Diffraction in nuclei

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Why interest in diffractive eA
More sensitive to nonlinear/black disk regime than inclusive
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Lot of work — 90’s, fixed target experiments, will not attempt to review here
Outline

Why interest in diffractive eA
More sensitive to nonlinear/black disk regime than inclusive

Lot of work –90’s, fixed target experiments, will not attempt to review here

- Classifications, computing things in the dipole picture
- Coherent exclusive Discussion based on Kowalski, Caldwell -09
- Incoherent $J/\Psi$ etc. Work together with H. Mäntysaari
- Diffractive structure functions Kowalski, T. L., Marquet and Venugopalan, -08
- Problem of $b_T/t$-dependence in BK
DIS in dipole frame

Use:
- $S$-matrix real
- optical theorem
- \[
\psi_{\gamma, T} \sim K_{0,1} \left( \sqrt{z(1-z)} Q |r_T| \right)
\]
- momentum scale $Q^2 \sim 1/r_T^2$
- Diffractive: $t$ is FT of $b_T$. 
Observables from dipole cross section

Predictive power!

\[ \sigma_{\text{dip}}(x, r_T, \Delta_T) = \int d^2 b_T \frac{d^2 \sigma_{\text{dip}}(x, r_T, b_T)}{d^2 b_T} e^{ib_T \cdot \Delta_T}, \quad \Delta_T^2 = -t \]

Inclusive

\[ \sigma_{L, T}^{\gamma^* p} = \int d^2 r_T \int dz \left| \psi_{L, T}^{\gamma}(Q^2, r_T, z) \right|^2 \sigma_{\text{dip}}(x, r_T) \]
Observables from dipole cross section

Predictive power!

\[ \sigma_{\text{dip}}(x, r_T, \Delta_T) = \int d^2 b_T \frac{-iA}{d^2 b_T} \frac{d^2 \sigma_{\text{dip}}(x, r_T, b_T)}{d^2 b_T} e^{ib_T \cdot \Delta_T}, \quad \Delta_T^2 = -t \]

\[ \nabla \]

Inclusive **diffractive** (elastic dipole–target)

\[ \sigma_{\gamma^* p}^{L, T} = \int d^2 r_T \int dz \left| \psi_{\gamma, T}^L(Q^2, r_T, z) \right|^2 \sigma_{\text{dip}}(x, r_T) \]

\[ \frac{\sigma_{\text{D, tot}}^{L, T}}{dt} = \frac{1}{16\pi} \int d^2 r_T \int dz \left| \psi_{\gamma, T}^L(Q^2, r_T, z) \right|^2 \sigma_{\text{dip}}^2(x, r_T, \Delta_T) \]
Observables from dipole cross section

Predictive power!

\[ \sigma_{\text{dip}}(x, r_T, \Delta_T) = \int d^2b_T \frac{d^2\sigma_{\text{dip}}(x, r_T, b_T)}{d^2b_T} e^{ib_T \cdot \Delta_T}, \quad \Delta_T^2 = -t \]

\[
\sigma_{\gamma^*p}^{L,T} = \int d^2r_T \int dz \left| \psi_{\gamma,L,T}(Q^2, r_T, z) \right|^2 \sigma_{\text{dip}}(x, r_T) \\
\frac{\sigma_{D,\text{tot}}^{L,T}}{dt} = \frac{1}{16\pi} \int d^2r_T \int dz \left| \psi_{\gamma,L,T}(Q^2, r_T, z) \right|^2 \sigma_{\text{dip}}^2(x, r_T, \Delta_T) \\
\sigma_{L,T}^{D,V} = \frac{1}{16\pi} \left| \int d^2r_T \int dz \left( \psi_{\gamma} \psi_{*V} \right)_{L,T} \sigma_{\text{dip}}(x, r_T, \Delta_T) \right|^2
\]

Inclusive diffractive (elastic dipole–target) exclusive diff.
Coherent vs. incoherent

