Quark Recombination with Quark Number Conservation

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Outline

• Problem of the current coalescence formulation: *violation of quark number conservation*

• New formulation
  *using rate equations to conserve quark number of each flavor*

• Example results for light quarks:
  *different scalings at high $P_\perp$ & low $P_\perp*

• Example results for charm quarks

• Conclusions

• Outlook
Current formulation of quark coalescence

Let \( f(p,x) \equiv (2\pi)^3 dN / (d^3 x d^3 p) \)

\( \alpha \beta \rightarrow M : \)

\[
E \frac{dN_M(p)}{d^3 p} = \int \frac{d\sigma^\mu}{(2\pi)^3} \int d^3 q g_M |\Psi_p(q)|^2 f_\alpha(p_\alpha,x) f_\beta(p_\beta,x)
\]

\( \alpha \beta \gamma \rightarrow B : \)

\[
E \frac{dN_B(p)}{d^3 p} = \int \frac{d\sigma^\mu}{(2\pi)^3} \int d^3 q_1 d^3 q_2 g_B |\Psi_p(q_1,q_2)|^2 f_\alpha(p_\alpha,x) f_\beta(p_\beta,x) f_\gamma(p_\gamma,x)
\]

→ Lead to \( n_q \)-scalings:

\( v_{2,M}(p_\perp) \propto 2v_{2,q}(p_\perp / 2) \)

\( v_{2,B}(p_\perp) \propto 3v_{2,q}(p_\perp / 3) \)

Also: effects on \( P_\perp \)-spectra

For multiplicities, we have

\[
N_M \propto N_q^2, \quad N_B \propto N_q^3
\]
Problem of the current formulation

The unitarity problem:
\[ N_M \propto N_q^2, \quad N_B \propto N_q^3 \]

violate quark number conservation.

For example:
if q=qbar only form meson M → \( N_M=N_q \)
if they only form baryon B or Bbar → \( N_B=N_q/3 \)
→ We expect linear scaling for the total multiplicities

\[ f_q(p_\perp) \]

At low \( P_\perp \) we may expect quark coalescence to dominate
→ need to include quark number conservation

Only valid for quarks above a scale \( P_\perp_0 \):
quarks’ coalescence probability \( \ll 1 \)
& fragmentation dominates coalescence
**New formulation:**
include quark number conservation (unitarity)
so that the formulation is valid for all $P_{\perp}$

If hadronization is fast so that the hyper-surface does not change much:

$$
\rightarrow f_M(p,x) = \int d^3q g_M |\Psi_p(q)|^2 f_\alpha(p_\alpha,x) f_\beta(p_\beta,x)
$$

Generalize to the time($t_c$)-evolution of the coalescence process:

$$
\frac{df_M(p,x,t_c)}{dt_c} = \int d^3q C_M(p,q,x,t_c) f_\alpha(p_\alpha,x,t_c) f_\beta(p_\beta,x,t_c)
$$

*Simplify notations: let’s omit label $x$, rewrite $t_c$ as $t$:

$$
\Rightarrow \frac{df_M(p,t)}{dt} = \int d^3q C_M(p,q,t) f_\alpha(p_\alpha,t) f_\beta(p_\beta,t)
$$

Neglect internal quark momentum in a hadron: let $C_M(p,q,t) \equiv c_M(p,t) \delta(q)$

$$
\Rightarrow \frac{df_M(p_M,t)}{dt} \equiv f_M'(p_M,t) = c_M(p_M,t) f_\alpha(p_\alpha,t) f_\beta(p_\beta,t)
$$

with $p_M = p_q N_{qM} = 2p_q$, $p_B = p_q N_{qB} = 3p_q$

*Coalescing rate for meson $M$*
An example for light quarks

Let’s consider:

• 1 quark flavor
• no baryon chemical potential (thus $q=q\bar{q}$)
• formation of 1 type of meson $M$ and 1 type of baryon $B$
  (including the anti-baryon): $q\bar{q} \rightarrow M, 3q \rightarrow B, 3\bar{q} \rightarrow \bar{B}$

→ Rate equations:

$$f'_M(p_M, t) = c_M(p_M, t)f^2_q(p_q, t)$$
$$f'_B(p_B, t) = c_B(p_B, t)f^3_q(p_q, t)$$

Quark number conservation is given by

$$\delta N_q(p_q, t) = -\delta N_M(p_M, t) - 3\delta N_B(p_B, t)$$

$$\Rightarrow f'_q(p_q, t) = -N_{qM}^3 f'_M(p_M, t) - 3N_{qB}^3 f'_B(p_B, t)$$

Initial conditions ($p_q$ is the quark momentum vector):

$$f_q(p_q, t_0) \equiv f_0(p_q), \quad f_M(p_M, t_0) = 0, \quad f_B(p_B, t_0) = 0$$
A) Solutions when coalescing rates for M & B have the same t-dependence

We can then define 
\[ r(p_q) \equiv \frac{3N_{qB}^3 c_B(p_B,t)}{N_{qM}^3 c_M(p_M,t)} \]
and obtain

\[ r(p_q) \ln \left[ \frac{1+r(p_q)f_0(p_q)}{1+r(p_q)f_q(p_q,t)} \right] \frac{f_q(p_q,t)}{f_0(p_q)} \]

\[ + \frac{1}{f_q(p_q,t)} - \frac{1}{f_0(p_q)} = N_{qM}^3 \int_{t_0}^{t} c_M(p_M,u)du \equiv I_M(p_M,t) \]

A1) The limit of 
\[ r(p_q) \to 0 : \]

\[ \Rightarrow f_q(p_q,t) \approx \frac{f_0(p_q)}{1 + f_0(p_q)I_M(p_M,t)} \]

at leading-order in \( r(p_q) \)

→ Probability of coalescence for quarks at \( Pq \) is:

\[ p_{coal.}(p_q) = 1 - \frac{f_q(p_q,t_F)}{f_0(p_q)} = 1 - \frac{1}{1 + f_0(p_q)I_M(p_M,t_F)} \]
Probability of coalescence of quarks

\[ p_{\text{coal.}}(p_q) = 1 - \frac{1}{1 + f_0(p_q) I_m(p_M, t_F)} \]

\( f_0(p_q) \)

\( p_{\text{coal.}}(p_q) \sim 1 \) for soft quarks

provided \( f_0(p_q) I_m(p_M, t_F) \gg 1 \)

The scale \( P_{\perp 0} \) that separate soft from hard quarks here corresponds to

\[ f_0(p_q) I_m(p_M, t_F) = O(1) \]

\( P_{\perp 0} \)

Rapid decrease at high \( P_T \)

\( p_{\text{coal.}}(p_q) \ll 1 \) for hard quarks
**Hadron solutions:**

\[ f_M(p_M,t) \approx \frac{f_0^2(p_q)I_M(p_M,t)}{N_{qM}^3[1 + f_0(p_q)I_M(p_M,t)]}, \]

\[ f_B(p_B,t) \approx \frac{f_0^2(p_q)r(p_q)}{6N_{qB}^3}\left[1 - \frac{1}{[1 + f_0(p_q)I_M(p_M,t)]^2}\right], \]

* For hard quarks: \( f_0(p_q)I_M(p_M,t_F) \ll 1 \)

\[ \Rightarrow f_M^{\text{hard}}(p_M,t_F) \approx \frac{f_0^2(p_q)I_M(p_M,t_F)}{N_{qM}^3}, \quad f_B^{\text{hard}}(p_B,t_F) \approx \frac{f_0^3(p_q)I_B(p_B,t_F)}{3N_{qB}^3} \]

\[ \text{with} \quad I_B(p_B,t_F) = r(p_q)I_M(p_M,t_F) \]

