DVCS \rightarrow GPDs

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Transverse imaging:
- $H(x, 0, -\Delta^2_\perp) \rightarrow q(x, b_\perp)$
- $\tilde{H}(x, 0, -\Delta^2_\perp) \rightarrow \Delta q(x, b_\perp)$
- $E(x, 0, -\Delta^2_\perp) \rightarrow \perp$ distortion of PDFs when the target is $\perp$ polarized
- Ji relation: $GPDs \rightarrow J_q$
- DVCS $\sim GPDs$
- $GPDs(\xi, \xi, t)$
- Summary
Generalized Parton Distributions (GPDs)

- GPDs: decomposition of form factors at a given value of $t$, w.r.t. the average momentum fraction $x = \frac{1}{2} (x_i + x_f)$ of the active quark

$$\int dx H_q(x, \xi, t) = F_1^q(t)$$
$$\int dx \tilde{H}_q(x, \xi, t) = G_A^q(t)$$

$$\int dx E_q(x, \xi, t) = F_2^q(t)$$
$$\int dx \tilde{E}_q(x, \xi, t) = G_P^q(t),$$

- $x_i$ and $x_f$ are the momentum fractions of the quark before and after the momentum transfer
- $2\xi = x_f - x_i$

- GPDs can be probed in deeply virtual Compton scattering (DVCS)
Impact parameter dependent PDFs

- define \( \perp \) localized state \([D. Soper, PRD 15, 1141 (1977)]\)

\[
|p^+, R_\perp = 0_\perp, \lambda\rangle \equiv \mathcal{N} \int d^2 p_\perp |p^+, p_\perp, \lambda\rangle
\]

Note: \( \perp \) boosts in IMF form Galilean subgroup \( \Rightarrow \) this state has

\[
R_\perp \equiv \frac{1}{p_+} \int dx^- d^2 x_\perp x_\perp T^{++}(x) = \sum_i x_i r_{i, \perp} = 0_\perp
\]

(cf.: working in CM frame in nonrel. physics)

- define impact parameter dependent PDF

\[
q(x, b_\perp) \equiv \int \frac{dx^-}{4\pi} \langle p^+, R_\perp = 0_\perp | \bar{q}(-\frac{x^-}{2}, b_\perp)\gamma^+ q(\frac{x^-}{2}, b_\perp) | p^+, R_\perp = 0_\perp \rangle e^{ixp^+ x^-}
\]

\[
\rightarrow
\]

\[
q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} H(x, 0, -\Delta_\perp^2),
\]

\[
\Delta q(x, b_\perp) = \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{i\Delta_\perp \cdot b_\perp} \tilde{H}(x, 0, -\Delta_\perp^2),
\]

DVCS \( \rightarrow \) GPDs – p.4/26
$q(x, \mathbf{b}_\perp)$ for unpol. $p$

\[ x = \text{momentum fraction of the quark} \]

\[ \mathbf{b} = \perp \text{position of the quark} \]
Transversely Deformed Distributions and $E(x, 0, -\Delta^2_\perp)$

So far: only unpolarized (or long. pol.) nucleon! In general ($\xi = 0$):

\[
\int \frac{dx^-}{4\pi} e^{ip^+ x^-} x^- <P + \Delta, \uparrow| \bar{q}(0) \gamma^+ q(x^-)| P, \uparrow> = H(x, 0, -\Delta^2_\perp)
\]

\[
\int \frac{dx^-}{4\pi} e^{ip^+ x^-} x^- <P + \Delta, \uparrow| \bar{q}(0) \gamma^+ q(x^-)| P, \downarrow> = -\frac{\Delta_x \Delta_y}{2M} E(x, 0, -\Delta^2_\perp).
\]

Consider nucleon polarized in $x$ direction (in IMF)

$|X\rangle \equiv |p^+, R_\perp = 0_\perp, \uparrow\rangle + |p^+, R_\perp = 0_\perp, \downarrow\rangle$.

$\rightarrow$ unpolarized quark distribution for this state:

\[
q(x, b_\perp) = \mathcal{H}(x, b_\perp) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2\Delta_\perp}{(2\pi)^2} E(x, 0, -\Delta^2_\perp)e^{-ib_\perp \cdot \Delta_\perp}
\]

Physics: $j^+ = j^0 + j^3$, and left-right asymmetry from $j^3$!

