- Sivers effect: asymmetry arises in the initial state due to a correlation between the intrinsic transverse momentum of the unpolarized quark and the transverse spin of the parent hadron

  $$\sim \frac{p_T \times S_T}{M} f_{1T}^\perp(x, p_T^2)$$

- Using the properties of quark fields under time reversal, it is found that without the gauge link, T-reversal invariance requires $$f_{1T}^\perp(x, p_T^2)$$ and $$h_{1T}^\perp(x, p_T^2)$$ are zero.

- Brodsky, Huang, Schmidt (2002) used a spectator diquark model to show that leading twist spin asymmetries can be generated also in the initial state.
Single Spin Asymmetries and $T$-invariance (contd.)

- Inclusion of Wilson lines → One can get nonzero $f_{1T}^\perp(x, p_T^2)$ and $h_{1T}^\perp(x, p_T^2)$ distribution functions
- Both are (naive) $T$-odd
- A phase difference can also occur in the final state of the hard scattering giving rise to single spin asymmetry → Collins fragmentation function $H_{1T}^\perp(z, k_T^2)$
- For a fragmentation correlator, due to the appearance of the hadronic final state $| P, X \rangle$, which cannot be represented by plane wave states: $T$-reversal changes 'out' states in the definition of the fragmentation correlator into 'in' states
- As a result $T$-invariance does not set the Collins function to zero; even without the gauge link $H_{1T}^\perp(z, k_T^2)$ is $T$-odd

Boer, Mulders, Pijlman (2003) for discussion on $T$-reversal
Including the gauge link, the correlator becomes

\[ \Phi^\pm (x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip \cdot \xi} \langle P, S | \bar{\psi}(0) U^{(\pm)}_{[0, \xi]} \psi(\xi) | P, S \rangle_{\xi^+ = 0} \]

- Axial gauge: \( U^n \) would reduce to 1
Constraints on the Quark Correlator

Hermiticity, parity and time reversal yield constraints for the correlators $\phi$

\[
\Phi^\dagger(p, P, S) = \gamma_0 \Phi(p, P, S) \gamma_0 \quad \text{[Hermiticity]}
\]
\[
\Phi(p, P, S) = \gamma_0 \Phi(\bar{p}, \bar{P}, -\bar{S}) \gamma_0 \quad \text{[Parity]}
\]
\[
\Phi^*(p, P, S) = (-i\gamma_5 C)\Phi(\bar{p}, \bar{P}, \bar{S})(-i\gamma_5 C) \quad \text{[TimeReversal]}
\]

where $\bar{p} = (p^0, -\vec{p})$, $C = i\gamma_2 \gamma_0$

Similar conditions arise for fragmentation correlators

When the gauge link is included, slightly different conditions apply

\[
\Phi^+^\dagger(x, p_T) = \gamma_0 \Phi^+(x, p_T)
\]
\[
\Phi^+(x, p_T) = \gamma_0 \Phi^+(x, -p_T) \gamma_0
\]
\[
\Phi^{+[\ast]}(x, p_T) = (-i\gamma_5 C)\Phi^-[x, -p_T](-i\gamma_5 C)
\]


Future and Past Pointing Wilson Lines (contd.)

- T-reversal related $\Phi^+$ and $\Phi^-$
- $\Phi^+$ enters in semi-inclusive DIS (SIDIS) whereas $\Phi^-$ enters in Drell-Yan (DY) scattering
- It follows that the Sivers’ functions are related by
  
  $$ f_{1T}^{+}(x, p_{T}^{2}) \big|_{DY} = - f_{1T}^{+}(x, p_{T}^{2}) \big|_{SIDIS} $$

- Sivers’ effect in SIDIS has been measured (see review by Efremov, Goeke, Schweitzer (2006))
Gauge Link in Integrated and Unintegrated Correlators

- If we neglect the transverse gauge link and choose axial gauge, the gauge link would become unity: then $T$-invariance would require the absence of $T$-odd terms

