TEMPORAL STRUCTURE OF THE SUPERNOVA NEUTRINO SIGNAL

Neutron Star Tomography

Sanjay Reddy
Luke Roberts, Vincenzo Cirigliano, Jose Pons
• Neutrinos are trapped during core collapse. Collapse is nearly adiabatic.

• Gravitational binding energy is stored as thermal energy and lepton degeneracy energy.
Supernova Neutrinos

Past:
SN 1987a: \( \sim 20 \) neutrinos \( \ldots \) in support of supernova theory

Future:
Can detect \( \sim 10,000 \) neutrinos from galactic supernova

\[
3 \times 10^{53} \text{ ergs} = 10^{58} \times 20 \text{ MeV Neutrinos}
\]

\[
\frac{dN_{\text{detect}}}{dt} \sim \frac{\sigma_{\text{ref}} \times n_p \times M_{\text{tons}}}{4\pi D^2} \frac{E^2_\nu}{m^2_e} \frac{dN_{\text{emit}}}{dt}
\]

Pons et al. (2002)
Protoneutron Star Evolution

Neutrino diffusion deleptonizes and cools the PNS.

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Typical time-scales:

\[ T(t) \approx T(t = 0) \left( 1 - \frac{t}{\tau_C} \right) \]

\[ \tau_C \approx C_v \frac{R^2}{c\langle \lambda_v \rangle} \]

\[ Y_v(t) \approx \frac{\mu_v^3}{6\pi^2} \]

\[ \approx Y(t = 0) \exp\left( -\frac{t}{\tau_D} \right) \]

\[ \tau_D = \frac{3}{\pi^2} \frac{\partial Y_L}{\partial Y_v} \frac{R^2}{c\langle \lambda_{ve} \rangle} \]

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Response of Interacting System

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]
Response of Interacting System

\[ \Delta R \]

\[ \omega = q \]

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

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Response of Interacting System

\[ \omega, q \]

\[ \Delta R \]

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Response of Interacting System

\[ \omega, q, \nu \]

\[ \Delta R \]

\[ \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \]

\[ \tau_{\text{collision}} = \text{Collision Time} \]

\[ q = \frac{2\pi}{\lambda} > \frac{1}{\Delta R} \]

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\[ \omega = \frac{2\pi}{\tau} > \frac{2\pi}{\tau_{\text{collision}}} \]

\[ \omega = \frac{2\pi}{\tau} < \frac{2\pi}{\tau_{\text{collision}}} \]

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Weak Interaction Rates

\[ L = \frac{G_F}{2\sqrt{2}} \, l_\nu(x) \, j^\mu(x) \]
\[ l_\nu = \bar{\nu}(x) \gamma_\nu (1 - \gamma_5) \nu(x) \]
\[ j^\mu = \bar{\psi}(x) (c_V \gamma^\mu - c_A \gamma^\mu \gamma_5 + iF_2 \sigma^{\mu\nu} \frac{q_\nu}{2M}) \psi(x) \]

\[ \frac{d^2 \sigma}{V \, d \cos \theta \, dE'} \approx G_F^2 \, \frac{E}{E'} \, \text{Im} \left[ L_{\mu\nu}(k, k + q) \, \Pi^{\mu\nu}(q) \right] \]
\[ L_{\mu\nu} = \text{Tr} \left[ l_\mu(k) \, l_\nu(k + q) \right] \]
\[ \Pi^{\mu\nu} = \int \frac{d^4 p}{(2\pi)^4} \, \text{Tr} \left[ j^\mu(p) \, j^\nu(p + q) \right] \]
Neutrino-Nucleon Scattering

Neutrinos couple to density and spin

\[ j^{\mu}(x) = \bar{\psi}(x) \gamma^{\mu}(c_{V} - c_{A} \gamma_{5}) \psi(x) \]

\[ \rightarrow c_{V} \psi^{+} \psi \delta^{\mu 0} - c_{A} \psi^{+} \sigma^{i} \psi \delta^{\mu i} \]

\[ \frac{d\Gamma}{d \cos \theta dE'_{\nu}} = \frac{G_{F}^{2}}{4\pi^{2}} (1 - f_{\nu}(E'_{\nu})) E'_{\nu}^{2} \]

\[ \times \left( c_{V}^{2} (1 + \cos \theta) S(|\vec{q}|, \omega) + c_{A}^{2} (3 - \cos \theta) S^{A}(|\vec{q}|, \omega) \right) \]

\[ S(|\vec{q}|, \omega) = \int_{-\infty}^{\infty} dt \; \text{exp}(i\omega t) \left\langle \rho(\vec{q}, t) \rho(-\vec{q}, 0) \right\rangle \]

\[ S^{A}(|\vec{q}|, \omega) = \int_{-\infty}^{\infty} dt \; \text{exp}(i\omega t) \delta_{ij} \left\langle \sigma_{i}(\vec{q}, t) \sigma(-\vec{q}, 0) \right\rangle \]

Iwamoto & Pethick (1982)
Computing Correlation Functions

No exact methods exist in strongly coupled quantum systems.