[X.Ji, PRL 91, 062001 (2003)]
Intuitive connection with $\vec{J}_q$

- DIS probes quark momentum density in the infinite momentum frame (IMF). Quark density in IMF corresponds to $j^+ = j^0 + j^3$ component in rest frame ($\vec{p}_{\gamma^*}$ in $-\hat{z}$ direction)

$\implies j^+$ larger than $j^0$ when quark current towards the $\gamma^*$; suppressed when away from $\gamma^*$

$\implies$ For quarks with positive angular momentum in $\hat{x}$-direction, $j^z$ is positive on the $+\hat{y}$ side, and negative on the $-\hat{y}$ side

- Details of $\perp$ deformation described by $E_q(x, 0, -\Delta^2_\perp)$

$\implies$ not surprising that $E_q(x, 0, -\Delta^2_\perp)$ enters Ji relation!

$$\langle J^i_q \rangle = S^i \int dx \left[ H_q(x, 0, 0) + E_q(x, 0, 0) \right] x.$$
Transversely Deformed PDFs and $E(x,0,-\Delta_{\perp}^2)$

- $q(x,b_{\perp})$ in $\perp$ polarized nucleon is deformed compared to longitudinally polarized nucleons!

$$q(x,b_{\perp}) = \mathcal{H}(x,b_{\perp}) - \frac{1}{2M} \frac{\partial}{\partial b_y} \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} E(x,0,-\Delta_{\perp}^2) e^{-i b_{\perp} \cdot \Delta_{\perp}}$$

- mean $\perp$ deformation of flavor $q$ ($\perp$ flavor dipole moment)

$$d^q_y \equiv \int dx \int d^2 b_{\perp} q(x,b_{\perp}) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa^p_q}{2M}$$

- $\kappa^p = 1.913 = \frac{2}{3} \kappa^p_u - \frac{1}{3} \kappa^p_d + \ldots$

- $\kappa^p_u = 2\kappa_p + \kappa_n = 1.673 \implies$ shift in $+\hat{y}$ direction
- $\kappa^p_d = 2\kappa_n + \kappa_p = -2.033 \implies$ shift in $-\hat{y}$ direction

- for proton polarized in $+\hat{x}$ direction
- $d^q_y = \mathcal{O}(\pm 0.2 \text{ fm})$
p polarized in $+\hat{x}$ direction

$u(x, b_\perp)$

$\hat{p}_\gamma^*$

$j^z > 0$

$j^z < 0$

lattice results (QCDSF)
The Ji-relation (poor man’s derivation)

What distinguishes the Ji-decomposition from other decompositions is the fact that $L_q$ can be constrained by experiment:

$$
\langle \vec{J}_q \rangle = \vec{S} \int_{-1}^{1} dx \ x \ [H_q(x, \xi, 0) + E_q(x, \xi, 0)]
$$

(nucleon at rest; $\vec{S}$ is nucleon spin)

$$
\leftrightarrow L^z_q = J^z_q - \frac{1}{2} \Delta q
$$

Derivation (MB-version):

- consider nucleon state that is an eigenstate under rotation about the $\hat{x}$-axis (e.g. nucleon polarized in $\hat{x}$ direction with $\vec{p} = 0$ (wave packet if necessary)

- for such a state, $\langle T^{00}_q y \rangle = 0 = \langle T^{zz}_q y \rangle$ and $\langle T^{0y}_q z \rangle = -\langle T^{0z}_q y \rangle$

$$
\leftrightarrow \langle T^{00}_q y \rangle = \langle T^{0y}_q z - T^{0z}_q y \rangle = \langle J^x_q \rangle
$$

- relate 2nd moment of $\bot$ flavor dipole moment to $J^x_q$
The Ji-relation (poor man’s derivation)

derivation (MB-version):

- consider nucleon state that is an eigenstate under rotation about the \( \hat{x} \)-axis (e.g. nucleon polarized in \( \hat{x} \) direction with \( \vec{p} = 0 \) (wave packet if necessary)

for such a state, \( \langle T_{q}^{00} y \rangle = 0 = \langle T_{q}^{zz} y \rangle \) and \( \langle T_{q}^{0y} z \rangle = -\langle T_{q}^{0z} y \rangle \)

\[ \langle T_{q}^{++} y \rangle = \langle T_{q}^{0y} z - T_{q}^{0z} y \rangle = \langle J_{q}^{x} \rangle \]

