Symmetry Origin of Observable Nonunitary Neutrino Mixing Matrix in TeV Scale Seesaw Models

Ernest Ma

Physics and Astronomy Department
University of California
Riverside, CA 92521, USA
Contents

• Seesaw Variations

• Zero Neutrino Mass from Cancellation

• Observable Nonunitary Neutrino Mixing Matrix

• Symmetry Origin of the Texture Hypothesis

• Concluding Remarks
Seesaw Variations

With 1 doublet neutrino $\nu$ and 1 singlet neutrino $N$, their $2 \times 2$ mass matrix is the well-known

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D \\ m_D & m_N \end{pmatrix},$$

resulting in the famous seesaw formula $m_\nu \simeq -m_D^2/m_N$. Hence $\nu - N$ mixing $\simeq m_D/m_N \simeq \sqrt{m_\nu/m_N} < 10^{-6}$, for $m_\nu < 1 \text{ eV}$ and $m_N > 1 \text{ TeV}$. 
Consider now 1 $\nu$ and 2 singlets: $N_{1,2}$. Their $3 \times 3$ mass matrix is then

$$\mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & 0 \\ m_D & m_1 & m_N \\ 0 & m_N & m_2 \end{pmatrix},$$

resulting in $m_\nu \simeq m_D^2 m_2/(m_N^2 - m_1 m_2)$. Since the limit $m_1 = 0$ and $m_2 = 0$ corresponds to lepton number conservation ($L = 1$ for $\nu$ and $N_2$, $L = -1$ for $N_1$), their smallness is natural $\Rightarrow$ the inverse seesaw [Mohapatra/Valle(1986)].
Here $\nu - N_1$ mixing $\sim m_\nu/m_D$, whereas $\nu - N_2$ mixing $\sim m_D/m_N$, and they are not constrained to be the same as was in the canonical seesaw.

For example, let $m_D \sim 10$ GeV, $m_N \sim 1$ TeV, $m_2 \sim 10$ keV, then $m_\nu \sim 1$ eV, and $\nu - N_1$ mixing $\sim 10^{-10}$ is very small, but $\nu - N_2$ mixing $\sim 10^{-2}$ is large enough to be observable.

Note that the geometric mean of the two mixings is again $10^{-6}$ as before.
**Linear Seesaw:**

[Barr(2004), Malinsky/Romao/Valle(2005)]

\[ \mathcal{M}_{\nu N} = \begin{pmatrix} 0 & m_D & m'_D \\ m_D & 0 & m_N \\ m'_D & m_N & 0 \end{pmatrix} \]

\[ \Rightarrow \ m_\nu \simeq -2m_Dm'_D/m_N, \text{ which is only linear in } m_D. \]

**Ma(2009):** For the linear seesaw to work, \( m'_D \) must be very small. In the limit it is zero, \( \mathcal{M}_{\nu N} \) is the same as that of the inverse seesaw in the same limit, so they must have the same origin.
To prove this, let $m'_D/m_D = \tan \theta$, then rotate the $(N_1, N_2)$ basis by $\theta$, we get

$$
M_{\nu N} = \begin{pmatrix}
0 & m_D/c & 0 \\
 m_D/c & m_N s_2 & m_N c_2 \\
0 & m_N c_2 & -m_N s_2 \\
\end{pmatrix}
$$

where $c = \cos \theta$, $c_2 = \cos 2\theta$, and $s_2 = \sin 2\theta$.

For small $\theta$, this is just the inverse seesaw, with

$$
m_{\nu} \simeq -\frac{2m_N m'_D}{m_D} \left( \frac{m_D^2}{m_N^2} \right) = -\frac{2m_D m'_D}{m_N}.
$$
Zero Neutrino Mass from Cancellation

If the inverse or linear seesaw mechanisms are invoked to get large $\nu - N$ mixing for each family, we will need 2 singlet neutrinos for each doublet neutrino, thus implying a $9 \times 9$ mass matrix. Is that really necessary?

