Nuclear effects throughout the \((x, Q^2)\) plane

Ian Cloët  
(University of Washington)

Collaborators

Wolfgang Bentz  
(Tokai University)

Anthony Thomas  
(Adelaide University)

Perturbative and Non-Perturbative Aspects of QCD at Collider Energies  
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Aspects of DIS on nuclear targets \[\Rightarrow\] nuclear structure functions

- Highlight opportunities provided by nuclear systems to study QCD
- Gain insight into nuclear structure from a QCD viewpoint

Present complementary approach to traditional nuclear physics

- Formulated as a covariant quark theory
- Grounded in good description of mesons and baryons
- At finite density self-consistent mean-field approach
- Bound nucleons differ from free nucleons

Possible answers to many long-standing questions: we address

- EMC effect & NuTeV anomaly

Highlight the unique opportunities provided by PV DIS on nuclei
**EMC Effect**

![Graph showing the EMC effect](image)

- Fundamentally challenged our understanding of nuclei.
- Immediate parton model interpretation:
  - valence quarks in nucleus carry less momentum than in nucleon.
- What is the mechanism? After more than 25 years no consensus.
- *nuclear structure, pions, 6 quark bags, rescaling, medium modification*
EMC Effect

- Understanding EMC effect critical for QCD based description of nuclei
- Need new experiments accessing different aspects of the EMC effect
- Important near term measurements
  - flavour decomposition & spin dependence of nuclear PDFs
- New experiments
  - semi-inclusive DIS, parity violating DIS, polarized DIS, Drell-Yan
Medium Modification

● 50 years of traditional nuclear physics tells us that the nucleus is composed of nucleon-like objects

● However if a nucleon property is not protected by a symmetry its value may change in medium – for example:
  ✦ mass, magnetic moment, size
  ✦ quark distributions, form factors, GPDs, etc

● There must be medium modification:
  ✦ nucleon propagator is changed in medium
  ✦ off-shell effects \( (p^2 \neq M^2) \)
  ✦ Lorentz covariance implies bound nucleon has 12 EM form factors

\[
\langle J^\mu \rangle = \sum_{\alpha, \beta = +, -} \Lambda^\alpha (p') \left[ \gamma^\mu f_1^{\alpha\beta} + \frac{1}{2M} i\sigma^{\mu\nu} q_\nu f_2^{\alpha\beta} + q^\mu f_3^{\alpha\beta} \right] \Lambda^\beta (p)
\]

● Need to understand these effects as first step toward QCD based understanding of nuclei
Medium Modification

- 50 years of traditional nuclear physics tells us that the nucleus is composed of nucleon-like objects
- However if a nucleon property is not protected by a symmetry its value may change in medium – for example:
  - mass, magnetic moment, size
  - quark distributions, form factors, GPDs, etc
- There must be medium modification:
  - nucleon propagator is changed in medium
  - off-shell effects ($p^2 \neq M^2$)
  - Becomes two form factors for on-shell nucleon

\[
\langle J^\mu \rangle = \bar{u}(p') \left[ \gamma^\mu F_1(Q^2) + \frac{1}{2M} i\sigma^{\mu\nu} q^\nu F_2(Q^2) \right] u(p)
\]

- Need to understand these effects as first step toward QCD based understanding of nuclei
- Pions play a fundamental role in traditional nuclear physics
  - therefore expect pion (anti-quark) enhancement in nuclei

- Drell-Yan experiment set up to probe anti-quarks in target nucleus
  - \( \bar{q}q \rightarrow \mu^+\mu^- \) — E906: run \( \sim 2011 \) FNAL, E772: Alde et al., PRL. 64, 2479 (1990).
  - no pionic enhancement – very unexpected – energy loss?