- $p_T$ dependent distribution function

$$
\Phi^{[U]}(x, p_T) = \int \frac{d(\xi \cdot P)d^2\xi_T}{(2\pi)^3} e^{ip\cdot\xi} \langle P | \bar{\psi}(0)U_{[0,\xi]}\psi(\xi) | P \rangle_{LF}
$$

- Here the gauge link is process dependent
- Collinear distribution functions are obtained from above by

$$
\Phi(x) = \int d^2p_T \Phi^{[U]}(x, p_T) = \int \frac{d(\xi \cdot P)}{2\pi} e^{ix\xi\cdot P} \langle P | \bar{\psi}(0)U_{[0,\xi]}^n\psi(\xi) | P \rangle_{LC}
$$

- Here, the gauge link is unique as the path is along $n$

$$
\Phi^{[\pm]}(x) = \Phi^{[-]}(x) = \Phi(x) = \int d^2p_T \Phi^{[\pm]}(x, p_T)
$$
Transverse Moments

- TMD correlator when expanded in terms of $p_T^2$, dependent distribution functions $f_i(x, p_T^2) \to$ contains both T-even and T-odd functions (as the correlator is not T-invariant due to the gauge link)

- Integrated over $p_T$: quark-gluon correlator that appear at leading twist in high energy processes contain only T-even operator combinations $\to$ parametrized in terms of T-even functions $f_i(x)$

- Azimuthal asymmetries: $p_T$ weighted cross sections contribute

- Introduce transverse moments

\[ \Phi^\alpha[U] = \int d^2p_T p_T^\alpha \Phi^U(x, p_T) \]

- This contains a nontrivial link dependence that prohibits the use of T invariance as a constraint

- One can decompose

\[ \Phi^\alpha[U] = \tilde{\Phi}^\alpha(x) + C^U_G \pi \Phi_G^\alpha(x, x) \]
Transverse Moments

- One can decompose

\[ \Phi^\alpha[U](x) = \tilde{\Phi}^\alpha(x) + C_G[U] \pi \Phi^\alpha_G(x, x) \]

- Here, \( \tilde{\Phi} \) is the process independent T-even part.
- \( \Phi_G \) contains the T-odd operator combination and is process independent
- \( C_G[U] \) is the calculable process dependent gluonic pole factor
- This is the soft limit \( (x_1 \to 0) \) of the quark-gluon correlator

\[
\Phi^\alpha_G(x, x-x_1) = n_\mu \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix_1(\eta \cdot P)} e^{i(x-x_1)(\xi \cdot P)}
\]

\[
\langle P | \bar{\psi}(0) U_{[0;\eta]}^{\mu} gG^{\mu\alpha}(\eta) U_{[\eta;\xi]}^{\nu} \psi(\xi) | P \rangle \bigg|_{LC} ,
\]
**Fragmentation Functions**

\[ \Delta_{ij}^{[U]}(z, k_T) = \sum_X \int \frac{d(\xi \cdot P_h) \, d^2 \xi_T}{(2\pi)^3} \, e^{i \, k \cdot \xi} \]

\[ \times \langle 0 | U_{[0, \xi]} \psi_i(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle \langle 0 | LF \rangle. \]

The collinear, \( k_T \)-integrated correlator

\[ \Delta(z) = \int d^2 k_T \Delta^{[U]}(z, k_T) \]

\[ = \sum_X \int \frac{d(\xi \cdot P)}{2\pi} \, e^{i \, z^{-1}(\xi, P)} \]

\[ \times \langle 0 | \mathcal{U}_{[0, \xi]}^n \psi_i(\xi) | P, X \rangle \langle P, X | \bar{\psi}_j(0) | 0 \rangle \langle 0 | LC \rangle, \]

This contains only T-even operator combinations, but \( | P, X \rangle \) is an 'out' state which is not T-invariant.

After \( k_T \) weighting

\[ \Delta_{\alpha}^{[U]}(z) = \int d^2 k_T \, k_T^\alpha \Delta^{[U]}(z, k_T) = \tilde{\Delta}_{\alpha}^{\alpha} \left( \frac{1}{z} \right) + C_{G}^{[U]} \pi \Delta_{\alpha}^{\alpha} \left( \frac{1}{z}, \frac{1}{z} \right). \]
Fragmentation Functions (contd.)