Buchmuller/Wyler(1990), Pilaftsis(1992,2005), Kersten/Smirnov(2007): Consider first two families: $\nu_{1,2}$ and $N_{1,2}$, with $M_N = \text{diag}(M'_1, M'_2)$, where

$$M_D = \begin{pmatrix} a_1 b_1 & a_1 b_2 \\ a_2 b_1 & a_2 b_2 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} b_1 & b_2 \end{pmatrix}.$$
One neutrino mass is zero because the determinant of $M_D$ is zero by construction. If we now also impose the arbitrary condition $b_1^2/M'_1 + b_2^2/M'_2 = 0$, then the other neutrino mass is zero as well. However, $\nu - N$ mixing may be large, because $a_i b_j$ need not be small. In other words, zero mixing is now not the limit of zero neutrino mass as in the previous seesaw formulas.

If small deviations from this texture are present, small neutrino masses will appear, but the large $\nu - N$ mixing will remain. This appears to be fine tuning, but a lot of phenomenological studies have been done in this context.
Observable Nonunitary Neutrino Mixing Matrix

The addition of heavy singlet fermions $N_i$ to the Standard Model induces both the well-known dimension-five operator

$$\Lambda^{-1} f_{\alpha\beta} (L_\alpha \Phi)(L_\beta \Phi) \quad \text{[mass mixing]}$$

and the less-known dimension-six operator

$$\Lambda^{-2} f_{\alpha\beta} (\Phi^\dagger \bar{L}_\alpha) i \partial^\mu \gamma_\mu (L_\beta \Phi) \quad \text{[kinetic mixing]}.$$
Both operators will probe $m_D/m_N$, which is of course too small in the canonical seesaw, but if the inverse seesaw or linear seesaw or the texture hypothesis is used, then they will provide useful phenomenological constraints.


Let the neutrino mixing matrix be $(1 + \eta)U$, then

$$|\eta| < \begin{pmatrix} 2.0 \times 10^{-3} & 6.0 \times 10^{-5} & 1.6 \times 10^{-3} \\ \sim & 8.0 \times 10^{-4} & 1.1 \times 10^{-3} \\ \sim & \sim & 2.7 \times 10^{-3} \end{pmatrix}.$$
For the texture hypothesis, i.e.\n\[
M_D = \begin{pmatrix}
a_1 \\
a_2 \\
a_3 \\
\end{pmatrix}
\begin{pmatrix}
b_1 & b_2 & b_3 \\
\end{pmatrix}
\]
with the condition \( b_1^2/M_1' + b_2^2/M_2' + b_3^2/M_3' = 0 \), the parameters \( \eta_{e\tau} \) and \( \eta_{\mu\tau} \) are correlated:\n\[
\left| \frac{\eta_{e\tau}}{1.6 \times 10^{-3}} \right|^2 + \left| \frac{\eta_{\mu\tau}}{1.1 \times 10^{-3}} \right|^2 < 1.
\]
Symmetry Origin of the Texture Hypothesis

He/Ma(2009): To understand the mechanism and symmetry of the texture hypothesis, change the neutrino basis to

$$
\mathcal{M}_{\nu N} = \begin{pmatrix}
0 & 0 & m_1 & 0 \\
0 & 0 & 0 & m_2 \\
m_1 & 0 & M_1 & M_3 \\
0 & m_2 & M_3 & M_2
\end{pmatrix}.
$$
If $M_D$ of the texture hypothesis is now rotated on the left with $-\tan^{-1}(a_1/a_2)$ and on the right with $\tan^{-1}(b_1/b_2)$, then $m_1 = 0$ automatically.

Furthermore, $M_1 = \cos^2 \theta_R M'_1 + \sin^2 \theta_R M'_2 = (b_1^2/M'_1 + b_2^2/M'_2)M'_1M'_2/(b_1^2 + b_2^2) = 0$ as well.

Hence $\nu_1$ and $\nu'_2 = (M_3\nu_2 - m_2N_1)/\sqrt{M_3^2 + m_2^2}$ are massless, the latter showing also how the large mixing occurs between $\nu_2$ and $N_1$, i.e. through the inverse seesaw mechanism. However, lepton number conservation would not only forbid $M_1$ but also $M_2$, which is arbitrary here. Where is the symmetry which does this?
Let $\nu_{1,2}, N_{1,2}$ have $L = 1, 1, 3, -1$.

Add the usual Higgs doublet $(\phi_1^+, \phi_1^0)$ with $L = 0$ and the Higgs singlet $\chi_2$ with $L = 2$. Then $m_2$ comes from $\langle \phi_1^0 \rangle$, $M_2$ from $\langle \chi_2 \rangle$, and $M_3$ from $\langle \chi_2^\dagger \rangle$.

