Perturbative description of jet-medium interaction

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Quantifying properties of Hot QCD Matter
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Jet-medium interactions

- Weakly coupled
  - Weakly interacting medium
    - e.g. AMY

- Weakly coupled
  - Strong interacting medium
    - e.g. LRW

- Strongly coupled
  - Strongly interacting medium
    - AdS drag string
Jet-medium interactions

• HT
  – Assume weak coupling between hard partons and the medium
  – No assumption about the transport properties of the medium
  – Global analysis of light and heavy flavor jet quenching
  – Extend to energy deposition to medium by jets
Factorization paradigm

The hard perturbative part is factorized from soft non-perturbative regimes
Build vacuum shower

\[ D(z, \mu^2) = \quad \text{Contains all emissions up to scale } \mu \]
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Increasing the scale => rebuilding the shower
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Increasing the scale => rebuilding the shower

The shower evolution is governed by DGLAP equations

\[
\frac{\partial \tilde{D}_{i\to h}(z, Q^2)}{\partial \ln Q^2} = \sum_j \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P_{i\to j}(y) D_{j\to h}(z/y, Q^2)
\]
Jet shower in medium

Jet shower is modified by multiple scatterings
Jet shower in medium

Jet shower is modified by multiple scatterings

\[ D(z, \mu^2 + \delta \mu^2) = \]

We have both vacuum and medium contribution
How does medium affect the jet?

The parton travels a soft gluon field and suffers transverse momentum diffusion, longitudinal drag and diffusion.

Assuming independent multiple scatterings

\[ \frac{\partial \phi}{\partial L^-} = \frac{1}{2} \hat{q} \nabla_{q\perp}^2 \phi + \hat{e} \frac{\partial \phi}{\partial q^-} + \frac{1}{4} \hat{e}_2 \frac{\partial^2 \phi}{\partial q^{-2}} \]

These transport coefficients can be calculated in a well-defined medium (i.e., HTL)

\[ \hat{q} = \left[ 4\pi^2 \alpha_s C_R / (N_c^2 - 1) \right] \int dy^- \langle F^{+\mu} (y^-) F^\mu_+ (0) \rangle \]

\[ \hat{e} = \left[ 4\pi^2 \alpha_s C_R / (N_c^2 - 1) \right] \int dy^- dy^+ \langle F^{++} (y^-) F^{++} (y^+) \rangle \]

\[ \hat{e}_2 = \left[ 4\pi^2 \alpha_s C_R / (N_c^2 - 1) \right] \int dy^- \langle F^{+-} (y^-) F^{+-} (0) \rangle \]

Majumder, Muller, PRC (2008)
Majumder, PRC (2009)
Typical single gluon emission kernel

Very hard in general: transverse momentum broadening, longitudinal momentum loss, large angle radiation, ...

Within collinear approx., single gluon emission cross section from multiple scattering of a hard parton in a dense medium: (covered by Majumder’s talk on Monday), arXiv:0912.2987
Single gluon emission single scattering

\[
\Delta \tilde{D}_{i\to h}(z, Q^2, q^-)|_{\xi_i}^{\xi_f} = \sum_j \int \frac{d\xi}{l_1^2} \frac{\alpha_s}{2\pi} \int \frac{dy}{y} P_{i\to j}(y) \int_{\xi_i}^{\xi_f} d\zeta \frac{\bar{q}(\zeta)}{\pi} \left[ 2 - 2 \cos \left( \frac{l_1^2 (\zeta - \xi_i)}{2q^- y (1 - y)} \right) \right] \tilde{D}_{j\to h}(z/y, l_1^2, q^- y)|_{\xi_{\zeta}}^{\xi_f}
\]
Multiple gluon emission

In the limit of $k_T << l_T$ and for $l_T$ ordered subsequent emissions, DGLAP for MMFF

\[
\frac{\partial \tilde{D}_{i \to h}(z, Q^2, q^-)}{\partial \ln Q^2} \bigg|_{\xi_i} = \sum_j \frac{\alpha_s}{2\pi} \int \frac{dy}{y} \int_{\xi_i}^{\xi_f} d\zeta \tilde{P}_{i \to j}(y, \zeta, Q^2, q^-) \tilde{D}_{j \to h}(z/y, Q^2, q^- y) \bigg|_{\zeta_f} + \text{vacuum part}
\]

\[
\tilde{P}_{i \to j} = P_{i \to j}(y) \frac{\hat{q}(\zeta)}{\pi Q^2} \left[ 2 - 2 \cos \left( \frac{\zeta - \zeta_i}{\tau_f} \right) \right]
\]

\[
\tau_f = \frac{2q^- y (1 - y)}{l^2_{\perp}}
\]
Heavy quark radiative energy loss

