Constraining the CGC in eA in order to predict AA

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Outline

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Links between eA and AA

Saturation and CGC
Factorization in AA
Factorization in eA
Saturation domain

\[ \log(x^{-1}) \]

\[ \log(Q^2) \]

\[ \Lambda_{\text{QCD}} \]
Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area:

\[ \rho \sim \frac{x G_A(x, Q^2)}{\pi R_A^2} \]

Recombination cross-section:

\[ \sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2} \]

Recombination happens if \( \rho \sigma_{gg \rightarrow g} \gtrsim 1 \), i.e. \( Q^2 \lesssim Q_s^2 \), with:

\[ Q_s^2 \sim \frac{\alpha_s x G_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}} \]

Note: At a given energy, the saturation scale is larger for a nucleus (for \( A = 200, A^{1/3} \approx 6 \))
CGC: Degrees of freedom

CGC = effective theory of small x gluons

- The fast partons \((k^+ > \Lambda^+)\) are frozen by time dilation
  - described as static color sources on the light-cone:

\[
J^\mu = \delta^{\mu+} \rho(x^-, \vec{x}_\perp) \quad (0 < x^- < 1/\Lambda^+)
\]

- Slow partons \((k^+ < \Lambda^+)\) cannot be considered static over the time-scales of the collision process
  - must be treated as standard gauge fields
  - eikonal coupling to the current \(J^\mu : A_\mu J^\mu\)

- The color sources \(\rho\) are random, and described by a distribution \(W_{\Lambda^+}[\rho]\), with \(\Lambda^+\) the longitudinal momentum that separates “soft” and “hard”
CGC: Degrees of freedom

- CGC effective theory with cutoff at the scale $\Lambda_0^+$:

\[
S = -\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} + \int (J_{1}^{\mu} + J_{2}^{\mu}) A_{\mu}
\]

\[S_{YM}\quad \text{fast partons}\]
CGC: renormalization group evolution

Independence w.r.t $\Lambda^+$ → evolution equation (JIMWLK):

\[
\frac{\partial W_{\Lambda^+}}{\partial \ln(\Lambda^+)} = \mathcal{H} \ W_{\Lambda^+}
\]

\[
\mathcal{H} = \frac{1}{2} \int d\vec{x}_\perp d\vec{y}_\perp \ \frac{\delta}{\delta \alpha(\vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \ \frac{\delta}{\delta \alpha(\vec{x}_\perp)}
\]

where \(-\partial_\perp^2 \alpha(\vec{x}_\perp) = \rho(1/\Lambda^+, \vec{x}_\perp)\)

- \(\eta(\vec{x}_\perp, \vec{y}_\perp)\) is a non-linear functional of \(\rho\)

- Resums all the powers of \(\alpha_s \ln(1/x)\) and of \(Q_s/\rho_\perp\) that arise in loop corrections

- Simplifies into the BFKL equation when the source \(\rho\) is small (expand \(\eta\) in powers of \(\rho\))
Links between eA and AA
Saturation and CGC
Factorization in AA
Factorization in eA
The Color Glass Condensate provides a framework to describe nucleus-nucleus collisions up to a time $\tau \sim Q_s^{-1}$.

Subsequent stages are described as fluid dynamics.
Reminder on hydrodynamics

Equations of hydrodynamics:

\[ \partial_\mu T^{\mu \nu} = 0 \]

Additional inputs:

EoS: \( p = f(\epsilon) \), Transport coefficients: \( \eta, \zeta, \cdots \)

- Required initial conditions: \( T^{\mu \nu}(\tau = \tau_0, \eta, \vec{x}_\perp) \)
Leading Order

- Expansion in $g^2$ in the saturated regime:

$$T^{\mu\nu} = \frac{Q_s^4}{g^2} \left[ c_0 + c_1 g^2 + c_2 g^4 + \cdots \right]$$

Leading Order \hspace{1cm} \text{given by classical fields :}

$$T^{\mu\nu}_{\text{LO}} \equiv c_0 \frac{Q_s^4}{g^2} = \frac{1}{4} g^{\mu\nu} \mathcal{F}^{\lambda\sigma} \mathcal{F}_{\lambda\sigma} - \mathcal{F}^{\mu\lambda} \mathcal{F}^{\nu\lambda}$$

$$[\mathcal{D}_\mu, \mathcal{F}^{\mu\nu}] = J_1^{\nu} + J_2^{\nu}$$

Yang–Mills equation

$$\lim_{t \to -\infty} A^\mu(t, \vec{x}) = 0$$
Leading Logs  [FG, Lappi, Venugopalan (2008)]

• The coefficients of the logs at NLO are given by the action of the JIMWLK Hamiltonians on the LO observable:

\[ \delta T_{\mu \nu}^{NLO} = \left[ \ln \left( \frac{\Lambda_0^-}{\Lambda_1^-} \right) \mathcal{H}_1 + \ln \left( \frac{\Lambda_0^+}{\Lambda_1^+} \right) \mathcal{H}_2 \right] T_{\mu \nu}^{LO} \]