\( \Rightarrow \) relate 2\(^{nd}\) moment of \( \perp \) flavor dipole moment to \( J_{q}^{x} \)

- effect sum of two effects:
  - \( \langle T^{++} y \rangle \) for a point-like transversely polarized nucleon
  - \( \langle T_{q}^{++} y \rangle \) for a quark relative to the center of momentum of a transversely polarized nucleon

- 2\(^{nd}\) moment of \( \perp \) flavor dipole moment for point-like nucleon

\[ \psi = \left( \frac{f(r)}{E + m} \right) \chi \quad \text{with} \quad \chi = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} \]
The Ji-relation (poor man’s derivation)

- derivation (MB-version):
  - \( T_{qz}^{0z} = i\bar{q} \left( \gamma^0 \partial^z + \gamma^z \partial^0 \right) q \)
  - since \( \psi^\dagger \partial_z \psi \) is even under \( y \to -y \), \( i\bar{q} \gamma^0 \partial^z q \) does not contribute to \( \langle T^{0z} y \rangle \)
  - using \( i\partial_0 \psi = E \psi \), one finds

\[
\langle T^{0z} b_y \rangle = E \int d^3r \psi^\dagger \gamma^0 \gamma^z \psi y = E \int d^3r \psi^\dagger \begin{pmatrix} 0 & \sigma^z \\ \sigma^z & 0 \end{pmatrix} \psi y \\
= \frac{2E}{E + M} \int d^3r \chi^\dagger \sigma^z \sigma^y \chi f(r)(-i)\partial_y f(r)y = \frac{E}{E + M} \int d^3r f^2(r)
\]

- consider nucleon state with \( \vec{p} = 0 \), i.e. \( E = M \) & \( \int d^3r f^2(r) = 1 \)
  - 2\(^{nd}\) moment of \( \perp \) flavor dipole moment \( \langle T_{q}^{++} y \rangle = \langle T_{0z}^{0z} b_y \rangle = \frac{1}{2} \)
  - ‘overall shift’ of nucleon COM yields contribution \( \frac{1}{2} \int dx xH_q(x, 0, 0) \) to \( \langle T_{q}^{++} y \rangle \)
The Ji-relation (poor man’s derivation)

- spherically symmetric wave packet for Dirac particle with $J_x = \frac{1}{2}$ centered around the origin has $\perp$ center of momentum $\frac{1}{M} \langle T^{++} b_y \rangle$ not at origin, but at $\frac{1}{2M}$!

- consistent with

\[ \frac{1}{2} = \langle J_x \rangle = \langle (T^0 z b^y - T^0 y b^z) \rangle = 2 \langle T^0 z b^y \rangle = \langle T^{++} b^y \rangle \]

- ‘overall shift of $\perp$ COM yields $\langle T^{++} b_y \rangle = \frac{1}{2} \int dx \ x H_q(x, 0, 0)$

- intrinsic distortion adds $\frac{1}{2} \int dx \ x E_q(x, 0, 0)$ to that
\( L_q \) for proton from Ji-relation (lattice)

- Calculate moments of GPDs using lattice QCD (\( \rightarrow \) Negele et al.)
- Insert in Ji-relation

\[
\langle J_q^i \rangle = S^i \int dx [H_q(x,0,0) + E_q(x,0,0)]\ x.
\]

\( \rightarrow L^z_q = J^z_q - \frac{1}{2} \Delta q \)

- \( L_u, L_d \) both large!
- Present calcs. show \( L_u + L_d \approx 0 \), but
  - Disconnected diagrams ..?
  - \( m^2_{\pi} \) extrapolation
  - Parton interpret. of \( L_q \)...
Ji relation \( J_q = \int_0^1 dx x [H(x, \xi, 0) + E(x, \xi, 0)] \) requires \( GPDs(x, \xi, 0) \) for (common) fixed \( \xi \) for all \( x \)

transverse imaging requires GPDs for \( \xi = 0 \)

\( A_{DVCS}(\xi, t) \rightarrow \int_{-1}^{1} dx \frac{GPD^{(+)}(x, \xi, t)}{x-\xi+i\epsilon} \)