- Important to understand anti-quarks in nuclei: Drell-Yan & PV DIS
**DIS on Nuclear Targets**

- **Why nuclear targets?**
  - only targets with $J \geq 1$ are nuclei
  - study QCD and nucleon structure at finite density

- **Hadronic Tensor:** in Bjorken limit & Callen-Gross ($F_2 = 2x \, F_1$)
  - For $J = \frac{1}{2}$ target
    
    $$W_{\mu \nu} = \left( g_{\mu \nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F_2(x, Q^2) + \frac{i \varepsilon_{\mu \nu \lambda \sigma} q^\lambda p^\sigma}{p \cdot q} g_1(x, Q^2)$$

  - For arbitrary $J$: $- J \leq H \leq J$    [2$J$ + 1 structure functions]
    
    $$W^H_{\mu \nu} = \left( g_{\mu \nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F^{2H}_{2A}(x_A, Q^2) + \frac{i \varepsilon_{\mu \nu \lambda \sigma} q^\lambda p^\sigma}{p \cdot q} g^H_{1A}(x_A, Q^2)$$

- **Parton model expressions**    [2$J$ + 1 quark distributions]

  $$F^{H}_{2A}(x_A) = \sum_q e_q^2 \, x_A \left[ q^H_A(x_A) + \bar{q}^H_A(x_A) \right] ; \quad \text{parity} \quad \Longrightarrow \quad F^H_{2A} = F^{-H}_{2A}$$
DIS on Nuclear Targets

- Hadronic Tensor: in Bjorken limit & Callen-Gross ($F_2 = 2x F_1$)
  - For $J = \frac{1}{2}$ target
    \[
    W_{\mu\nu} = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F_2(x, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_1(x, Q^2)
    \]
  - For arbitrary $J$: $-J \leq H \leq J$ [2$J$ + 1 structure functions]
    \[
    W^H_{\mu\nu} = \left( g_{\mu\nu} \frac{p \cdot q}{q^2} + \frac{p_\mu p_\nu}{p \cdot q} \right) F_2^H(x_A, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p \cdot q} g_1^H(x_A, Q^2)
    \]

- Parton model expressions [2$J$ + 1 quark distributions]
  \[
  F_2^H(x_A) = \sum_q e_q^2 x_A \left[ q_A^H(x_A) + \bar{q}_A^H(x_A) \right]; \quad \text{parity} \implies F_2^H = F_2^{-H}
  \]
  \[
  F_2(x) = \frac{1}{2J + 1} \sum_{H=-J}^{J} F_2^H(x)
  \]
Definition of finite nuclei quark distributions

\[
q_A^H(x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^- / A} \langle A, P, H | \bar{\psi}_q(0) \gamma^+ \psi_q(\xi^-) | A, P, H \rangle
\]

Approximate using a modified convolution formalism

\[
q_A^H(x_A) = \sum_{\alpha, \kappa, m} \int dy_A \int dx \, \delta(x_A - y_A x) f_{\alpha, \kappa, m}^{(H)}(y_A) q_{\alpha, \kappa}(x)
\]
**Definition of finite nuclei quark distributions**

\[
q^H_A(x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+x_A \xi^-/A} \langle A, P, H | \bar{\psi}_q(0) \gamma^+ \psi_q(\xi^-) | A, P, H \rangle
\]

**Approximate using a modified convolution formalism**

\[
q^H_A(x_A) = \sum_{\alpha, \kappa, m} \int dy_A \int dx \, \delta(x_A - y_A x) \, f^{(H)}_{\alpha, \kappa, m}(y_A) \, q_{\alpha, \kappa}(x)
\]

**Convolution formalism diagrammatically:**
Assume all spin is carried by the valence nucleons.

If \( A \gtrsim 8 \) and for example if:

\[
\begin{align*}
J &= \frac{3}{2} \\
\Rightarrow \quad F_{2A}^{3/2} &\approx F_{2A}^{1/2}
\end{align*}
\]

This is a model independent result within the convolution formalism.

Introduce multipole quark distributions:

\[
q^{(K)}(x) \equiv \sum_{J} (-1)^{J-H} \sqrt{2K+1} \left( \frac{J}{H} - \frac{J}{H} \frac{K}{0} \right) q^{H}(x), \quad K = 0, 2, \ldots, 2J
\]

Example:

\[
J = \frac{3}{2} \quad \Rightarrow \quad q^{(0)} = q^{\frac{3}{2}} + q^{\frac{1}{2}} \quad q^{(2)} = q^{\frac{3}{2}} - q^{\frac{1}{2}}
\]

Higher multipoles encapsulate difference between helicity distributions.
Some multipole quark distributions result

- Large $K > 1$ multipole PDFs would be very surprising
- $\rightarrow$ large off-shell effects &/or non-nucleon components, etc
New Sum Rules