$\hat{\Delta}^\alpha_G \left( \frac{1}{z} \right)$ and $\Delta^\alpha_G \left( \frac{1}{z}, \frac{1}{\bar{z}} \right)$ are process independent; contain T-even and T-odd operator combinations respectively.

But because of the hadronic states $| P, X \rangle$, each of the above correlators contains T-even and T-odd functions.

$C^{[\mathcal{U}]}_G$ is the calculable process dependent gluonic pole factor.

Gluonic pole correlator is the soft limit $z_1^{-1} = x_1 \to 0$, of the quark-gluon correlator

$$\Delta^\alpha_{Gij} (x, x - x_1) = \sum_X \int \frac{d(\xi \cdot P)}{2\pi} \frac{d(\eta \cdot P)}{2\pi} e^{ix_1(\eta \cdot P)} e^{i(x-x_1)(\xi \cdot P)}$$

$$\times \langle 0 | \mathcal{U}^n_{[0,\eta]} g G^{n\alpha}(\eta) \mathcal{U}^n_{[\eta,\xi]} \psi_i(\xi) | P, X \rangle \langle P, X | \overline{\psi}_j(0) | 0 \rangle \bigg|_{LC}$$
Summary of the discussion so far

- At leading twist, TMD distribution correlator gets a T-odd part which comes due to the link and is process dependent.

- TMD fragmentation has two sources for T-odd structure: process dependent links and that due to the state $|P, X\rangle$.

- These two effects are distinguishable.

- Take $p_T$ weighted moments: if $\Delta G(k_1 = 0) = 0$ then the only T-odd part of the fragmentation correlator will come from $\tilde{\Delta}_G^\alpha$, and this is process independent (universal).

- In a model calculation by doing a spectral analysis we show that this is indeed true.
Gluonic Pole Matrix Elements

- Model calculation for SIDIS and $e^+e^-$ annihilation
  
  A. Metz (2002); Collins, Metz (2004)

- More recently in hadron-hadron scattering: $\Delta_G(k_1 = 0) = 0$ and $\Phi_G(k_1 = 0) \neq 0$
  
  F. Yuan (2008)

- T-odd part of the distribution functions come only from the gluonic pole factors (process dependent)

- In these works, full process has been considered and carefully studying the cuts to extract the gluonic pole contributions

- One may also consider the two-parton correlator and generate the T-odd contribution at one loop by performing a perturbative expansion of the gauge link
  
  Gamberg, Goldstein, Schegel (2008), Bacchetta, Gamberg, Goldstein, Mukherjee (2008)

- Here in contrast we look at the soft part only by starting directly the color gauge invariant multiparton correlators $\Phi_G$ and $\Delta_G$ (with resummed gauge link) and take soft limit to extract the gluonic pole matrix elements

- Tree level matrix elements $\rightarrow$ first moment wrt $p_T$ : contributes to SSA
Spectator Model Approach: Distribution and Fragmentation Functions
**Spectator Model Approach**

Consider spectator model with spectator mass $M_s$. Result for the cut diagram as in the fig.

$$\Phi(x, k_T) \sim \int d(k \cdot P) \frac{F(k^2, k \cdot P)}{(k^2 - m^2 + i\epsilon)^2} \delta ((k - P)^2 - M_s^2).$$

$F(k, k \cdot P)$ is the numerator of propagators and/or traces of them in the presence of Dirac gamma matrices; as well as vertex form factors in the model Lightcone components of the momenta:

$$P = \left[ \frac{M^2}{2}, 1, 0_T \right],$$

$$P - k = \left[ \frac{M_s^2 - k_T^2}{2(1 - x)}, 1 - x, -k_T \right],$$

$$k = \left[ \frac{(1 - x)M^2 - M_s^2}{2(1 - x)}, x, k_T \right].$$
Here we have implemented the constraint from $\delta((k - P)^2 - M_s^2)$