In the Fermi gas:

In the mean field approximation:

Corrections to single particle energy

Vertex Corrections
Computing Correlation Functions

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In the Fermi gas:

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Vertex Corrections

\[
\begin{align*}
\text{RPA}
\end{align*}
\]
Mean Free Paths in the RPA

\[ \Gamma_\mu + \Gamma_\nu = \Gamma_\mu G_{\mu\nu}(p + q) + G_{\nu\mu}(p) + V_{\sigma\tau}(p + q) \]

\[ V_\sigma \tau \]

Interactions move strength to higher energy

Reddy, Pons, Prakash, Lattimer (1999)
Mean Free Paths in the RPA

\[ \Gamma_\mu \Gamma_\nu = \Gamma_\mu G_{\mu\nu}(p+q) + \Gamma_\nu G_{\nu\mu}(p) \]

Interactions move strength to higher energy

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Classical Dense Gases

Exact method exists - Molecular dynamics

Eg. Nuclei interacting with screened Coulomb forces

\[ V(r) = \frac{Z^2 e^2}{r} \exp \left( -\frac{r}{\lambda} \right) \]

Importance of multi particle-hole excitations.

Including Multi-Pair Strength

Spin relaxation time in neutron or nuclear matter is small due to non-central forces.

Strength is spread due to coupling to multi particle-hole states—pushed to higher energy.

Expect larger reduction in cross-sections.

\[
\text{Im} \tilde{\chi}_\sigma(\omega, q \to 0) = \frac{\omega \tau_\sigma}{(1 + G_0)^2 + (\omega \tau_\sigma)^2}
\]

Lykasov, Pethick, Schwenk (2008)
Sum Rules:

Thermodynamic (compressibility) sum Rule:

\[
\lim_{q \to 0} \int \frac{d\omega}{2\pi} S_\rho(q,\omega) \frac{(1 - e^{\beta\hbar\omega})}{\hbar \omega} = \rho \left( \frac{\partial \rho}{\partial p} \right)_T
\]

This relates response to the equation of state - provides a good consistency check.

F-Sum Rule:

\[
\int \frac{d\omega}{2\pi} \omega S_\rho(q,\omega) \frac{(1 - e^{\beta\hbar\omega})}{2\hbar} = \frac{1}{2\hbar^2} \langle[[H,\rho(q)],\rho(q)]\rangle
\]
Phase Transitions

How different are the neutrino cross-sections in other phases of matter at high density?

Novel High Density Phases:
- Hyperons
- Kaons
- Quark Matter

QGP

Nuclear Matter

Neutron Stars

Nuclei

930

~1200 ?

?$\mu_B$ (MeV)

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Novel High Density Phases: Hyperons, Kaons, Quark Matter..
Phase Transitions

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Novel High Density Phases:
- Hyperons, Kaons
- Quark Matter

QGP

PNS Evolution

Nuclear Matter

Neutron Stars

Collapse

T (MeV)

μ_B (MeV)

170

130

930

~1200 ?

?.

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First-Order Transitions

A first-order phase transition generically leads to a heterogeneous phase.

It is energetically favorable to phase separate oppositely charged phases if the surface tension is not too large.

Mixed Phase: Heterogeneous phase with structures of size 5-10 fm.

Glendenning (1992) (nuclear -> quark)
Neutrino Mean Free Path in a Mixed Phase

Scattering from quark droplets in a quark-hadron transition.

\[
\frac{d\sigma}{d\cos(\theta)} = N_D \frac{G_F^2}{16\pi} S_q Q_W^2 E_{\nu}^2 (1 + \cos(\theta))
\]

Conclusions

• Neutrino diffusion is one key input that sets the time-scale of a supernova neutrino signal.
• The density dependence of cross-sections are likely to affect the time structure.
• Neutrino cross-section in hot and dense nuclear matter are likely suppressed by factors of 2-4.
• Phase transitions are likely to change the neutrino cross-sections.
Conclusions

• Neutrino diffusion is one key input that sets the time-scale of a supernova neutrino signal.
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• Phase transitions are likely to change the neutrino cross-sections.

Would a future galactic supernova signal allow us to map out the neutrino opacity as function of depth in the neutron star?