\( \xi \) longitudinal momentum transfer on the target \( \xi = \frac{p^{++'}-p^{+}}{p^{++'}+p^{+}} \)

\( x \) (average) momentum fraction of the active quark \( x = \frac{k^{++'}+p^{+}}{p^{++'}+p^{+}} \)

\( \Im A_{DVCS}(\xi, t) \rightarrow GPD^{(+)}(\xi, \xi, t) \)

only sensitive to ‘diagonal’ \( x = \xi \)

limited \( \xi \) range, e.g. \( -t = \frac{4\xi^2 M^2 + \Delta^2}{1-\xi^2} \) implies \( \xi > \xi_{min} \) for fixed \( t \)

\( \Re A_{DVCS}(\xi, t) \rightarrow \int_{-1}^{1} dx \frac{GPD^{(+)}(x, \xi, t)}{x-\xi} \) probes GPDs off the diagonal, but ...
\( A(\xi, t) \leftrightarrow GPD^{(+)}(\xi, \xi, t), \Delta(t) \)

- (Anikin, Teryaev, Diehl, Ivanov, ...): dispersion relation for DVCS amplitude

\[ \Re A(\nu, t, Q^2) = \frac{\nu^2}{\pi} \int_0^\infty \frac{d\nu^\prime}{\nu^\prime} \Im A(\nu^\prime, t, Q^2) \nu - \nu^\prime + \Delta(t, Q^2) \]

- In combination with LO factorization (\( A = \int_{-1}^1 dx H(x, \xi, t, Q^2) \))

\[ \Re A(\xi, t, Q^2) = \int_{-1}^1 dx \frac{H(x, \xi, t, Q^2)}{x - \xi} = \int_{-1}^1 dx \frac{H(x, x, t, Q^2)}{x - \xi} + \Delta(t, Q^2) \]

- Earlier derived from polynomiality (Goeke, Polyakov, Vanderhaegheh)

\[ \leftrightarrow \text{‘Condense’ information contained in } A_{DVCS} \text{ (fixed } Q^2) \text{ into } GPD(x, x, t, Q^2) \text{ & } \Delta(t, Q^2) \]
\[ A(\xi, t) \leftrightarrow GPD(\xi, \xi, t), \Delta(t) \]

- \( \Re A(\xi, t) = \int_{-1}^{1} dx \frac{H(x, \xi, t)}{x-\xi} \) probes GPDs for \( x \neq \xi \), but new information can be ‘projected back’ onto diagonal plus \( D \)-term!

- remaining ‘new’ (not in \( \Im A \)) info on GPDs after ‘projecting back’ onto diagonal:
  - \( D \)-form factor
  - constraints from \( \int dx \frac{GPD(x, x, t)}{x-\xi} \) on \( GPD(\xi, \xi, t) \) in kinematically inaccessible range \( \xi < \xi_{\text{min}} \) \& \( \xi > \xi_{\text{max}} \)

- Information away from diagonal \( (x = \xi) \): \( Q^2 \) evolution: changes \( x \) distribution in a known way for fixed \( \xi \)
DVCS $\rightsquigarrow$ GPD($x, \xi, t$) (a mathematical exercise)

$GPD(x, \xi, t, Q^2) = (1 - x^2) \sum_{n=0}^{\infty} C_n^{3/2}(x) \sum_{m=0(even)}^{n} a_{nm}(\xi) C_{n-m}(\xi, t, Q^2)$

- $C_n^{3/2}(x)$ Gegenbauer polynomials; $a_{nm}(\xi)$ known polynomial
- $C_k(\xi, t, Q^2)$ unknown, but evolve with known power $\sim \gamma_k$ of $\alpha_s(Q^2)$
- consider $x = \xi$ (relabel: $k = n - m$)

$$GPD(\xi, \xi, t, Q^2) = (1 - \xi^2) \sum_{k=0}^{\infty} C_k(\xi, t, Q^2) f_k(\xi) \quad (1)$$

with $f_k(\xi) = \sum_{m=0(even)}^{\infty} a_{m+k,m}(\xi) C_{m+k}^{3/2}(\xi)$ known function.