One finds

$$\Phi(x, k_T) \sim \frac{(1 - x)^2 F(x, k_T)}{(\mu^2(x) - k_T^2)^2},$$

with

$$\mu^2(x) = x M_s^2 + (1 - x) m^2 - x(1 - x) M^2.$$

Details of the function $F(x, k_T^2)$ depend on the model: including vertices. These are chosen such a way not to produce unphysical results, like a decaying proton if $M \geq m + M_s$

$m$ is the constituent mass rather than the bare mass

Useful feature: ability to produce reasonable valence and sea quark distributions using the freedom in the model connected to an intuitive picture

We do not consider specific form of $F(x, k_T^2)$

Result for fragmentation is obtained by substituting $x \rightarrow \frac{1}{z}$
Quark-quark-gluon correlator for distribution : in spectator model
Quark-quark-gluon correlator for fragmentation: in spectator model

\[ \Delta_G(k, k-k_1) \]
We parametrize the gluon momentum as

\[ k_1 = \left[ k_1^-, x_1, k_{1T} \right], \]

where \( k_1^- = k_1 \cdot P - \frac{1}{2} x_1 M^2 \) first component to be integrated over

Relevant momenta (implementing the on-shell condition for \( P - k \)):

\[ k - k_1 = \left[ -k_1^- + \frac{(1 - x) M^2 - M_s^2 + k_{1T}^2}{2(1 - x)}, x - x_1, k_T - k_{1T} \right], \]

\[ P - k + k_1 = \left[ k_1^- + \frac{M_s^2 - k_{1T}^2}{2(1 - x)}, 1 - x + x_1, -k_T + k_{1T} \right], \]

\[ P - k_1 = \left[ -k_1^- + \frac{M^2}{2}, 1 - x_1, -k_{1T} \right] \]
Quark-gluon Matrix Elements in spectator model (contd.)

Basic result for the quark-gluon correlators $\Phi_G(x, x - x_1, k_T, k_T - k_1T)$

$$\Phi_G \sim \frac{1}{(k^2 - m^2)} \left\{ \int \frac{dk_1^-}{2\pi i} \frac{F_1 \left( k_1^-, x, x_1, k_T, k_1T \right)}{(k_1^2 - m_1^2 + i\epsilon)((k - k_1)^2 - m^2 + i\epsilon)((P - k + k_1)^2 - M_{s1}^2 + i\epsilon)} \right. $$

$$+ \left. \int \frac{dk_1^-}{2\pi i} \frac{F_2 \left( k_1^-, x, x_1, k_T, k_1T \right)}{(k_1^2 - m_1^2 + i\epsilon)((k - k_1)^2 - m^2 + i\epsilon)((P - k_1)^2 - M_{s1}^2 + i\epsilon)} \right\} \left\{ \int \frac{dk_1^-}{2\pi i} \frac{F_1 \left( k_1^-, x, x_1, k_T^2, k_1T^2 \right)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x) k_1^- - A_2 + i\epsilon)((1 - x + x_1) k_1^- - B_1 + i\epsilon)} \right. $$

$$+ \left. \int \frac{dk_1^-}{2\pi i} \frac{F_2 \left( k_1^-, x, x_1, k_T^2, k_1T^2 \right)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x) k_1^- - A_2 + i\epsilon)((x_1 - 1) k_1^- - B_2 + i\epsilon)} \right\} \right. $$

$$\sim \frac{1 - x}{(\mu^2 - k_T^2)} \left\{ \int \frac{dk_1^-}{2\pi i} \frac{F_1 \left( k_1^-, x, x_1, k_T^2, k_1T^2 \right)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x) k_1^- - A_2 + i\epsilon)((1 - x + x_1) k_1^- - B_1 + i\epsilon)} \right. $$

$$+ \left. \int \frac{dk_1^-}{2\pi i} \frac{F_2 \left( k_1^-, x, x_1, k_T^2, k_1T^2 \right)}{(x_1 k_1^- - A_1 + i\epsilon)((x_1 - x) k_1^- - A_2 + i\epsilon)((x_1 - 1) k_1^- - B_2 + i\epsilon)} \right\} ,$$
where we use the quantities

\[ 2 A_1 = m_1^2 - k_{1T}^2, \]

\[ 2 A_2 = m^2 - (x - x_1) M^2 + \frac{x - x_1}{1 - x} (M_s^2 - k_T^2) - (k_T - k_{1T})^2, \]

\[ 2 B_1 = M_{s1}^2 - \frac{1 - x + x_1}{1 - x} (M_s^2 - k_T^2) - (k_T - k_{1T})^2, \]

\[ 2 B_2 = M_{s2}^2 - (1 - x_1) M^2 - k_{1T}^2. \]