- Sum rules for multipole quark distributions

\[ \int dx \, x^{n-1} \, q^{(K)}(x) = 0, \quad K, n \text{ even}, \quad 2 \leq n < K, \]
\[ \int dx \, x^{n-1} \, \Delta q^{(K)}(x) = 0, \quad K, n \text{ odd}, \quad 1 \leq n < K. \]

- Examples:

\[ J = \frac{3}{2} \implies \langle \Delta q^{(3)}(x) \rangle = 0 \]
\[ J = 2 \implies \langle \Delta q^{(3)}(x) \rangle = \langle q^{(4)}(x) \rangle = 0 \]
\[ J = \frac{5}{2} \implies \langle \Delta q^{(3)}(x) \rangle = \langle q^{(4)}(x) \rangle = \langle \Delta q^{(5)}(x) \rangle = \langle x^2 \Delta q^{(5)}(x) \rangle = 0 \]

- Sum rules place tight constraints on multipole PDFs

Nambu–Jona-Lasinio Model

- Interpreted as low energy chiral effective theory of QCD

  \[ \frac{Z(k^2)}{k^2} \]

- Can be motivated by infrared enhancement of quark–gluon interaction
e.g. DSEs and Lattice QCD

- Investigate the role of quark degrees of freedom

- NJL has same symmetries as QCD

- Lagrangian:  
  \[ \mathcal{L}_{NJL} = \overline{\psi} \left( i\gamma^\mu \partial_\mu - m \right) \psi + G \left( \overline{\psi} \Gamma \psi \right)^2 \]
**Nucleon in the NJL model**

- Nucleon approximated as quark-diquark bound state
- Use relativistic Faddeev approach:

\[
P - k = P - k
\]

- Nucleon quark distributions

\[
q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ix \cdot p^+ \xi^-} \langle p, s | \bar{\psi}_q(0) \gamma^+ \psi_q(\xi^-) | p, s \rangle_c, \quad \Delta q(x) = \langle \gamma^+ \gamma_5 \rangle
\]

- Associated with a Feynman diagram calculation

\[
\mathbf{[q(x), \Delta q(x), \Delta_T q(x)]} \rightarrow \mathbf{X} = \delta \left(x - \frac{k^+}{p^+}\right) \left[\gamma^+, \gamma^+ \gamma_5, \gamma^+ \gamma_1 \gamma_5\right]
\]
**Results: proton quark distributions**

- Covariant, correct support, satisfies baryon and momentum sum rules

\[ \int dx \left[ q(x) - \bar{q}(x) \right] = N_q, \quad \int dx \, x \left[ u(x) + d(x) + \ldots \right] = 1 \]

- Satisfies positivity constraints and Soffer bound

\[ |\Delta q(x)|, \quad |\Delta_T q(x)| \leq q(x), \quad q(x) + \Delta q(x) \geq 2|\Delta_T q(x)| \]

Asymmetric Nuclear Matter

- Finite density Lagrangian: add $\bar{q}q$ interaction in $\sigma$, $\omega$, $\rho$ channels

\[ L = \bar{\psi}_q \left( i \not\! \partial - M^* - V_q \right) \psi_q + L'_I \]

- Fundamental physics: mean fields couple to the quarks in nucleons

- Finite density quark propagator

\[ S(k)^{-1} = \not{k} - M - i\varepsilon \quad \rightarrow \quad S_q(k)^{-1} = \not{k} - M^* - V_q - i\varepsilon \]

- Hadronization + mean-field $\Rightarrow$ effective potential that provides

\[ V_{u(d)} = \omega_0 \pm \rho_0, \quad \omega_0 = 6 G_\omega (\rho_p + \rho_n), \quad \rho_0 = 2 G_\rho (\rho_p - \rho_n) \]

- $G_\omega \Leftrightarrow Z = N$ saturation & $G_\rho \Leftrightarrow$ symmetry energy
Isovector EMC effect

- **EMC ratio:**
  \[
  R = \frac{F_{2A}}{F_{2A,\text{naive}}} = \frac{F_{2A}}{Z F_{2p} + N F_{2n}} \approx \frac{4 u_A(x) + d_A(x)}{4 u_f(x) + d_f(x)}
  \]

