Tensor-polarized quark distributions in a spin-one hadron

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Summary and Prospects
Situation

- **Spin structure of the spin-1/2 nucleon**

  *Nucleon spin puzzle:* This issue is not solved yet, but it is rather well studied theoretically and experimentally.

- **Spin-1 hadrons (e.g. deuteron)**

  There are some theoretical studies especially on tensor structure in electron-deuteron deep inelastic scattering.

  ➔ HERMES experimental results ➔ JLab proposal

  **No investigation has been done for**

  *hadron (p, π, ...) - polarized deuteron processes.*

  ➔ hadron facility (J-PARC, RHIC, COMPASS, GSI, ...) experiment?
Purposes of studying polarized deuteron reactions

(1) Neutron information
   • Polarized PDFs in the neutron

(2) New structure functions
   • Tensor structure function $b_1$
     → (1) Test of our hadron description in another spin
     (2) Description of tensor structure by quark-gluon degrees of freedom

(3) Asymmetries in polarized light-antiquark distributions
   • $\Delta \bar{u} / \Delta \bar{d}$, $\Delta_T \bar{u} / \Delta_T \bar{d}$
Nucleon spin

Naïve Quark Model

Almost none of nucleon spin is carried by quarks!

Nucleon spin crisis!? 

“old” standard model

Tensor structure $b_1$ (e.g. deuteron)

only S wave $b_1 = 0$

standard model $b_1 \neq 0$

Tensor-structure crisis!? 

Sea-quarks and gluons?  
Orbital angular momenta?

$b_1$ experiment $\neq b_1$ “standard model”
Electron scattering from a spin-1 hadron


\[
W_{\mu\nu} = -F_1 g_{\mu\nu} + F_2 \frac{p_\mu p_\nu}{v} + g_1 \frac{i}{v} \epsilon_{\mu\nu\lambda\sigma} q^\lambda s^\sigma + g_2 \frac{i}{v^2} \epsilon_{\mu\nu\lambda\sigma} q^\lambda \left( p \cdot q s^\sigma - s \cdot q p^\sigma \right) + b_1 r_{\mu\nu} + \frac{1}{6} b_2 \left( s_{\mu\nu} + t_{\mu\nu} + u_{\mu\nu} \right) + \frac{1}{2} b_3 \left( s_{\mu\nu} - u_{\mu\nu} \right) + \frac{1}{2} b_4 \left( s_{\mu\nu} + t_{\mu\nu} \right)
\]

\(n = p \cdot q, \quad \kappa = 1 + M^2 Q^2 / v^2, \quad E^2 = -M^2, \quad s^\sigma = -\frac{i}{M^2} \epsilon^{\sigma\alpha\beta\gamma} E^*_\alpha E^*_\beta p_\gamma\)

\[
r_{\mu\nu} = \frac{1}{v^2} \left( q \cdot E^* p_\mu E_\nu + q \cdot E^* p_\nu E_\mu + q \cdot E p_\mu E^*_\nu + q \cdot E p_\nu E^*_\mu - \frac{4}{3} v p_\mu p_\nu \right)
\]

\[
t_{\mu\nu} = \frac{1}{2v^2} \left( q \cdot E p_\mu E^*_\nu + q \cdot E^* p_\mu E_\nu + q \cdot E p_\nu E^*_\mu + q \cdot E p_\nu E^*_\mu - \frac{4}{3} v p_\mu p_\nu \right)
\]

\[
u_{\mu\nu} = \frac{1}{v} \left( E^*_\mu E_\nu + E^*_\nu E_\mu + 2M^2 g_{\mu\nu} - 2 p_\mu p_\nu \right)
\]

Note: Obvious factors from \(q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0\) are not explicitly written. \(E^\mu = \) polarization vector

\(b_1, \ldots, b_4\) tems are defined so that they vanish by spin average.

\(b_1, b_2\) tems are defined to satisfy \(2xb_1 = b_2\) in the Bjorken scaling limit.