- Nucleon positions $b_{Ti}$, $i = 1 \ldots A$
- Average

$$\langle \cdot \rangle_N = \int \prod_{i=1}^{A} d^2b_{Ti} |\varphi(b_{T1} \ldots b_{TA})|^2$$

nuclear N-body wavef.
Coherent vs. incoherent

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- Average

$$\langle \cdot \rangle_N = \int \prod_{i=1}^{A} d^2 b_{Ti} |\varphi(b_{T1} \ldots b_{TA})|^2$$

Coherent: Nucleus intact

Incoherent: Nucleus disintegrates
Coherent vs. incoherent

- Nucleon positions $b_{Ti}, i = 1 \ldots A$
- Average

\[
\langle \cdot \rangle_N = \int \prod_{i=1}^{A} d^2 b_{Ti} \left| \varphi(b_{T1} \ldots b_{TA}) \right|^2
\]

**Coherent:**
Nucleus intact

\[
\left| \langle A(\Delta T) \rangle_N \right|^2
\]
Average gluon density

**Incoherent:**
Nucleus disintegrates

\[
\left( \langle |A(\Delta T)|^2 \rangle_N - \left| \langle A(\Delta T) \rangle_N \right| \right)^2
\]
fluctuations
Coherent vs. incoherent in practice

Coherent: $\langle \sigma_{\text{dip}} (\Delta_T) \rangle_N$ smooth

$\implies$ FT at small scale $- t \sim 1/R_A^2$
Coherent vs. incoherent in practice

Coherent: $\langle \sigma_{\text{dip}}(\Delta_T) \rangle_N$ smooth
$\implies$ FT at small scale $-t \sim 1/R_A^2$

Incoherent: FT $\sigma_{\text{dip}}(\Delta_T)$ before $\langle \cdot \rangle_N$
$\implies$ lumpy, larger $-t \sim 1/R_p^2$
Coherent vs. incoherent in practice

Coherent: $\langle \sigma_{\text{dip}}(\Delta T) \rangle_N$ smooth
$\implies$ FT at small scale $-t \sim 1/R^2_A$

Incoherent: FT $\sigma_{\text{dip}}(\Delta T)$ before $\langle \cdot \rangle_N$
$\implies$ lumpy, larger $-t \sim 1/R^2_p$

Result: $t$-distributions

\[ t = -\Delta T^2 \]

\begin{itemize}
  \item $\text{Ca, with breakup}$
  \item $\text{Ca, no breakup}$
  \item $p$ (x 40)
\end{itemize}

\[ \frac{d\sigma}{dt} \text{[fm}^2/\text{GeV}^2] \]

\begin{axis}[
    width=0.7\textwidth,
    xlabel=-t [GeV$^2$],
    ylabel=d\sigma/dt [fm$^2$/GeV$^2$],
    xtick={0,0.1,0.2,0.3,0.4,0.5},
    ytick={10^{-5},10^{-4},10^{-3},10^{-2},10^{-1}},
    legend entries={Ca, with breakup, Ca, no breakup, p (x 40)},
]
\end{axis}
Nuclear structure from $t$-distribution

Kowalski, Caldwell

Measure coherent $t$-distribution at EIC!
Is such a resolution in $t$ possible?

Kowalski, Caldwell

- Measure outgoing electron (at least at $Q^2 = 0$) very accurately
- Reconstruct $J/\psi$ with $\sigma_{p_T} < 1$MeV
- Do not need to measure recoil nucleus $p_T$
Incoherent diffraction

Approach from dilute end:

- At high $Q^2$ everything is linear
- Lower $Q^2 \sim Q_s^2 \implies$ gluon saturation, nonlinearity!
- Saturation more prominent in nuclei
Incoherent diffraction

Approach from dilute end:
- At high $Q^2$ everything is linear
- Lower $Q^2 \sim Q_s^2 \implies$ gluon saturation, nonlinearity!
- Saturation more prominent in nuclei

Approach from hadronic end:
- Start from hadronic scattering, parametizations respecting unitarity
- Increase $Q^2$, objects become more compact
- Discover **color transparency**
Motivation:

- Baseline for nuclear dependence
- small $x$ limit ($\infty$ coh. length), simplicity
- Parameter-free “clean” estimates