- Density is fixed only changing $Z/N$ ratio
- EMC effect essentially a consequence of binding at the quark level
- **proton excess:** $u$-quarks feel more repulsion than $d$-quarks ($V_u > V_d$)
- **neutron excess:** $d$-quarks feel more repulsion than $u$-quarks ($V_d > V_u$)
Weak mixing angle and the NuTeV anomaly

- **NuTeV**: \( \sin^2 \theta_W = 0.2277 \pm 0.0013^{(\text{stat})} \pm 0.0009^{(\text{syst})} \)

- World average \( \sin^2 \theta_W = 0.2227 \pm 0.0004 : 3 \sigma \rightarrow \) “NuTeV anomaly”

- Huge amount of experimental & theoretical interest [over 400 citations]

- No universally accepted complete explanation
Paschos-Wolfenstein ratio

- Paschos-Wolfenstein ratio motivated the NuTeV study:

\[ R_{PW} = \frac{\sigma_{NC}^{\nu A} - \sigma_{NC}^{\bar{\nu} A}}{\sigma_{CC}^{\nu A} - \sigma_{CC}^{\bar{\nu} A}}, \quad NC \Rightarrow Z^0, \quad CC \Rightarrow W^\pm \]

- For an isoscalar target \( u_A \simeq d_A \) and if \( s_A \ll u_A + d_A \)

\[ R_{PW} = \left( \frac{1}{2} - \sin^2 \theta_W \right) + \left( 1 - \frac{7}{3} \sin^2 \theta_W \right) \frac{\langle x u_A^- - x d_A^- \rangle}{\langle x u_A^- + x d_A^- \rangle} \]

- NuTeV “measured” \( R_{PW} \) on an Fe target (\( Z/N \simeq 26/30 \))
- Correct for neutron excess ⇔ flavour dependent EMC effect
- Use our medium modified “Fe” quark distributions

\[ \Delta R_{PW} = \Delta R_{PW}^{\text{naive}} + \Delta R_{PW}^{\text{Fermi}} + \Delta R_{PW}^{\rho^0} \]

\[ = - (0.0107 + 0.0004 + 0.0028). \]

- Isoscalarity \( \rho^0 \) correction can explain up to 65% of anomaly
**NuTeV anomaly cont’d**

- Also correction from $m_u \neq m_d$ - Charge Symmetry Violation
  - CSV + $\rho_0$ $\implies$ no NuTeV anomaly
  - No evidence for physics beyond the Standard Model

- Instead “NuTeV anomaly” is evidence for medium modification
  - Equally interesting
  - EMC effect has over 850 citations

- Model dependence?
  - sign of correction is fixed by nature of vector fields
    \[
    q(x) = \frac{p^+}{p^+-V^+} q_0 \left( \frac{p^+}{p^+-V^+} x - \frac{V^+_q}{p^+-V^+} \right), \quad N > Z \implies V_d > V_u
    \]
  - $\rho^0$-field shifts momentum from $u$- to $d$-quarks
  - size of correction is constrained by Nucl. Matt. symmetry energy

- $\rho_0$ vector field reduces NuTeV anomaly – Model Independent!!
Includes NuTeV functionals

Small increase in systematic error

NuTeV anomaly interpreted as evidence for medium modification

Equally profound as evidence for physics beyond Standard Model
**Consistent with other observables?**

- We claim isovector EMC effect explains $\sim 1.5\sigma$ of NuTeV result
  - is this mechanism observed elsewhere?
- Yes!! Parity violating DIS: $\gamma Z^0$ interference

\[
A_{PV} = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L} \propto \left[ a_2(x) + \frac{1 - (1 - y)^2}{1 + (1 - y)^2} a_3(x) \right]
\]

\[
a_2(x) = -2g_A^e \frac{F_2^{\gamma Z}}{F_2^\gamma} = \frac{6u^+ + 3d^+}{4u^+ + d^+} - 4\sin^2\theta_W
\]

\[
a_3(x) = -2g_V^e \frac{F_3^{\gamma Z}}{F_2^\gamma} = 3 \left(1 - 4\sin^2\theta_W\right) \frac{2u^- + d^-}{4u^+ + d^+}
\]