\(2xb_1 = b_2\) in the scaling limit \(\sim O(1)\)

\(b_3, b_4 = \) twist-4 \(\sim \frac{M^2}{Q^2}\)
**Structure Functions**

\[ F_1 \propto \langle d\sigma \rangle \]

\[ g_1 \propto d\sigma(\uparrow, +1) - d\sigma(\uparrow, -1) \]

\[ b_1 \propto d\sigma(0) - \frac{d\sigma(+1) + d\sigma(-1)}{2} \]

note: \( \sigma(0) - \frac{\sigma(+1) + \sigma(-1)}{2} = 3\langle \sigma \rangle - \frac{3}{2} [\sigma(+1) + \sigma(-1)] \)

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**Parton Model**

\[ F_1 = \frac{1}{2} \sum_i e_i^2 (q_i + \bar{q_i}) \]

\[ q_i = \frac{1}{3} (q_i^{+1} + q_i^0 + q_i^{-1}) \]

\[ g_1 = \frac{1}{2} \sum_i e_i^2 (\Delta q_i + \Delta \bar{q}_i) \]

\[ \Delta q_i = q_i^{+1} - q_i^{-1} \]

\[ b_1 = \frac{1}{2} \sum_i e_i^2 (\delta_T q_i + \delta_T \bar{q}_i) \]

\[ \delta_T q_i = q_i^0 - \frac{q_i^{+1} + q_i^{-1}}{2} \]
Personal studies

- **Sum rule for $b_1$**

- **Polarized proton-deuteron Drell-Yan: General formalism**

- **Polarized proton-deuteron Drell-Yan: Parton model**

- **Extraction of $\Delta\bar{u}/\Delta d$ and $\Delta_T\bar{u}/\Delta_T d$ from polarized pd Drell-Yan**

- **Projections to $b_1, \ldots, b_4$ from $W^{\mu\nu}$**

- **Tensor-polarized distributions from HERMES data**

Motived by the following works:

- **Hoodbhoy-Jaffe-Manohar (1989)**
- **Polarized deuteron acceleration at RHIC:**
- **HERMES measurement on $b_1$ (2005)**
- **Future possibilities at JLab, J-PARC, RHIC, …**

This talk

- **JLab proposal in preparation**
Analysis of HERMES data to obtain tensor-polarized quark distributions


Purposes

• Understanding of current situation on tensor-polarized distributions
• Useful for future proposals at JLab, J-PARC, …
• Test of theoretical model estimates
• Description of tensor structure in terms of quark-gluon degrees of freedom
• Understanding of hadron spins with orbital angular momenta

…
HERMES measurements on $b_1$

A. Airapetian et al. (HERMES), PRL 95 (2005) 242001.

$27.6 \text{ GeV/c} \leftrightarrow 0$

positron $\rightarrow$ deuteron

$b_1$ measurements in the kinematical region
$0.01 < x < 0.45$, $0.5 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$
**Constraint on valence-tensor polarization (sum rule)**

\[
\int dx \left\{ \begin{array}{c}
\text{valence quark} \\
\text{parton} \\
\text{parton}
\end{array} \right\} \leftrightarrow \begin{array}{c}
\text{valence quark} \\
\text{parton}
\end{array}
\]

\[
\int dx b_i^D(x) = \frac{5}{18} \int dx [\delta_T u_v + \delta_T d_v] + \frac{1}{18} \int dx [8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D]
\]

**Elastic amplitude in a parton model**

\[
\Gamma_{H,H} = \langle p, H | J_0(0) | p, H \rangle = \sum_i e_i \int dx \left[ q_i^H + q_i^{H*} - \bar{q}_i^H - \bar{q}_i^{H*} \right]
\]

\[
\frac{1}{2} \left[ \Gamma_{0,0} - \frac{1}{2} (\Gamma_{1,1} + \Gamma_{-1,-1}) \right] = \frac{1}{3} \int dx [\delta_T u_v(x) + \delta_T d_v(x)]
\]