! No doubt more refined treatments already exist
Motivation:

- Baseline for nuclear dependence
- Small $x$ limit ($\infty$ coh. length), simplicity
- Parameter-free “clean” estimates

Assume:

- Independent scatterings off nucleons:
  \[ S_A(r_T, b_T) = \prod_{i=1}^{A} S_p(r_T, b_T - b_{Ti}) \]
- Factorized $b_T$ dependence:
  \[ 1 - S_p(b_T, r_T, x) = T_p(b_T) N(r_T, x) \]
- Neglect NN-correlations, just $T_A(b_T)$
- Use existing HERA-fitted parametrizations.
Diffractive VM cross section

Result

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi} \int d^2 r_T \frac{dz}{4\pi} \frac{(\psi_V^* \psi) (r, z, Q)}{\det A} \times \int d^2 r'_T \frac{dz'}{4\pi} \frac{(\psi_V^* \psi) (r', z', Q)}{\det A} \times |A_{q\bar{q}}|^2 (x, r, r', \Delta T)
\]

\[
|A_{q\bar{q}}|^2 = 16\pi B_p \int d^2 b_T \sum_{i=1}^{A} \binom{A}{n} \times \exp \left\{ -B_p \Delta T^2 / n \right\} \exp \left\{ -2\pi A T_A(b) \left[ N(r) + N(r') \right] \right\} \times \left( \frac{\pi B_p N(r) N(r') T_A(b)}{1 - 2\pi B_p T_A(b) \left[ N(r) + N(r') \right]} \right)^n
\]
Diffractive VM cross section

Result

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi} \int d^2r_T \frac{dz}{4\pi} (\psi^*_V \psi)(r, z, Q) \times \int d^2r'_T \frac{dz'}{4\pi} (\psi^*_V \psi)(r', z', Q) \times |A_{q\bar{q}}|^2 (x, r, r', \Delta T)
\]

\[
|A_{q\bar{q}}|^2 = 16\pi B_p \int d^2b_T \sum_{i=1}^{A} \binom{A}{n} \times \exp \left\{ -B_p \Delta T^2 / n \right\} \exp \left\{ -2\pi AT_A(b) \left[ \mathcal{N}(r) + \mathcal{N}(r') \right] \right\} \propto \left( \frac{\pi B_p \mathcal{N}(r) \mathcal{N}(r') T_A(b)}{1 - 2\pi B_p T_A(b) \left[ \mathcal{N}(r) + \mathcal{N}(r') \right]} \right)^n
\]

(Really only \( n = 1 \) contributes)
Diffractive VM cross section

Result

\[
\frac{d\sigma}{dt} = \frac{1}{16\pi} \int d^2 r_T \frac{dz}{4\pi} (\psi_V^* \psi)(r, z, Q)
\]

\[
\times \int d^2 r_T' \frac{dz'}{4\pi} (\psi_V^* \psi)(r', z', Q)
\]

\[
\times \left| A_{q\bar{q}} \right|^2 (x, r, r', \Delta T)
\]

\[
\left| A_{q\bar{q}} \right|^2 = 16\pi B_p \int d^2 b_T \sum_{i=1}^{A} \binom{A}{n} \text{ nucl. suppression}
\]

\[
\times \exp \left\{ -B_p \Delta T^2 / n \right\} \exp \left\{ -2\pi A T_A(b) \left[ N(r) + N(r') \right] \right\}
\]

\[
\times \left( \frac{\pi B_p N(r) N(r') T_A(b)}{1 - 2\pi B_p T_A(b) [N(r) + N(r')]^n} \right)
\]
$Q^2$-dependence

“Transparency”