$m_1 = 0$ at tree level, because there is no Higgs doublet with $L = -4$, and $M_1 = 0$ at tree level, because there is no Higgs singlet with $L = \pm 6$.

In one loop, $M_1$ will be induced, thus giving $\nu_2'$ an inverse seesaw mass $= M_1 m_2^2 / M_3^2$. Once $\nu_2$ is massive, $\nu_1$ also gets a two-loop radiative mass from the exchange of two $W$’s [Babu/Ma(1988)].
Figure 1: One-loop generation of $M_1$. 
Figure 1: Two-$W$ generation of neutrino mass.
With small $m_1$ and $M_1$, the $4 \times 4$ neutrino mass matrix $\mathcal{M}_{\nu N}$ is reduced to the $2 \times 2$

$$\mathcal{M}_\nu \simeq \begin{pmatrix} m_1^2 M_2 / M_3^2 & -m_1 m_2 / M_3 \\ -m_1 m_2 / M_3 & M_1 m_2^2 / M_3^2 \end{pmatrix}.$$ 

Since $M_2 \sim M_3$ in this hypothesis, the (1,1) entry is a canonical seesaw, whereas the (2,2) entry is an inverse seesaw and the (1,2) or (2,1) entry is a linear seesaw. In this basis, only $\nu_2$ has possible large mixing with $N$. Similarly, if 3 families are considered with 3 $N$, only one linear combination of the 3 $\nu$ may mix significantly with $N$. 

For three families, let

\[ \mathcal{M}_{\nu N} = \begin{pmatrix}
0 & 0 & 0 & m_1 & 0 & 0 \\
0 & 0 & 0 & 0 & m_2 & 0 \\
0 & 0 & 0 & 0 & 0 & m_3 \\
m_1 & 0 & 0 & M_1 & M_4 & M_5 \\
m_2 & 0 & M_4 & M_2 & M_6 \\
m_3 & M_5 & M_6 & M_3 \\
\end{pmatrix}. \]

The texture hypothesis is equivalent to \( m_1 = m_2 = 0 \) and \( M_1 = M_4 = 0 \).
To enforce this pattern at tree level, use $L = 1$ for $\nu_{1,2,3}$ as usual, but $L = 3, -2, -1$ for $N_{1,2,3}$.

Add one Higgs doublet with $L = 0$ and three Higgs singlets with $L = 2, 3, 4$. Then $M_1 = M_4 = 0$, because there is no Higgs singlet with $L = \pm 6$ or $L = \pm 5$, but will become nonzero in one loop, whereas $m_1 = m_2 = 0$ to all orders.

To obtain nonzero $m_{1,2}$, a second Higgs doublet with $L = -4, 1$ may be added. $\nu_{1,2}$ is then massive in one loop, and $\nu_{2,1}$ becomes massive in two loops.
Lepton number has been used as a global U(1) symmetry, but a discrete version (e.g. $Z_7$) also works.

The global U(1) may also be gauged, using either $U(1)_{B-L}$ or $U(1)_\chi$ from $E_6$ where $Q_\chi = 5(B - L) - 4Y$. Many possible signatures of these extensions may be looked for at the LHC.

For example, with $U(1)_\chi$, it is possible to produce $N$ in pairs through $q\bar{q} \rightarrow Z' \rightarrow N\bar{N}$, if kinematically allowed. The final states to be analyzed are $l^\pm l^\mp W^\pm W^\mp$ and $l^\pm l^\pm W^\mp W^\mp$. Without new physics, $N$ is not likely to be observable. [Ibarra/Molinaro/Petcov(2010).]
Concluding Remarks

TeV scale seesaw models may allow $\nu - N$ mixing to be large enough, so that the nonunitarity of the neutrino mixing matrix, i.e. $(1 + \eta)U$, may become observable.

In the texture hypothesis, special choices of lepton symmetry are required to maintain the assumed pattern, in which case extra Higgs particles must appear.

The phenomenological constraints on $\eta_{e\tau}$ and $\eta_{\mu\tau}$ are of order $10^{-3}$. Neutrino oscillations are not much affected in such models.