Assuming that the numerator does not grow with \( k_1^- \) one can perform the \( k_1^- \) integrations using the generalized theta functions

\[ \Theta_{n_1 n_2 \ldots}^m (x_1, x_2, \ldots) = \int \frac{d\alpha}{2\pi i} \frac{\alpha^m}{(\alpha x_1 - 1 + i\epsilon)^{n_1}(\alpha x_2 - 1 + i\epsilon)^{n_2} \ldots} \]
\[ \Phi_G(x, x - x_1) = \int d^2 k_T \, d^2 k_{1T} \left\{ \left( \frac{1 - x) F_1(x, x_1, k_T, k_{1T})}{(\mu^2 - k_T^2) (\text{denom1})} \right) \times \left[ \left( x_1 (B_1 - A_2) - x B_1 - (1 - x) A_2 \right) x_1 \theta(x_1) ight. \\
+ \left. \left( (1 - x) A_1 + x_1 (A_1 - B_1) \right) (x_1 - x) \theta(x_1 - x) \right] \\
+ \left[ (x A_1 + x_1 (A_2 - A_1)) (1 - x + x_1) \theta(1 - x + x_1) \right] \\
+ \left( \frac{1 - x) F_2(x, x_1, k_T, k_{1T})}{(\mu^2 - k_T^2) (\text{denom2})} \right) \times \left[ \left( A_2 - x B_2 + x_1 (B_2 - A_2) \right) x_1 \theta(x_1) ight. \\
+ \left. \left( x_1 (A_1 - B_2) - A_1 \right) (x_1 - x) \theta(x_1 - x) \right] \\
+ \left[ (x A_1 + x_1 (A_2 - A_1)) (x_1 - 1) \theta(x_1 - 1) \right] \right\} \]
Gluonic Pole Contributions

Taking $x_1 \to 0$ we get the gluonic pole correlators, for distribution functions ($0 \leq x \leq 1$),

$$
\Phi_G(x, x) = -\int d^2k_T d^2k_1T \frac{(1 - x) F_1(x, 0, k_T, k_1T) \theta(1 - x)}{(\mu^2 - k_T^2)(x B_1 + (1 - x) A_2) A_1} .
$$

and for fragmentation functions ($x = 1/z \geq 1$)

$$
\Delta_G(x, x) = 0 .
$$

Gamberg, Mukherjee, Mulders, PRD(2008)

Result supported by Meissner, Metz, PRL (2009)

- So far we did model calculation with minimal assumption, need model independent proof
A Model Independent Analysis

\[ x < -1 \]

\[ P + k \]

\[ u \]

\[ -k \]

\[ P \]

\[ -k \]

\[ \]

\[ 0 < x < 1 \]

\[ P - k \]

\[ s \]

\[ k \]

\[ P \]

\[ k \]

\[ \]

\[ x < -1 \]

\[ -(k + P) \]

\[ u \]

\[ -k \]

\[ P \]

\[ -k \]

\[ \]

\[ x > 1 \]

\[ k - P \]

\[ s \]

\[ k \]

\[ P \]

\[ \]

\[ k = xP \]
**A Model Independent Analysis**

- $k^-$ and $k_1^-$-integrations in the quark-quark and quark-quark-gluon correlators lead to light-front correlators, for which time-ordering is irrelevant

  Landshoff and Polkinghorne (1972), Jaffe(1983)

- Integrating parton correlators over $k^-$ allows connecting them to a single anti-parton - hadron scattering four-point function $\mathcal{A}(k^2; s, u)$

- Depending on the value of $x$, the imaginary part of this amplitude represents the parton/antiparton distribution or fragmentation correlators