![Graph showing $d\sigma^A / dt / d\sigma^p / dt$ as a function of $Q^2$ at $t = 0.5$ GeV$^2$. The graph includes curves for $\text{ipsat, } x = 0.0001$, $\text{ipsat, } x = 0.01$, $\text{ipsat_nonsatp, } x = 0.0001$, $\text{ipsat_nonsatp, } x = 0.01$, $\text{iim, } x = 0.0001$, and $\text{iim, } x = 0.01$. Each curve represents a different parameter setting, with the $Q^2$ scale ranging from $10^0$ to $10^2$ GeV$^2$. The graph demonstrates the dependence of $d\sigma^A / dt / d\sigma^p / dt$ on $Q^2$.}
Inclusive diffraction: structure function \( F_2^D(x_P, \beta, Q^2) \)

\[
\beta = \frac{Q^2}{M_X^2 + Q^2} \quad x_P = \frac{M_X^2 + Q^2}{W^2 + Q^2} \quad x = \beta x_P
\]

\[
x_P F_2^D = x_P F_{T,q\bar{q}}^D + x_P F_{L,q\bar{q}}^D + x_P F_{T,q\bar{g}g}^D + \text{higher Fock states}
\]

Essential regimes:

- \( \beta \ll 1 \): dominated by higher Fock (\( q\bar{q}g \) etc.)
- \( \beta \sim 0.5 \): dominated by transverse \( q\bar{q} \)
- \( \beta \to 1 \): longitudinal \( q\bar{q} \).

(Proton, \( Q^2 = 5 \text{GeV}^2 \), \( x_P = 10^{-3} \))
$b_T$-dependence of different components

Dominant impact parameters different

\[ b_{\text{diff}}(q\bar{q}) < b_{\text{incl}} < b_{\text{diff}}(q\bar{q}g) \]

Integrand vs. $b_T$ for

\[ F_{2,b}^{D} \quad \beta = 0.9 \]
\[ F_{2,T}^{D} \quad \beta = 0.5 \]
\[ F_{2,qg}^{D} \beta = 0 \text{ (GBW)} \]
\[ F_{2,qg}^{D} \beta = 0 \text{ (MS)} \]

$Q^2 = 100 \text{GeV}^2$

$Q^2 = 1 \text{GeV}^2$

($x_F = 10^{-3}$)
\[ F_2^D \beta\text{-dependence} \]

Ratios of nucleus/proton

\[ \frac{F_{2A}(x)}{AF_{2p}^D(x)} \text{ with } x = L, T(q\bar{q}), q\bar{q}g, \text{ tot.} \]

- \( \beta \ll 1: q\bar{q}g \) strongly suppressed (black disk limit)
- \( \beta \sim 0.5: \) transverse \( q\bar{q} \) enhanced.
- \( \beta \to 1: \) longitudinal \( q\bar{q} \) very much enhanced.

(\( \text{Au at } Q^2 = 5\text{GeV}^2 \) and \( x_p = 10^{-3} \))
$F^D_2$: $Q^2$-dependence

Nuclei have smaller $Q^2/Q_s^2$ at same $x_P$, $Q^2$

- Nuclear enhancement of $F^D_2$'s grows with $Q^2$

- Non breakup = coherent
- Breakup = coherent + incoherent

(Au at $x_P = 10^{-3}$)
$F^D_2$ $A$-dependence

Small $A$ more dilute than $p$: coherent diff. suppressed

(Different components at $x_F = 10^{-3}$ and $Q^2 = 5$GeV$^2$)
$F_2^D$ $A$-dependence

Small $A$ more dilute than $p$: coherent diff. suppressed

(Different components at $x_F = 10^{-3}$ and $Q^2 = 5\text{GeV}^2$)

(Different nuclei vs $\beta$)
\( b_T \)-integral for \( t \)-dependence in incoherent is difficult

E.g. vector meson production

\[
\frac{\sigma_{D,V}^{L,T}}{dt} = \frac{1}{16\pi} \left| \int d^2 r_T \int dz \left( \psi \gamma \psi^* V \right)_{L,T} (Q^2, r_T, z) \right| ^2
\]

\[
\int d^2 b_T \sigma_{dip}(x, r_T, b_T) e^{i \Delta T \cdot b_T} \right| ^2
\]
\[ b_T - \text{integral for } t - \text{dependence in incoherent is difficult} \]

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\]

\[
\int d^2b_T \sigma_{\text{dip}}(x, r_T, b_T) e^{i \Delta \cdot b_T} \right|^2
\]

