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Quantifying the Properties of Hot QCD Matter
1. **Introduction/Review**
   - Hydrodynamic simulations
   - Azimuthal correlations—$v_2$, $v_4$
   - Previous results

2. **Results from Hydrodynamics**
   - $v_4/(v_2)^2$ at RHIC energies
     - Ideal hydrodynamics / parameter dependence
     - Viscous hydrodynamics / dependence on $\delta f$
   - LHC prediction

3. **Conclusions**
OUTLINE

1 INTRODUCTION/REVIEW
   - Hydrodynamic simulations
   - Azimuthal correlations—\( \nu_2, \nu_4 \)
   - Previous results

2 RESULTS FROM HYDRODYNAMICS
   - \( \nu_4/(\nu_2)^2 \) at RHIC energies
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3 CONCLUSIONS
Can use (viscous) hydrodynamics to model evolution of a collision

Initial conditions and freeze out procedure are both important.
At freeze out the fluid is converted into particles

\[ \frac{dN}{dY d^2p_t} \propto \int p_\mu d\Sigma^\mu f(p_\mu) \]

with a distribution function given by kinetic theory

\[ f = f_0 + \delta f = e^{(-E/T)} \left[ 1 + \left( \frac{\chi(p)}{p^2} \right) p_i p_j \Pi^{ij} \right] \]

Momentum dependence of \( \delta f \) not universal—depends on particle dynamics (but difficult to calculate for a realistic hadron system at freezeout.)

The quadratic form used (until recently) by all viscous hydro groups may not be correct.
Our Hydro Model  (thanks to P. and U. Romatschke for the code)

- Second order conformal viscous hydrodynamics with constant $\eta/s$ in 2+1 D
  - no bulk viscosity
  - no chemical potential
- “Realistic” QCD equation of state (Laine and Schröder ’06)
  - no treatment of chemical non-equilibrium
- Glauber and Color Glass Condensate (fKLN) initial conditions
  - optical models (no fluctuations)
  - just two simple models that represent roughly the possible range of initial eccentricity
- Cooper-Frye freeze out prescription (with resonance feeddown—code by J. Sollfrank and P. Kolb)
  - including alternate choices for momentum dependence of $\delta f$
\[ \frac{dN}{dY\, d^2p_t} = v_0 \left[ 1 + \sum_n 2v_n \cos(n\phi) \right] \]
To compare to experiment, compute azimuthal moments:

$$\frac{dN}{dY \, d^2 p_t} = v_0 \left[ 1 + \sum_n 2v_n \cos(n\phi) \right]$$

Elliptic flow: $v_2 \equiv \langle \cos(2\phi) \rangle$
$v_2, v_4, \text{ETC.}$

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Elliptic flow: $v_2 \equiv \langle \cos(2 \phi) \rangle$

Hexadecapole flow: $v_4 \equiv \langle \cos(4 \phi) \rangle$
Large measured $v_2$ indicates small viscosity, but exact value depends on initial eccentricity.
**Ideal Hydrodynamics Prediction:** $v_4/(v_2)^2 = 1/2$

Perform a saddle point approximation:

$$\frac{dN}{dY d^2p_t} = \frac{d}{(2\pi)^3} \int p_\mu d\Sigma^\mu e\left(-\frac{p \cdot u}{T}\right)$$

For large $p_t$, saddle point is at the maximum $u$ parallel to momentum:

$$u_{\text{max}}(\phi) = U(1 + 2V_2 \cos(2\phi) + 2V_4 \cos(4\phi) + \ldots)$$

$$\implies v_2(p_t) = \frac{V_2U}{T} (p_t - m_t v)$$

$$v_4(p_t) = \frac{1}{2} \left( \frac{V_2U}{T} \right)^2 (p_t - m_t v)^2 + \frac{V_4U}{T} (p_t - m_t v)$$

$$= \frac{1}{2} v_2(p_t)^2 + \frac{V_4}{V_2} v_2(p_t)$$

($v \equiv U/\sqrt{1+U^2}$)

Experimental results are larger than 1/2.

Most of the discrepancy can be understood from fluctuations.
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RESULTS FROM HYDRODYNAMICS

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CONCLUSIONS
COMPARING TO PREVIOUS RESULTS: IDEAL HYDRO WITH $T_f = 100$ MeV

- Like previous calculations: Asymptotes to $\sim 1/2$ with corrections like $1/p_t$
- Some unexpected impact parameter dependence, but **not** due to initial eccentricity.
Sensitivity to $T_f$

- $v_4/(v_2)^2$ sensitive to $T_f$
- (For ideal hydro) best-fit $T_f$ for other observables (140 MeV) also results in flat $v_4/(v_2)^2$
**Identified Particles**

- Identified particles have same $v_4/(v_2)^2$
Viscous results at realistic $T_f = 140$ MeV

- Viscosity lowers $v_4/(v_2)^2$
- Standard “quadratic ansatz” for $\delta f$ destroys flat curve and increases dependence on $b$
- Using a different $\delta f$ can fix some of this:
RESULTS FROM HYDRODYNAMICS $v_4/(v_2)^2$ AT RHIC ENERGIES

$\delta f$ DEPENDENCE

Glauber

CGC

PHENIX $\eta/s = 0.0001$
Linear
$p^{1.5}$
Quadratic

PHENIX/1.37 $\eta/s = 0.0001$
Linear
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PHENIX/1.37 $\eta/s = 0.0001$
Linear
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Quadratic
**Quadratic ansatz for $\delta f$ difficult to reconcile with data.**

**$\chi(p)$ most likely somewhere between linear and quadratic.**

**Could provide stronger constraint as hydro models improve, giving insight into hadron dynamics.**
Similar to RHIC results at slightly lower freeze out temperature
Viscous corrections are smaller, making choice for $δf$ less relevant.
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Summary/Conclusions

- Hydrodynamic simulations confirm expectation of \( \frac{v_4}{(v_2)^2} \sim 1/2 \)
- Sensitive to \( T_f \), with 140 MeV giving a flat dependence on \( p_t \) in ideal hydro (and for \( \delta f \) with weak \( p_t \) dependence)
- \( \frac{v_4}{(v_2)^2} \) is insensitive to initial eccentricity (unlike \( v_2 \))
- Viscosity tends to decrease \( \frac{v_4}{(v_2)^2} \) for a realistic \( T_f \)
- Viscosity increases impact parameter dependence for standard quadratic ansatz for \( \delta f \), but decreases it for weaker \( p_t \) dependence.
- \( \frac{v_4}{(v_2)^2} \) at LHC should be similar to RHIC
Like in transport calculations, viscosity increases $\nu_4/(\nu_2)^2$ (but only with a small $T_f$)