- Parton model expressions

\[
F_2^{\gamma Z} = 2 \sum e_q g_V^q x \left(q + \bar{q}\right), \quad g_V^q = \pm \frac{1}{2} - 2e_q \sin^2\theta_W
\]

\[
F_3^{\gamma Z} = 2 \sum e_q g_A^q \left(q - \bar{q}\right), \quad g_A^q = \pm \frac{1}{2}
\]
Parity Violating DIS: Carbon

- Ignoring quark mass differences, $s$-quarks and EW corrections

- For a $N = Z$ target:

\[ a_2(x) = \frac{6u_A^+ + 3d_A^+}{4u_A^+ + d_A^+} - 4\sin^2\theta_W \rightarrow \frac{9}{5} - 4\sin^2\theta_W \]

\[ a_3(x) = 3\left(1 - 4\sin^2\theta_W\right) \frac{2u^- + d^-}{4u_A^+ + d_A^+} \rightarrow \frac{9}{5}\left(1 - 4\sin^2\theta_W\right) \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} \]

- Measurement of $a_2(x)$ at each $x$ \(\rightarrow\) a NuTeV experiment!
**Parity Violating DIS: Carbon**

![Graphs showing the behavior of $a_2(x)$ and $a_3(x)$ for $Z/N = 1$ (Carbon) with $Q^2 = 5 \text{ GeV}^2$ and $\sin^2 \theta_W$](image)

- Ignoring quark mass differences, $s$-quarks and EW corrections
  - For a $N = Z$ target:
    
    $$a_2(x) = \frac{6 u_A^+ + 3 d_A^+}{4 u_A^+ + d_A^+} - 4 \sin^2 \theta_W \rightarrow \frac{9}{5} - 4 \sin^2 \theta_W$$
    
    $$a_3(x) \rightarrow \frac{9}{5} \left(1 - 4 \sin^2 \theta_W\right) \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} = \frac{9}{5} \left(1 - 4 \sin^2 \theta_W\right) \left[1 + 2 \frac{\bar{u}_A + \bar{d}_A}{u_A^- + d_A^-}\right]^{-1}$$

- Measurement of $a_2(x)$ at each $x \implies$ a NuTeV experiment!
Parity Violating DIS: Iron

For a $N \sim Z$ target:

\[
a_2(x) = \frac{9}{5} - 4 \sin^2 \theta_W - \frac{12}{25} \frac{u_A^+(x) - d_A^+(x)}{u_A^+(x) + d_A^+(x)}
\]

\[
a_3(x) = \frac{9}{5} (1 - 4 \sin^2 \theta_W) \left\{ \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} - \frac{1}{3} \left[ \frac{12}{5} \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} \frac{u_A^+ - d_A^+}{u_A^+ + d_A^+} - \frac{u_A^- - d_A^-}{u_A^+ + d_A^+} \right] \right\}
\]

“Naive” result has no medium corrections

Sizeable medium effects in $a_2(x)$
For a $N \simeq Z$ target:

$$a_2(x) = \frac{9}{5} - 4 \sin^2 \theta_W - \frac{12}{25} \frac{u_A^+(x) - d_A^+(x)}{u_A^+(x) + d_A^+(x)}$$

$$a_3(x) = \frac{9}{5} \left(1 - 4 \sin^2 \theta_W\right) \left\{ \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} - \frac{1}{3} \left[ \frac{12}{5} \frac{u_A^- + d_A^-}{u_A^+ + d_A^+} \frac{u_A^+ - d_A^+}{u_A^+ + d_A^+} - \frac{u_A^- - d_A^-}{u_A^+ + d_A^+} \right] \right\}$$

After naive isoscalarity corrections medium effects still very large

Large $x$ dependence of $a_2(x) \rightarrow$ evidence for medium modification
Flavour Dependence of EMC effect

- Flavour dependence determined by measuring $F_{2A}^{\gamma}$ and $F_{2A}^{\gamma Z}$

- $N > Z \implies d$-quarks feel more repulsion than $u$-quarks: $V_d > V_u$

$$q(x) = \frac{p^+}{p_{+ +} - V^+} q_0 \left( \frac{p^+}{p_{+ +} - V^+} x - \frac{V^+_{q^+}}{p_{+ +} - V^+} \right)$$