- for fixed $\xi$, each term in (1) evolves with different $\gamma_k$

$\leftrightarrow$ from $Q^2$-dependence of $GPD(\xi, \xi, t, Q^2)$ (fixed $\xi$ and $t$) over ‘wide’ range of $Q^2$, in principle possible to determine $C_k(\xi, t, Q^2)$

$\leftrightarrow$ $GPD(x, \xi, t, Q^2)$ for $x \neq \xi$ model-independently!
DVCS $\rightarrow$ $GPD(x, \xi, t)$ (a mathematical exercise)

- issues:
  - higher twist ‘contamination’
  - higher order evolution kernel
  - limited coverage in $Q^2$ (here, an EIC would be a giant leap!)
    and $\xi$
  - singular shape of GPDs (cusp at $x = \xi$) requires many polynomials in Gegenbauer expansion
Application of $\int_{-1}^{1} dx \frac{H(x, \xi, t)}{x-\xi} = \int_{-1}^{1} dx \frac{H(x, x, t)}{x-\xi} + \Delta(t)$

- take $\xi \to 0$ (should exist for $-t$ sufficiently large)

$$\int_{-1}^{1} dx \frac{H^+(x, 0, t)}{x} = \int_{-1}^{1} dx \frac{H^+(x, x, t)}{x} + \Delta(t)$$

$\leftarrow$ DVCS allows access to same generalized form factor $\int_{-1}^{1} dx \frac{H^+(x, 0, t)}{x}$ also available in WACS (wide angle Compton scattering), but $t$ does not have to be of order $Q^2$

$\leftarrow$ after flavor separation, $\frac{1}{F_1(t)} \int_{-1}^{1} dx \frac{H^+(x, 0, t)}{x}$ at large $t$ provides information about the ‘typical $x$’ that dominates large $t$ form factor
Overlap Representation for GPDs \((x > \zeta)\)

\[
GPD(x, \zeta, t) = \sum_{n, \lambda_i} (1 - \zeta)^{1 - \frac{n}{2}} \int \prod_{i=1}^{n} \frac{dx_i dk_{\perp i}}{16\pi^3} 16\pi^3 \delta \left( 1 - \sum_{j=1}^{n} x_j \right) \delta \left( \sum_{j=1}^{n} k_{\perp j} \right) \delta(x - x_1) \\
\times \psi^s_{(n)}(x'_i, k'_{\perp i}, \lambda_i) \ast \psi^s_{(n)}(x_i, k_{\perp i}, \lambda_i),
\]

- \(GPD(x, \zeta, t) = \frac{\sqrt{1 - \zeta}}{1 - \frac{\zeta}{2}} H(x, \zeta, t) - \frac{\zeta^2}{4(1 - \frac{\zeta}{2})^{1/2}} E(x, \zeta, t)\), for \(s' = s\)
- \(GPD(x, \zeta, t) = \frac{1}{\sqrt{1 - \zeta}} \frac{\Delta^1 - i \Delta^2}{2M} E(x, \zeta, t)\), for \(s' = \uparrow\) and \(s = \downarrow\)

\(\Delta\) is the transverse momentum transfer.

- \(x'_1 = \frac{x_1 - \zeta}{1 - \zeta}\) and \(k'_{\perp 1} = k_{\perp 1} - \frac{1 - x_1}{1 - \zeta} \Delta_{\perp}\) for the active quark, and
- \(x'_i = \frac{x_i}{1 - \zeta}\) and \(k'_{\perp i} = k_{\perp i} + \frac{x_i}{1 - \zeta} \Delta_{\perp}\) for the spectators \(i = 2, \ldots, n\).
GPDs in \( \perp \) position space \( (n = 2) \)

\[
GPD(x, \zeta, t) = \sum_{\lambda_i} \int \frac{d{k_{\perp}}}{16\pi^3} \psi^{s'}(x_1', k_{\perp 1}', \lambda_i)^* \psi^s(x_1, k_{\perp 1}, \lambda_i),
\]

- \( x_1' = \frac{x_1 - \zeta}{1 - \zeta} \) and \( k_{\perp 1}' = k_{\perp 1} - \frac{1-x_1}{1-\zeta} \Delta_{\perp} \) for the active quark
- spectator momentum constrained by momentum conservation: \( x_2 = 1 - x_1 \) and \( k_{\perp 2} = -k_{\perp 1} \)

Diagonalize by Fourier transform

\[
\tilde{\psi}^s(x, r_{\perp}) = \int \frac{d^2{k_{\perp}}}{2\pi} \psi^s(x, k_{\perp}) e^{i{k_{\perp}} \cdot r_{\perp}}
\]