**Macroscopically**

\[
\Gamma_{0,0} = \lim_{t \to 0} F_c(t) - \frac{t}{3} F_\bar{q}(t), \quad \Gamma_{1,1} = \lim_{t \to 0} F_c(t) + \frac{t}{6} F_\bar{q}(t)
\]

\[
\frac{1}{2} \left[ \Gamma_{0,0} - \frac{1}{2} (\Gamma_{1,1} + \Gamma_{-1,-1}) \right] = -\lim_{t \to 0} \frac{t}{2} F_\bar{q}(t)
\]

\[
\int dx b_i^D(x) = \frac{5}{9} \int dx \left[ \Gamma_{0,0} \left( \Gamma_{1,1} + \Gamma_{-1,-1} \right) \right] + \frac{1}{18} \int dx [8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D]
\]

\[
= -\frac{5}{6} \lim_{t \to 0} t F_\bar{q}(t) + \frac{1}{18} \int dx [8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D]
\]

\[
= 0 \text{ (valence)} + \frac{1}{18} \int dx [8\delta_T \bar{u}^D + 2\delta_T \bar{d}^D + \delta_T \bar{s}^D]
\]

**Constraint on tensor-polarized valence quarks:**

\[
\int dx \delta_T q_v(x) = 0
\]
**Functional form of parametrization**

Assume flavor-symmetric antiquark distributions: \( \delta \bar{q}^D = \delta \bar{u}^D = \delta \bar{d}^D = \delta s^D = \delta \bar{s}^D \)

\[
b_1^D(x)_{LO} = \frac{1}{18} \left[ 4 \delta_T u_v^D(x) + \delta_T d_v^D(x) + 12 \delta_T \bar{q}^D(x) \right]
\]

At \( Q_0^2 = 2.5 \text{ GeV}^2 \), \( \delta_T q_v^D(x,Q_0^2) = \delta_T w(x)q_v^D(x,Q_0^2) \), \( \delta_T \bar{q}^D(x,Q_0^2) = \alpha_q \delta_T w(x)\bar{q}^D(x,Q_0^2) \)

Certain fractions of quark and antiquark distributions are tensor polarized and such probabilities are given by the function \( \delta_T w(x) \) and an additional constant \( \alpha_{\bar{q}} \) for antiquarks in comparison with the quark polarization.

\[
b_1^D(x,Q_0^2)_{LO} = \frac{1}{18} \left[ 4 \delta_T u_v^D(x,Q_0^2) + \delta_T d_v^D(x,Q_0^2) + 12 \delta_T \bar{q}^D(x,Q_0^2) \right]
\]

\[
= \frac{1}{36} \delta_T w(x) \left[ 5 \{ u_v(x,Q_0^2) + d_v(x,Q_0^2) \} + 4 a_{\bar{q}} \left\{ 2 \bar{u}(x,Q_0^2) + 2 \bar{d}(x,Q_0^2) + s(x,Q_0^2) + \bar{s}(x,Q_0^2) \right\} \right]
\]

\[
\delta_T w(x) = ax^b (1-x)^c (x_0 - x)
\]

Two types of analyses

Set 1: \( \delta_T \bar{q}^D(x) = 0 \) Tensor-polarized antiquark distributions are terminated \( (\alpha_{\bar{q}} = 0) \),

Set 2: \( \delta_T \bar{q}^D(x) \neq 0 \) Finite tensor-polarized antiquark distributions are allowed \( (\alpha_{\bar{q}} \neq 0) \).
Theoretical background for the parametrization

(1) Tensor-polarized valence quarks: $\int dx \delta_r q_v(x) = 0$

(2) Standard convolution approach

Convolution model: $A_{\h H, hH} (x) = \int \frac{dy}{y} \sum_s f_s^H (y) \hat{A}_{hs,hs} (x / y) \equiv \sum_s f_s^H (y) \otimes \hat{A}_{hs,hs} (y)$