For heavy quarks, still similar evolution equation

$$\frac{\partial \tilde{D}_{i \to h}(z, Q^2, q^-)|_{\zeta_i}}{\partial \ln Q^2} = \sum_j \frac{\alpha_s}{2\pi} \int \frac{dy}{y} \int_{\zeta_i}^{\zeta_f} d\zeta \tilde{P}_{i \to j}(y, \zeta, Q^2, q^-) \tilde{D}_{j \to h}(z/y, Q^2, q^-y)|_{\zeta_i}^{\zeta_f}$$

with an additional overall suppression factor due to the finite mass

$$f_{Q/q} = \left[ \frac{l_{\perp}^2}{l_{\perp}^2 + (1-y)^2 M_Q^2} \right]^4$$

and smaller formation time

$$\tau_f = \frac{2q^-y(1-y)}{l_{\perp}^2 + (1-y)^2 M_Q^2}$$

Brick (only radiative energy loss)

\[ q_{\text{hat}} = 2 \text{GeV}^2/\text{fm} \]

Note different shapes of vacuum FF for light and heavy quarks
Methodology of jet E-loss calculation

\[d\sigma_h = \sum_{ab} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab\rightarrow jd} \otimes \tilde{D}_{h/j}\]
Methodology of jet E-loss calculation

\[ d\sigma_h = \sum_{a,b,d} f_{a/A} \otimes f_{b/B} \otimes d\sigma_{ab\rightarrow jd} \otimes \tilde{D}_{h/j} \]
Modeling medium

- Wood-Saxon for nuclear density function

- Entropy density $s \sim T^3 \sim \rho_{\text{part}}$

- Medium thermalized at $\tau_0 = 0.6\text{fm}/c$, with $T_0 = 400\text{MeV}$, $T_c = 160\text{MeV}$

- 1-D Bjorken expansion for the medium

- Hard jets vertices determined by binary collision density
For LO HTL medium

Elastic loss included with both longitudinal drag and diffusion

$$D'(z) = \int d\Delta z P(\Delta z) D(z/(1 - \Delta z))/(1 - \Delta z)$$

$$Q_{\text{min}} = \sqrt{2}\text{GeV}$$ for central collisions

Wicks, Horowitz, Djordjevic, Gyulassy, NPA (2007)
What else we can do?

• LO HTL result

\[ \hat{q} = \frac{g^4 T^3 C_s}{2\pi} \left[ 1.28 \log \left( \frac{q_{\perp}^{\text{max}}}{4T} \right) + 0.71 \right] \]

• NLO HTL calculation from Caron-Hout, arXiv:0811.1603; NPA (2009)

\[ \hat{q} = \frac{g^4 T^3 C_s}{2\pi} \left[ 1.28 \log \left( \frac{q_{\perp}^{\text{max}}}{4T} \right) + 1.25 \right] \]

• Calculation for jet weakly coupled to strongly interacting medium from Liu, Rajagopal, Wiedemann, PRL (2006)

\[ \hat{q}_{\text{SYM}} = \frac{\pi^2}{a} \sqrt{\Lambda} T^3 = \frac{\pi^{3/2} \Gamma(\frac{3}{4})}{\Gamma(\frac{5}{4})} \sqrt{\Lambda} T^3 \approx 26.69 a_{\text{SYM}} N_c T^3 \]

• These results may suggest log(energy)-independent transport coefficients
Results

Only one parameter

\[ \hat{q} \propto T^3 \]

Three transport coefficients related by

\[ \frac{d(\Delta p_{\perp})^2}{dt} \approx 2 \frac{d(\Delta p_z)^2}{dt} \approx \frac{4T}{|v|} \frac{dp_z}{dt} \]

\[ \hat{q}_0 \approx 2 \text{ GeV}^2/\text{fm} \text{ at } T = 400 \text{ MeV} \quad \tau_0 = 0.6 \text{ fm}/c \]
Where does the lost energy go?

• Jet lose energy => medium gain energy

• Energy loss is roughly:

\[
\Delta E_{\text{rad}} \simeq \frac{1}{2} \hat{q} L^2, \quad \Delta E_{\text{coll}} \simeq \hat{e} L \Rightarrow \frac{dE_{\text{lost}}}{dL} = \hat{q} L + \hat{e}
\]

• The energy lost by jet = the energy gained by medium?

• What about the space-time pattern of the energy deposition?

\[
\frac{dE_{\text{deposit}}}{dL} = ?
\]

• How much fraction of energy is deposited in the medium when jet gets out of the medium?
How does a jet deposit energy?