- $\rho^0$ field has shifted momentum from $u$ to $d$ quarks
- $u$ quarks are more bound than $d$ quarks

- If observed $\implies$ very strong evidence for medium modification
Finite nuclei EMC effects

- **EMC ratio**
  \[ R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z F_{2p} + N F_{2n}} \]

- **Polarized EMC ratio**
  \[ R_s^H = \frac{g_{1A}^H}{g_{1A}^{H,\text{naive}}} = \frac{g_{1A}^H}{P_p^H g_{1p} + P_n^H g_{1n}} \]

- **Spin-dependent cross-section is suppressed by 1/A**
  - Must choose nuclei with \( A \lesssim 27 \)
  - Protons should carry most of the spin e.g. \( \rightarrow \) \(^7\text{Li}, \(^{11}\text{B}, \ldots \)

- **Ideal nucleus is probably \(^7\text{Li} \)**
  - From Quantum Monte–Carlo: \( P_p^J = 0.86 \) & \( P_n^J = 0.04 \)

- **Ratios equal 1 in non-relativistic and no-medium modification limit**
**EMC ratio $^7$Li, $^{11}$B and $^{27}$Al**

- **$^7$Li**
  - $Q^2 = 5 \text{ GeV}^2$
  - Experiment: $^9$Be
  - Unpolarized EMC effect
  - Polarized EMC effect

- **$^{11}$B**
  - $Q^2 = 5 \text{ GeV}^2$
  - Experiment: $^{12}$C
  - Unpolarized EMC effect
  - Polarized EMC effect

- **$^{27}$Al**
  - $Q^2 = 5 \text{ GeV}^2$
  - Experiment: $^{27}$Al
  - Unpolarized EMC effect
  - Polarized EMC effect
**Is there medium modification**

![Graph showing EMC Ratios for 27Al](image)

- **Experiment:** 27Al
- Unpolarized EMC effect
- Polarized EMC effect
- $Q^2 = 5 \text{ GeV}^2$
Is there medium modification

- Medium modification of nucleon has been switched off
- Relativistic effects remain
- Large splitting would be strong evidence for medium modification
# Nuclear Spin Sum

<table>
<thead>
<tr>
<th>Proton spin states</th>
<th>$\Delta u$</th>
<th>$\Delta d$</th>
<th>$\Sigma$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0.97</td>
<td>-0.30</td>
<td>0.67</td>
<td>1.267</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>0.91</td>
<td>-0.29</td>
<td>0.62</td>
<td>1.19</td>
</tr>
<tr>
<td>$^{11}\text{B}$</td>
<td>0.88</td>
<td>-0.28</td>
<td>0.60</td>
<td>1.16</td>
</tr>
<tr>
<td>$^{15}\text{N}$</td>
<td>0.87</td>
<td>-0.28</td>
<td>0.59</td>
<td>1.15</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
<td>0.87</td>
<td>-0.28</td>
<td>0.59</td>
<td>1.15</td>
</tr>
<tr>
<td>Nuclear Matter</td>
<td>0.79</td>
<td>-0.26</td>
<td>0.53</td>
<td>1.05</td>
</tr>
</tbody>
</table>

- **Angular momentum of nucleon:** \( J = \frac{1}{2} \Delta \Sigma + L_q + J_g \)
  - in medium \( M^* < M \) and therefore quarks are more relativistic
  - lower components of quark wavefunctions are enhanced
  - quark lower components usually have larger angular momentum
  - \( \Delta q(x) \) very sensitive to lower components

- **Conclusion:** quark spin $\rightarrow$ orbital angular momentum in-medium
Conclusion

* Illustrated the inclusion of quarks into a traditional description of nuclei
  - complementary approach to traditional nuclear physics

* Major discrepancy with SM predictions for $Z^0$ is NuTeV anomaly
  - may be resolved by CSV and isovector EMC effect corrections

* EMC effect and NuTeV anomaly are interpreted as evidence for medium modification of the bound nucleon wavefunction
  - result can be tested using PV DIS

* Some important remaining challenges:
  - polarized EMC effect  
    [quark spin converted $\rightarrow L_q$ in nuclei]
  - flavour dependence of EMC effect