- Forward amplitude has singularities (cuts) for positive parton virtuality $k^2$ and in the Mandelstam variables, $s = (P - k)^2$ and $u = (P + k)^2$

  \[
  k^- = \frac{s + i\epsilon}{2(x - 1)} = \frac{u + i\epsilon}{2(x + 1)} = \frac{k^2 + i\epsilon}{2x},
  \]

- Integration contours can be wrapped around the $s$ and $u$-cuts for positive and negative $x$-values respectively for $|x| < 1$

- Masses, or transverse momenta of the partons do not matter and the support properties are valid for collinear and TMD PDFs
\[ k^- \text{ integration contours for the scattering amplitude} \]

\[ \Phi(x) = \theta(x) \theta(1-x) \text{Disc}_{[s]}A + \theta(-x) \theta(1+x) \text{Disc}_{[u]}A. \]

\[ \Delta(x) = \theta(x-1) \text{Disc}_{[s]}A + \theta(-1-x) \text{Disc}_{[u]}A. \]
For given positive $s$ (s-channel, $x > 0$) or positive $u$ (u-channel, $x < 0$) we define additional invariants $s_1 = (P \mp k \pm k_1)^2$ and $u_1 = (P \mp k_1)^2$. 
Multiparton scattering amplitude

\begin{align*}
\text{(a) } \Phi_G(x, x) \\
\text{(b) } \Delta_G(x, x)
\end{align*}

For given values of $s$ and $u$, there are cuts along $s_1 > 0$ ($u_1 < 0$) and $u_1 > 0$ ($s_1 < 0$) as well as for positive parton virtualities. Only when $x_1 \in [x-1, 1]$ (for positive $x$) or $x_1 \in [-1, x+1]$ (for negative $x$) the singularities in $s_1$ and $u_1$ are relevant.
The relevant singularities for $k_1^-$ are found from

\[ k_1^- = \frac{s_1 + i\epsilon}{2(x_1 - (x \mp 1))} + k^- = \frac{u_1 + i\epsilon}{2(x_1 \mp 1)} \]

\[ = \frac{k_1^2 + i\epsilon}{2x_1} = \frac{(k - k_1)^2 + i\epsilon}{2(x_1 - x)} + k^- \]

with $\mp$ referring to $s$- and $u$-channel cuts, respectively.

we consider the limit $x_1 \to 0$. For $0 < x < 1$, the value $x_1 = 0$ lies in the interval for which the $s_1$ and $u_1$ discontinuities can contribute.

For the case $x > 1$ only these parton virtualities matter.

For $\Delta G(x, x)$ the $k_1^-$ integration can be wrapped around the $k_1^2$ cut, which smoothly vanishes for $x_1 \to +0$

\[ \Phi_G(x, x) = \theta(x) \theta(1 - x) \text{Disc}_{[s,s_1]} A \]

\[ + \theta(-x) \theta(1 + x) \text{Disc}_{[u,u_1]} A, \]

\[ \Delta_G(x, x) = 0, \]
Conclusions

- Instead of doing a quantitative analysis involving details of a phenomenological model, we first limit ourselves to a spectral analysis within the spectator framework, in order to understand the basic features of these objects.

- Advantage: we are able to investigate only the soft parts at tree level and take the zero momentum limit of the gluon involved.

- Assumed that masses and vertices do not spoil our analysis → limits on the mass distributions of the spectators, use of vertices that cancel the bare-mass poles in the quark and gluon propagators and behavior of vertices that assures sufficient convergence of integrations.

- Result: within realistic assumptions, the gluonic pole contributions in the case of fragmentation correlators vanish whereas these contributions do not vanish for distribution correlators.
The result for fragmentation correlators at nonzero gluon momentum is nonzero.

We next do a model independent analysis based on the analytic properties of the scattering amplitude, and show the same result.

Result is important in the study of universality of distribution and fragmentation functions: the fact the gluonic pole matrix element is non-zero for distribution means that T-odd part there is process dependent.

That it is zero for fragmentation means that the T-odd part of the fragmentation correlator is process independent.

Gluonic pole cross sections need not be considered for fragmenting final-state partons, but only for the distribution functions involving initial-state partons.