One can show \( \int_{t_0}^{t_F} c_H(p_H,u)du = g_H \) (H=Meson or Baryon) for hard quarks

\[ \Rightarrow I_M(p_M,t_F) \& I_B(p_B,t_F) \text{ are just constants for hard quarks} \]

\[ \Rightarrow f_M^{\text{hard}}(p_M,t_F) \propto f_0^2(p_q), \quad f_B^{\text{hard}}(p_B,t_F) \propto f_0^3(p_q) \]

**Reproduces \( n_q \)-scalings**

\[ \rightarrow v_{2,M}^{\text{hard}}(p_\perp) \propto 2v_{2,q}(p_\perp/N_{qM}), \quad v_{2,B}^{\text{hard}}(p_\perp) \propto 3v_{2,q}(p_\perp/N_{qB}) \]
Hadron solutions to leading-order in $r(p_q)$:

$$f_M(p_M,t) \approx \frac{f_0^2(p_q)I_M(p_M,t)}{N_{qM}^3 \left[ 1 + f_0(p_q)I_M(p_M,t) \right]} ,$$

$$f_B(p_B,t) \approx \frac{f_0^2(p_q)r(p_q)}{6N_{qB}^3 \left[ 1 - \frac{1}{\left[ 1 + f_0(p_q)I_M(p_M,t) \right]^2} \right]}$$

* For soft quarks: $f_0(p_q)I_M(p_M,t_F) \gg 1$

$$\Rightarrow f_M^{\text{soft}}(p_M,t_F) \approx \frac{f_0(p_q)}{N_{qM}^3} , \quad f_B^{\text{soft}}(p_B,t_F) \approx \frac{f_0^2(p_q)r(p_q)}{6N_{qB}^3}$$

**Linear scaling due to unitarity;**

**Meson dominates here:** $N_M \approx N_q$

**Different scaling for Baryons**

If $r(p_q)$ does not depend on $\varphi(p_q)$, as in the case for hard quarks:

$$\rightarrow \nu_{2,M}^{\text{soft}}(p_\perp) \propto \nu_{2,q}(p_\perp / N_{qM}) , \quad \nu_{2,B}^{\text{soft}}(p_\perp) \propto 2\nu_{2,q}(p_\perp / N_{qB})$$

**Scaling is weaker than for hard quarks in this case**

**INT 2010: Quantifying the Properties of Hot QCD Matter**

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A) Solutions when coalescing rates for M & B have the same t-dependence

\[ f_q(p_q, t) \approx \frac{f_0(p_q)}{\sqrt{1 + 2f_0^2(p_q)I_B(p_B, t)}} , \]

\[ f_M(p_M, t) \approx \frac{\ln[1 + 2f_0^2(p_q)I_B(p_B, t)]}{2N_{qM}^3 r(p_q)} , \]

\[ f_B(p_B, t) \approx \frac{f_0(p_q)}{3N_{qB}^3} \left( 1 - \frac{1}{\sqrt{1 + 2f_0^2(p_q)I_B(p_B, t)}} \right) \]

at leading-order in \(1/r(p_q)\)

The scale \(P_{\perp 0}\) that separate soft from hard quarks here corresponds to

\[ f_0^2(p_q)I_B(p_B, t_F) = O(1) \]
The limit of \( r(p_q) \to \infty \):

* For hard quarks:
  
  same \( n_q \)-scalings are reproduced

* For soft quarks:

\[
\Rightarrow f_{M,\text{soft}}(p_M,t_F) \approx \frac{\ln[2f_0^2(p_q)I_B(p_B,t_F)]}{2N_{qM}^3r(p_q)} \\
, \quad f_{B,\text{soft}}(p_B,t_F) \approx \frac{f_0(p_q)}{3N_{qB}^3}
\]

Linear scaling due to unitarity;
Baryons dominates here: \( N_B \approx N_q/3 \)