* Exciting new experiments:
  - PV DIS, pion induced Drell-Yan, neutron knockout

* Slowly building a QCD based understanding of nuclear structure
Model Parameters

- **Free Parameters:**
  \[ \Lambda_{IR}, \Lambda_{UV}, M_0, G_\pi, G_s, G_a, G_\omega \text{ and } G_\rho \]

- **Constraints:**
  - \( f_\pi = 93 \text{ MeV}, \ m_\pi = 140 \text{ MeV} \ \& \ \ M_N = 940 \text{ MeV} \)
  - \( \int_0^1 dx \ (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267 \)
  - \( (\rho, E_B/A) = (0.16 \text{ fm}^{-3}, -15.7 \text{ MeV}) \)
  - \( a_4 = 32 \text{ MeV} \)
  - \( \Lambda_{IR} = 240 \text{ MeV} \)

- We obtain [MeV]:
  - \( \Lambda_{UV} = 644 \)
  - \( M_0 = 400, \ M_s = 690, \ M_a = 990, \ldots \)

- Can now study a very large array of observables:
  - e.g. **meson and baryon**: quark distributions, form factors, GPDs, finite temp. and density, neutron stars
**Regularization**

- **Proper-time regularization**

\[
\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau \tau^{n-1} e^{-\tau X} \rightarrow \frac{1}{(n-1)!} \int_{1/(\Lambda_{IR})^2}^{1/(\Lambda_{UV})^2} d\tau \tau^{n-1} e^{-\tau X}
\]

- \(\Lambda_{IR}\) eliminates unphysical thresholds for the nucleon to decay into quarks: \(\rightarrow\) simulates confinement

- E.g.: Quark wave function renormalization
  - \[Z(k^2) = e^{-\Lambda_{UV}(k^2-M^2)} - e^{-\Lambda_{IR}(k^2-M^2)}\]
  - \(\rightarrow\) \(Z(k^2 = M^2) = 0 \implies \) no free quarks

- Needed for: **nuclear matter saturation, \(\Delta\) baryon, etc**
Gap Equation & Mass Generation

\[ -1 = -1 + \frac{1}{p - m + i\varepsilon} \Rightarrow \frac{1}{p - M + i\varepsilon} \]

- **Quark Propagator:**

- **Mass is generated via interaction with vacuum**

- **Dynamically generated quark masses** \( \iff \langle \bar{\psi}\psi \rangle \neq 0 \iff D\chi_{SB} \)
- **Proper-time regularization:** \( \Lambda_{IR} \) and \( \Lambda_{UV} \)

\[ \Rightarrow \text{No free quarks} \Rightarrow \text{Confinement} \quad [Z(k^2 = M^2) = 0] \]
Off-Shell Effects

- For an off-shell nucleon, photon–nucleon vertex given by

\[ \Gamma_{N}^{\mu}(p', p) = \sum_{\alpha, \beta = +, -} \Lambda^{\alpha}(p') \left[ \gamma^{\mu} f_{1}^{\alpha\beta} + \frac{i\sigma^{\mu\nu}q_{\nu}}{2M} f_{2}^{\alpha\beta} + q^{\mu} f_{3}^{\alpha\beta} \right] \Lambda^{\alpha}(p) \]

- In-medium nucleon is off-shell, extremely difficult to quantify effects
  - However must understand to fully describe in-medium nucleon

- Simpler system: off-shell pion form factors
  - relax on-shell constraint \( p'^{2} = p^{2} = m_{\pi}^{2} \)
  - Very difficult to calculate in many approaches, e.g. Lattice QCD

\[ (p' + p)^{\mu} F_{\pi,1}(p'^{2}, p^{2}, Q^{2}) + (p' - p)^{\mu} F_{\pi,2}(p'^{2}, p^{2}, Q^{2}) \]

- For \( p'^{2} = p^{2} = m_{\pi}^{2} \) we have \( F_{\pi,1} \rightarrow F_{\pi} \) and \( F_{\pi,2} = 0 \)
Results: Nuclear Matter

- $\rho_p + \rho_n = \text{fixed}$ – Differences arise from:
  - naive: different number protons and neutrons
  - medium: $p$ & $n$ Fermi motion and $V_{u(d)}$ differ $\rightarrow u_p(x) \neq d_n(x), \ldots$