Jet loses/deposits energy and momentum through elastic collisions. This will lead to a constant energy deposition rate.
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Jets lose energy also by radiations
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Jets lose energy also by radiations

The radiations serve as additional sources for energy deposition
Not always all lost energy are deposited into medium
Similar to evolution of FF

Suppose at some low scale $\mu$, we know the collisional energy deposit rate $dE/dL$

$$\Delta E(\mu^2)|_{\zeta_i}^{\zeta_f} = \int_{\zeta_i}^{\zeta_f} d\zeta \frac{dE}{dL}(\zeta) = \zeta_i \overbrace{\ldots}^{\zeta_f}$$

This contains the contribution to energy deposition from all emissions up to $\mu$
Similar to evolution of FF

Suppose at some low scale $\mu$, we know the collisional energy deposit rate $\frac{dE}{dL}$

$$\Delta E(\mu^2)|_{\zeta_i}^{\zeta_f} = \int_{\zeta_i}^{\zeta_f} d\zeta \frac{dE}{dL}(\zeta) = \zeta_i \quad \zeta_f$$

This contains the contribution to energy deposition from all emissions up to $\mu$

Now increase the virtuality, jet will drop virtuality by radiation

The total energy deposition would be from the parent parton from $z_i$ to $z$, and two splitting partons from $z$ to $z_f$
Similar to evolution of FF

Suppose at some low scale $\mu$, we know the collisional energy deposit rate $dE/dL$

$$\Delta E(\mu^2)|^{\zeta_f}_{\zeta_i} = \int_{\zeta_i}^{\zeta_f} d\zeta \frac{dE}{dL}(\zeta) = \zeta_i \rightarrow \zeta_f$$

This contains the contribution to energy deposition from all emissions up to $\mu$

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The total energy deposition would be from the parent parton from $z_i$ to $z$, and two splitting partons from $z$ to $z_f$

One may write down similar evolution equation for energy deposition

$$\frac{d\Delta E_q(E, Q^2)|^{\zeta_f}_{\zeta_i}}{d \ln Q^2} = \frac{\alpha_s}{2\pi} \int dy \int_{\zeta_i}^{\zeta_f} d\zeta P_{q\rightarrow qg}(y, \zeta, Q^2, E) \left\{ \Delta E_q(E, Q^2|^{\zeta_f}_{\zeta_i} + \Delta E_q(yE, Q^2|^{\zeta_f}_{\zeta_i} + \Delta E_q((1 - y)E, Q^2|^{\zeta_f}_{\zeta_i}) \right\}$$
Energy deposition result

Use HTL result as input at scale $\mu_0 = 4T$

$$\frac{d\Delta E(\mu_0, E)}{d\zeta} = \frac{C_R\alpha_s(\mu_0^2)m_D^2}{4} \ln \left[ \frac{4ET}{m_D^2} \right]$$

$E = 20\text{GeV}, T = 300\text{MeV}$

Energy deposition rate grows with length/time

Only a part of total energy loss is deposited in the medium

$$\Delta E_{\text{deposit}} < \Delta E_{\text{lost}}$$

GYQ, Majumder, Song, Heinz, PRL (2009)
Transverse momentum deposition

Use HTL result as input at scale \( \mu_0 = 4T \)

\[
\frac{d\langle p_{\perp}^2 \rangle(\mu_0, E)}{d\xi} = C_R \alpha_s(\mu_0^2) m_D^2 T \ln \left( \frac{4ET}{m_D^2} \right)
\]

\( E = 20\text{GeV}, T = 300\text{MeV} \)

Similar to energy deposition, transverse momentum deposition rate also grows with length/time

GYQ, Majumder, Song, Heinz, PRL (2009)
What does it do to the medium?

• Use a simple linear hydro with a relaxation time $\tau_R=1/m_D$

• The source term (only energy deposition included here)

$$J^\mu \equiv \left[ \frac{d\Delta E(\mu, E)}{d\zeta}, 0, 0, \frac{dp_z(\mu, E)}{d\zeta} \right] \delta^2(r_\perp)\delta(t-z).$$

• Assume energy deposited is a small perturbation, solving linear hydro equation

$$T^{\mu\nu} \approx T_0^{\mu\nu} + \delta T^{\mu\nu}; \quad \partial_\mu T_0^{\mu\nu} = 0, \quad \partial_\mu \delta T^{\mu\nu} = J^\nu.$$

• Decompose the small excess energy-momentum tensor as

$$\delta T^{00} \equiv \delta \epsilon, \quad \delta T^{0i} \equiv g^i,$$

$$\delta T^{ij} = \delta_{ij} \epsilon_s^2 \delta \epsilon - \frac{\nu}{sT} \left( \partial^i g^j + \partial^j g^i - \frac{2}{3} \delta_{ij} \nabla \cdot \vec{g} \right).$$
Our result

Multiple emissions increase the energy deposition, producing an enhanced Mach cone.
Another perturbative calculation

FIG. 2: (Color online) Result for the energy density wave (GeV/fm$^3$) excited by back to back quarks propagating along the $\hat{z}$ axis (as indicated by the black arrows). The plots, which show the collisional and radiative contributions separately, are shown in the $x$-$z$ plane, however, the results are cylindrically symmetric about the $\hat{z}$ axis. The Mach cone formation is visible in both contributions. The radiative induced excitation leads to a $t$ growth in the source strength, which can be seen from the plot.

Neufeld, Muller, PRL (2009)
Summary

- Both energy loss and deposition are computed consistently in the same perturbative formalism.
- A global description of both light and heavy flavor quenching.
- A large part of the energy is deposited in later history of the jet, leading to enhanced Mach cone like structure.