$A_{\h H, h' H'} = \varepsilon_{\mu}^* W_{\mu \nu}^{H H'} \varepsilon_{\nu}^y$

$b_1 = A_{+0,+0} - \frac{A_{+0,+0} + A_{-0,-0}}{2}$

$\hat{A}_{+\uparrow,\uparrow} = F_1 - g_1$

$b_1 = A_{+0,+0} - \frac{A_{+0,+0} + A_{+0,+0}}{2} = \int \frac{dy}{y} \sum_s \left[ f^0(y) - \frac{f^+(y) + f^-(y)}{2} \right] F_1 (x / y)$

where $f^H (y) \equiv f^H_+ (y) + f^H_- (y)$

Momentum distribution of a nucleon: $f^H (y) = \int d^3 p |\phi^H (\vec{p})|^2 \delta \left( y - \frac{E + p_z}{M} \right)$

D-state admixture: $\phi^H (\vec{p}) = \phi^H (\vec{p})^{t=0} \cos \alpha + \phi^H (\vec{p})^{t=2} \sin \alpha$

$= \cos \alpha \psi_0(p) Y_00(\hat{p}) \chi_H + \sin \alpha \sum_{m_L} \langle 2m_L : 1m_s | 1H \rangle \psi_2(p) Y_{2m_L} (\hat{p}) \chi_{m_s}$

Numerical estimates indicate the oscillatory function with $\int dx b_1 (x) = 0$. 

\( x \)
Results

Two types of fit results:

- set-1: $\chi^2 / \text{d.o.f.} = 2.83$
  Without $\delta_T q$, the fit is not good enough.

- set-2: $\chi^2 / \text{d.o.f.} = 1.57$
  With finite $\delta_T q$, the fit is reasonably good.

Obtained tensor-polarized distributions $\delta_T q(x), \delta_T \bar{q}(x)$ from the HERMES data.

→ They could be used for
  - experimental proposals,
  - comparison with theoretical models.

Finite tensor polarization for antiquarks:

$$\int_0^1 \! dx b_1(x) = 0.058$$

$$= \frac{1}{9} \int_0^1 \! dx \left[ 4\delta_T \bar{u}(x) + \delta_T \bar{d}(x) + \delta_T \bar{s}(x) \right]$$
Summary

(1) The tensor-polarized distributions: $\delta_T q(x)$, $\delta_T \bar{q}(x)$ were obtained from the HERMES data on $b_1$.

(2) Finite tensor polarization was obtained for antiquarks: $\int dx \delta_T \bar{q}(x) \neq 0$.

Prospects

Future experimental possibilities at JLab, EIC, J-PARC, RHIC, COMPASS, GSI-FAIR, ...

Experimental proposal is considered at JLab.

Unpolarized proton+ polarized deuteron

Spin asymmetry in $p + \vec{d} \rightarrow \mu^+ \mu^- + X$

$$A_{UQ_0} = \frac{\sum_a e_a^2 \left[ q_a(x_A) \delta_T \bar{q}_a(x_B) + \bar{q}_a(x_A) \delta_T q_a(x_B) \right]}{\sum_a e_a^2 \left[ q_a(x_A) \bar{q}_a(x_B) + \bar{q}_a(x_A) q_a(x_B) \right]}$$

Unique advantage of J-PARC ($\delta \bar{q}$ measurement)

$$\int dx \ b_1^D(x) = 0 + \frac{1}{9} \int dx \ \delta_T \bar{q}(x)$$

$$A_{UQ_0} (\text{large } x_F) \approx \frac{\sum_a e_a^2 q_a(x_A) \delta \bar{q}_a(x_B)}{\sum_a e_a^2 q_a(x_A) \bar{q}_a(x_B)}$$

Gottfried: $\int \frac{dx}{x} \left[ F_2^n(x) - F_2^u(x) \right] = \frac{1}{3} + \frac{2}{3} \int \left[ \bar{u} - \bar{d} \right]$
The End

The End