- Coherent: \( \int dr \int dz \int db J_0(b|\Delta_T|) \rightarrow 3 \text{ integrals}, \)

\( b \)-dependence smooth
$b_T$-integral for $t$-dependence in incoherent is difficult

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\]

- **Coherent:** \( \int dr \int dz \int db J_0(b|\Delta_T|) \rightarrow 3 \) integrals, 
  \( b \)-dependence smooth

- **Incoherent:** \( \int dr \int dz \int d^2b_T e^{i \Delta \cdot b_T} \rightarrow 4 \) integrals, 
  \( b_T \)-dependence lumpy
**b**-integral for \( t \)-dependence in incoherent is difficult

E.g. vector meson production

\[
\frac{\sigma_{D,V}^{L,T}}{dt} = \frac{1}{16\pi} \left| \int d^2r_T \int dz \left( \psi \gamma \psi^* V \right)_{L,T} (Q^2, r_T, z) \right|^2 
\int d^2b_T \sigma_{dip}(x, r_T, b_T) e^{i \Delta_T \cdot b_T} \right| ^2
\]

▶ Coherent: \( \int dr \int dz \int db J_0(b|\Delta_T|) \rightarrow 3 \) integrals, 
\( b \)-dependence smooth

▶ Incoherent: \( \int dr \int dz \int d^2b_T e^{i \Delta_T \cdot b_T} \rightarrow 4 \) integrals, 
\( b_T \)-dependence lumpy

▶ Integrated over \( t \) easier \((\int d^2\Delta_T): \int d^2b_T (\int dr dz \cdots)^2 \)

\( (4 \) integrals, but pos. def., non oscillatory integrand in \( b_T \))
$b_T$-dependence beyond Glauber

Evolution and independent $\gamma^* N$ scatterings don’t commute.

\[ p : \quad S_p(b_T) \sim e^{-Q^p_s(b_T)^2 r^2} \]

\[ A : \quad S_A(b_T) \sim e^{-AT_A(b_T) Q^p_s r^2} \]

Origin of ⊥ shape confinement, HE evolution weak coupling:
how to combine?
$b_T$-dependence beyond Glauber

Evolution and independent $\gamma^* N$ scatterings don’t commute.

Evolution $x \rightarrow 0$

\[ p: \quad S_p(b_T) \sim e^{-Q_s^p(b_T)^2 r^2} \quad \text{►} \quad S_p(b_T) \sim e^{-(Q_s^p(b_T)^2 r^2) \gamma} \]

\[ \text{▼} \]

\[ A: \quad S_A(b_T) \sim e^{-AT_A(b_T) Q_s^p r^2} \quad \text{►} \quad S_A(b_T) \sim e^{-(AT_A(b_T) Q_s^p r^2) \gamma} \]

Origin of $\perp$ shape confinement,

HE evolution weak coupling:

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Evolution and independent $\gamma^* N$ scatterings don’t commute.

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\[ p : \quad S_p(b_T) \sim e^{-Q_s^p(b_T)^2r^2} \quad \blacktriangleleft \quad S_p(b_T) \sim e^{-(Q_s^p(b_T)^2r^2)\gamma} \]

\[ A : \quad S_A(b_T) \sim e^{-AT_A(b_T)Q_s^p r^2} \quad \blacktriangleleft \quad S_A(b_T) \sim e^{-(AT_A(b_T)Q_s^p r^2)\gamma} \]

 Origin of $\perp$ shape confinement,
HE evolution weak coupling:
how to combine?
Conclusions

- Nuclear diffraction: very sensitive to gluon saturation
- Coherent: experimental challenge, theory clean; measure averages over transverse plane
- Incoherent: measure nucleon fluctuations
- Inclusive diffraction: integrate over $t$
Conclusions

- Nuclear diffraction: very sensitive to gluon saturation
- Coherent: experimental challenge, theory clean; measure averages over transverse plane
- Incoherent: measure nucleon fluctuations
- Inclusive diffraction: integrate over $t$

Further questions:
- $t/b$-dependence in nuclei

Thank you.