• By iterating this process, one arrives at:

\[ \langle T_{\mu \nu}(\tau, \eta, \vec{x}_\perp) \rangle_{LLog} = \int \left[ D\rho_1 \right] \left[ D\rho_2 \right] W_1[\rho_1] W_2[\rho_2] \underbrace{T_{\mu \nu}^{LO}(\tau, \vec{x}_\perp)}_{\text{for fixed } \rho_{1,2}} \]

\[ \text{for fixed } \rho_{1,2} \]
Correlations in $\eta$ and $\vec{x}_\perp$

- The factorization valid for $\langle T^{\mu\nu} \rangle$ can be extended to multi-point correlations:

$$\langle T^{\mu_1\nu_1}(\tau, \eta_1, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}(\tau, \eta_n, \vec{x}_{n\perp}) \rangle_{\text{LLlog}} = $$

$$= \int \left[ D\rho_1 D\rho_2 \right] W_1[\rho_1] W_2[\rho_2]$$

$$\times T^{\mu_1\nu_1}_{\text{LO}}(\tau, \vec{x}_{1\perp}) \cdots T^{\mu_n\nu_n}_{\text{LO}}(\tau, \vec{x}_{n\perp})$$

- With this formula, one can in principle build an event generator for the initial conditions in AA collisions

- For this, we need to know the distributions $W[\rho]$
Conditions for factorizability

i. The observable must be **inclusive**, i.e. should not impose a condition that is realized only in a certain subclass of events

At a technical level, it should be an observable that can be rewritten in terms of fields that are specified only by retarded boundary conditions

ii. Factorization has only be proven either for local operators, or for non-local operators where all the points are separated by space-like intervals

If the points have time-like intervals, things becomes more complicated because the measurement done at the first point influences what happens at the other points
What do we need to know about $W[\rho]$?

- 2-point correlations $\langle U(\vec{x}_\perp)U^\dagger(\vec{y}_\perp) \rangle$
- Rapidity dependence
- Impact parameter dependence
- Fluctuations of the nuclear shape
- Ideally, higher-point correlations (but there is some evidence that they may be a small correction)
Links between eA and AA

Saturation and CGC

Factorization in AA

Factorization in eA
Inclusive DIS at Leading Order

- CGC effective theory with cutoff at the scale $\Lambda_0^{-}$:

  ![Diagram showing fields and sources]

- At Leading Order, DIS can be seen as the interaction between the target and a $q\bar{q}$ fluctuation of the virtual photon:
Inclusive DIS at Leading Order

- Forward dipole amplitude at leading order:

\[
T_{\text{LO}}(\vec{x}_\perp, \vec{y}_\perp) = 1 - \frac{1}{N_c} \text{tr} \left( U(\vec{x}_\perp) U^\dagger(\vec{y}_\perp) \right)
\]

\[
U(\vec{x}_\perp) = \text{P exp} i g \int \frac{1}{x P^−} dz^+ A^−(z^+, \vec{x}_\perp)
\]

\[
[D_\mu, F^{\mu\nu}] = \delta^{\nu−} \rho(x^+, \vec{x}_\perp)
\]

▷ at LO, the scattering amplitude on a saturated target is entirely given by classical fields
Inclusive DIS at Leading Log

- At leading log accuracy, the NLO contribution is:

\[ \delta T_{NLO}(\vec{x}_\perp, \vec{y}_\perp) = \ln \left( \frac{\Lambda_0}{\Lambda_1} \right) \mathcal{H} T_{LO}(\vec{x}_\perp, \vec{y}_\perp) \]

- By resumming all the slow field modes at leading log accuracy:

\[ \sigma_{\gamma^* T} = \int_0^1 dz \int d^2 \vec{r}_\perp |\psi(q|z, \vec{r}_\perp)|^2 \sigma_{\text{dipole}}(x, \vec{r}_\perp) \]

\[ \sigma_{\text{dipole}}(x, \vec{r}_\perp) \equiv 2 \int d^2 \vec{X}_\perp \int [D\rho] W_{xP-}[\rho] T_{LO}(\vec{x}_\perp, \vec{y}_\perp) \]
Anatomy of an AA calculation

From color sources to AA observables

What can be probed in eA collisions?

How to constrain \( W[\rho] \) in eA collisions?
Calculating the observable at \( \tau \)

- In the CGC at Leading Log accuracy, the relevant “observables” are expressible in terms of the classical gauge field \( A^\mu \):

\[
\mathcal{O}(\tau) \equiv \int [D\mathcal{A}] \ P_\tau[\mathcal{A}] \ \mathcal{O}[\mathcal{A}]
\]

- \( P_\tau[\mathcal{A}] = \) probability of the configuration \( \mathcal{A} \) of the gauge field at the time \( \tau \)

Requirement for a Monte-Carlo:
ensemble of \( \mathcal{A} \)’s distributed according to the distribution \( P_\tau[\mathcal{A}] \)
Going from $P_0[A]$ to $P_\tau[A]$