Different scaling for Mesons
B) Solutions when mesons dominate over baryons
(without assuming the same t-dependence for coalescing rates for M & B)

\[ f_q(p_q, t) \approx \frac{f_0(p_q)}{1 + f_0(p_q)I_M(p_M, t)}, \]

\[ f_M(p_M, t) \approx \frac{f_0^2(p_q)I_M(p_M, t)}{N_{qM}^3[1 + f_0(p_q)I_M(p_M, t)]}, \]

\[ f_B(p_B, t) \approx \int_{t_0}^{t} \frac{f_0^3(p_q)c_B(p_B, u)}{[1 + f_0(p_q)I_M(p_M, u)]^3} du \]

E.g., if

\[ c_B(p_B, t) = \varepsilon c_M(p_M, t)I_M(p_M, t) \]

\[ \Rightarrow f_B(p_B, t) \approx \frac{\varepsilon f_0^3(p_q)I_M^2(p_M, t)}{2N_{qM}^3[1 + f_0(p_q)I_M(p_M, t)]^2} \]

same as A1) The limit of \( r(p_q) \rightarrow 0 \)

\[ f_B(p_B, t) \approx \frac{\varepsilon f_0(p_q)}{2N_{qM}^3} \]

\[ \nu_{2,B}^{soft}(p_\perp) \propto \nu_{2,q}(p_\perp / N_{qB}) \]

Compare with case A1)

\[ c_B(p_B, t) = \varepsilon c_M(p_M, t) \quad \nu_{2,B}^{soft}(p_\perp) \propto 2\nu_{2,q}(p_\perp / N_{qB}) \]

Scaling at low Pt depend on coalescence dynamics for non-leading hadrons
Below certain scale $P_{\perp 0}$, $n_q$-scaling does not represent parton $v_2$ because the scaling at low $P_{\perp}$ is different.
Consider

\[ q\bar{q} \rightarrow M, \quadqc \rightarrow D, \quad \bar{q}\bar{c} \rightarrow \bar{D}, \quad \bar{c}\bar{c} \rightarrow J/\psi \]

assuming \( q=q\bar{q}, \) thus \( c=c\bar{c} \)

\[
\begin{align*}
 f_M'(p_M,t) &= c_M(p_M,t)f_q^2(p_q,t) \\
 f_D'(p_D,t) &= c_D(p_D,t)f_q(p_q,t)f_c(p_c,t) \\
 f_{\psi}'(p_\psi,t) &= c_\psi(p_\psi,t)f_c^2(p_c,t) \\
 f_q'(p_q,t) &= -N_qM^3 f_M'(p_M,t) - f_D'(p_D,t)/z_{qD}^3 \\
 f_c'(p_c,t) &= -f_D'(p_D,t)/z_{cD}^3 - N_M^3 f_\psi'(p_\psi,t)
\end{align*}
\]

\[
\begin{align*}
 p_q &= z_{qD}p_D \equiv \frac{m_q}{m_q + m_c} p_D \\
 p_c &= z_{cD}p_D \equiv \frac{m_c}{m_q + m_c} p_D \\
 p_\psi &= 2p_c
\end{align*}
\]
Charm mesons: II

When the coalescing rates have the same time-dependence:

we define \( r_D(p_D) = \frac{c_D(p_D,t)}{z_{cD} N_{qM}^3 c_M(p_M,t)} \), \( r_\psi(p_\psi) = \frac{z_{cD}^3 N_{qM}^3 c_\psi(p_\psi,t)}{c_D(p_D,t)} \)

\[ \Rightarrow f_c(p_c,t) \approx \frac{f_{c0}(p_c)}{1 + f_0(p_q) I_M(p_M,t)} \]