- \( r_{\perp} \) is the \( \perp \) distance between active quark and spectator

\[\Rightarrow GPD(x, \zeta, t) \propto \int d^2{r_{\perp}} \tilde{\psi}^*(x', r_{\perp}) \tilde{\psi}^*(x', r_{\perp}) e^{-i\frac{1-x}{1-\zeta} r_{\perp} \cdot \Delta_{\perp}}\]
repeating the same steps in the general case \((n \geq 3)\) yields......

\[
GPD(x, \zeta, t) = \sum_n (1 - \zeta)^{1 - \frac{n}{2}} \int \prod_{i=1}^{n} \frac{d^2 \mathbf{r}_{\perp i}}{2\pi} \bar{\psi}_{(n)}(x_i', \mathbf{r}_\perp i) \bar{\psi}_{s}^{(n)}(x_i, \mathbf{r}_\perp i) e^{-i \frac{1-x}{1-\zeta} (\mathbf{r}_{\perp 1} - \mathbf{R}_\perp s) \cdot \Delta_\perp}
\]

\(\mathbf{R}_\perp s\) is the center of momentum of the spectators.

\(\mapsto\) FT of GPD w.r.t. \(\Delta_\perp\) gives overlap when active quark and spectators are distance \(\frac{1-x}{1-\zeta} \mathbf{r}_\perp\) apart
general case: $\Delta_\perp$ conjugate to $\frac{1-x}{1-\zeta} r_\perp$

special case: $\zeta = 0 \implies \frac{1-x}{1-\zeta} r_\perp = (1-x) r_\perp = b_\perp = \text{distance between active quark and center of momentum of hadron.}$

special case: $x = \zeta \implies \frac{1-x}{1-\zeta} r_\perp = r_\perp$

$\rightarrow$ for $x = \zeta$, the variable that is (Fourier) conjugate to $\Delta_\perp$ is the distance between the active quark and the center of momentum of the spectators $r_\perp$

unlike the $b_\perp$ distribution, which must become point-like for $x \rightarrow 1$, the $r_\perp$-distribution does not have to become narrow for $x \rightarrow 1$

Note: the $t$-slope still has to go to zero as $\zeta \rightarrow 1$, as

$$-t = \frac{\zeta^2 M^2 + \Delta^2_\perp}{1-\zeta}$$

$\rightarrow$ $t$-slope $B$ and $\Delta^2_\perp$-slope $B_\perp$ related via $B = (1-\zeta) B_\perp$
**Summary**

- GPDs $\xrightarrow{FT} IPDs$ (impact parameter dependent PDFs)
- $E(x, 0, -\Delta_{\perp}^2) \rightarrow \perp$ deformation of PDFs for $\perp$ polarized target
- $\kappa q/p \Rightarrow$ sign of deformation
- Information content of DVCS amplitude can be ‘condensed’ onto $GPD(\xi, \xi, t, Q^2) \& \Delta(t, Q^2)$
- $GPD(\xi, \xi, t, Q^2) \forall Q^2 \Rightarrow GPD(x, \xi, t, Q^2)$
  - How large $Q^2$ for LO factorization to be valid?
  - Still need long lever arm in $Q^2$ to use evolution...
- Fourier transform of GPDs w.r.t. $\Delta_{\perp}$ provide dependence of overlap matrix element on $\frac{1-x}{1-\zeta} r_{\perp}$ where $r_{\perp}$ is separation between active quark and the COM of spectators
- for $x = \zeta$, variable conjugate to $\Delta_{\perp}$ is $r_{\perp}$
Summary

- distribution of $\perp$ polarized quarks in unpol. target described by chirally odd GPD $\bar{E}_T^q = 2\bar{H}_T^q + E_T^q$

- attractive FSI $\Rightarrow$ measurement of $h_1^{\perp,q}$ (DY,SIDIS) provides information on $\bar{E}_T^q$ and hence on spin-orbit correlations

- expect:
  
  $h_1^{\perp,q} < 0$ \hspace{1cm} $|h_1^{\perp,q}| > |f_{1T}^q|$

- $x^2$-moment of chirally odd twist-3 PDF $e(x) \rightarrow$ transverse force on transversely polarized quark in unpolarized target ($\rightarrow$ Boer-Mulders)