- Each field configuration $\mathcal{A}$ evolves according to the classical Yang-Mills equation:

$$\left[ D_\mu, F^{\mu\nu} \right] = 0$$

- Notes:
  
  i. there are no sources at $\tau > 0 \quad \triangleright \quad \text{r.h.s.}=0$
  
  ii. equivalently, $P_\tau[A]$ obeys the classical Liouville equation:

$$\partial_\tau \mathcal{A} + \{ P_\tau, \mathcal{H}_{YM} \} = 0$$

**Requirement at $\tau = 0^+$:**

ensemble of field configurations $\mathcal{A}$ right after the collision
From the sources \( \rho_{1,2} \) to \( A \) at \( \tau = 0^+ \)

- Gauge fields of the nuclei prior to the collision:

\[
A_{1,2}^i = \frac{i}{g} U_{1,2}^i \partial^i U_{1,2}
\]

\[
U_{1,2} = T \exp ig \int dx^\pm \nabla_{\perp}^{-2} \rho_{1,2}
\]

- Then, one gets the fields at \( \tau = 0^+ \) via

\[
A^i = A_1^i + A_2^i
\]

\[
A^\eta = \frac{i}{g} [A_1^i, A_2^i]
\]

For each projectile, we need:

ensembles of configurations for \( \rho_1 \) and \( \rho_2 \)
(or equivalently for \( A_{1,2} \) or \( U_{1,2} \))
Evolution of the $\rho$’s from $y_0$ to $y$

- The distribution of $\rho$’s obeys the JIMWLK equation:

$$\partial_y W_y[\rho] = \mathcal{H} W_y[\rho]$$

- The JIMWLK equation is a diffusion equation (in a functional space), and has an equivalent formulation as a Langevin equation (i.e. in terms of random walks):
  
  i. this form is more appropriate for a Monte-Carlo generator
  
  ii. it keeps track of the correlations of the $\rho$’s at different rapidities (important for things such as the ridge)

Requirement at the rapidity $y_0$:

initial ensemble of $\rho$’s (usually at $x \sim 10^{-2}$)

Note: this is where the $b$-dependence and the shape fluctuations should go
2 Anatomy of an AA calculation

From color sources to AA observables
What can be probed in eA collisions?
How to constrain $W[\rho]$ in eA collisions?
Testable aspects in eA collisions

- Anything that has to do with what happens when two $\rho$’s collide is off-limits

- The initial distribution of $\rho$’s and its evolution with rapidity affect observables in eA collisions, and can in principle be constrained/tested

Important things to keep in mind:

1. $W[\rho]$ is a functional
2. There is no fundamental theory that tells us what is a good initial distribution at $y_0$
3. Observables are sensitive only to certain “moments” of the distribution
4. Once the initial distribution is fixed, the JIMWLK equation leaves no freedom for the $y$ dependence
Anatomy of an AA calculation
From color sources to AA observables
What can be probed in eA collisions?
How to constrain $W[\rho]$ in eA collisions?
General fitting strategy

i. Set up a model of the initial condition $W_0[\rho]$

ii. Evolve it to $W_\gamma[\rho]$ with the JIMWLK equation

iii. Compute the eA observable of interest

iv. Compare to measurements at an EIC. Vary the parameters of the initial condition to minimize the $\chi^2$

▷ similar in spirit to what is done in DGLAP fits, except that the technology is more cumbersome to implement and that the spectrum of possible models is wider
What could be a model for the initial $W[ρ]$?

• The McLerran-Venugopalan model is often used as an initial condition at moderate $x_0$ for a large nucleus:

  - partons distributed randomly
  - many partons in a small longitudinal tube
  - no correlations at different impact parameters

• The MV model assumes that the density of color charges $ρ(\vec{x}_⊥)$ has a Gaussian distribution:

$$W_{y_0}[ρ] = \exp \left[ - \int d^2 \vec{x}_⊥ ρ_a(\vec{x}_⊥)ρ_a(\vec{x}_⊥) \frac{2μ^2}{2μ^2} \right]$$

(justified by the central limit theorem for a large nucleus)
Simpler version: neglect higher correlations

**i.** Assume Gaussian correlations only. Set up a model of the initial $\langle U(\vec{x}_\perp)U^\dagger(\vec{y}_\perp)\rangle_0$

**ii.** Evolve it with the BK equation

**iii.** Compute the eA observable of interest, assuming Gaussian correlations only

**iv.** Compare to measurements at an EIC. Vary the parameters of the initial condition to minimize the $\chi^2$

▷ Steps i. and ii. are now much simpler to implement

▷ Impact parameter dependence is still a problem in BK. A kludge is to use the $b$-independent BK, with a fixed $b$ profile (and then solve BK independently at each $b$)
Summary

- In the non-linear saturated regime, inclusive observables in AA collisions are expressible in a factorized way that involves the same distributions as in eA reactions.

- eA collisions can be used to constrain the initial distribution of color sources, and to test its rapidity evolution.

- The fitting procedure is similar in spirit to DGLAP, but much more cumbersome.

- An important aspect of AA collisions are the effects related to the shape of the collision zone (e.g. elliptic flow). Thus, the ensemble of initial $\rho$’s used in an AA event generator must be aware of the geometry of a nucleus (size, shape, falloff at the edges) and its fluctuations.
  - inclusive reactions at an EIC are not going to be sufficient to pin this down.
  - diffraction?