→ The scale \( P_{\perp,0,c} \) that separate soft from hard charm quarks is

\[ P_{\perp,0,c} = P_{\perp,0,q} \frac{m_c}{m_q} \quad \text{where } P_{\perp,0,q} \text{ corresponds to } f_0(p_q) I_M(p_M,t_F) = O(2^{1/r_D(p_D)} - 1) \]

This \( P_{\perp,0,q} \) is different from \( P_{\perp,0} \) of light quarks given by \( f_0(p_q) I_M(p_M,t_F) = O(1) \)

Also: charm quarks coalesce

at a different rate than light quarks

\[ f_q(p_q,t) \approx \frac{f_0(p_q)}{1 + f_0(p_q) I_M(p_M,t)} \]

\[ f_{c0}(p_c) \]

\[ f_0(p_q) \]

\( t_0 \quad t_F \quad t \)
Charm mesons: III

* For hard quarks:
  
  same $n_q$-scalings are reproduced.
  
  For example:
  
  $$v_{2,D}^{hard}(p_\perp) = v_{2,q}(z_{qD}p_\perp) + v_{2,c}(z_{cD}p_\perp)$$

• For soft quarks, general results reduce to
  (when $r_D$, $r_\psi$ have no $\phi$-dependence):

  $$\Rightarrow f_{D}^{soft}(p_M, t_F) \approx f_{c0}(p_c)z_{cD}^3 \quad \text{Linear scaling due to unitarity}$$

  $$f_{\psi}^{soft}(p_\psi, t_F) \propto f_{c0}^2(p_c) / f_0^a(p_q) \quad (a > 0)$$

  $$\rightarrow v_{2,D}^{soft}(p_\perp) \approx v_{2,c}(z_{cD}p_\perp)$$

  $$v_{2,\psi}^{soft}(p_\perp) \approx 2v_{2,c}(p_\perp / N_{qM}) - a v_{2,q}(p_\perp m_q / N_{qM} / m_c)$$

  For example: $v_{2,c} = 0 \rightarrow v_{2,\psi} < 0$
Charm mesons: IV

For soft quarks:

\[ v_{2,c} = 0 \rightarrow v_{2,\psi} < 0 \quad \text{(when } r_D, r_\psi \text{ have no } \phi\text{-dependence)} \]

\[ f_c(p_c, t) \approx \frac{f_{c0}(p_c)}{[1 + f_0(p_q)I_M(p_M, t)]^{r_D(p_D)}} \]

\text{shows } f_c(p_c) \text{ and } f_q(p_q) \text{ are inversely correlated when } I_{M(p_M, t)} \text{ has no } \phi\text{-dependence}

\text{c-quarks coalescence with } q \text{ to form D mesons}

q has \textbf{positive } v_2 \rightarrow \text{charm quarks with positive } v_2 \text{ are being used;}

Total charm quark \( v_2 \) is 0

\rightarrow \textbf{negative } v_2 \text{ for remaining c-quarks}

\rightarrow \textbf{negative } J/\psi \ v_2
Conclusions

• A new formation is developed: satisfies quark number conservation of each flavor, applies to both low and high $P_\perp$

• Results for hard quarks reproduce the $n_q$-scaling between hadron and quark distributions

• Results for soft quarks show different scalings: linear for the hadron that dominates a quark flavor, depend on coalescence dynamics for other hadrons

• Charm quarks show unique scalings at low $P_\perp$
Outlook: issues with quark coalescence

*Need to relax assumptions:*

1) extended beyond rare process by including unitarity

2) Coalescence rates $C_B, C_M$ at low $P_\perp$

3) space-momentum correlations before coalescence

4) factorization of 2-parton distribution function

5) effect of binding energy

6) energy-momentum violation

   \[ 2q \rightarrow 1\text{meson} \text{ or } 3q \rightarrow 1\text{meson}+1q? \]

7) effect of multi-parton coalescence

   \[ 2q+2\overline{q} \rightarrow 2\text{mesons}, \ldots \]

*Further tests:*

consistency between hadron spectra and $v_2$