Lattice based Equation(s) of State and its (their) effect(s) on the hydrodynamical evolution

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Quantifying the properties of Hot QCD matter

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arXiv:0912:2541
QCD equation of state

lattice QCD (Karsch & Laermann, hep-lat/0305025):

\[ T_c = (173 \pm 15) \text{ MeV} \]
\[ \varepsilon_c \sim 0.7 \text{ GeV/fm}^3 \]

- EoS from first principles
- until recently, seldom used in hydro calculations
EoS by hotQCD collaboration

Bazavov et al. arXiv:0903.4379 [hep-lat]

- evaluate interaction measure \((\varepsilon - 3P)/T^4\)
- obtain pressure via

\[
\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} \frac{dP}{T^5} \varepsilon - 3P \frac{T}{T^5}
\]

- What is \(P(T_0)\)?
- What is \((\varepsilon(T_0) - 3P(T_0))/T_0^4\)?
- How good is lattice below \(T_c\)?
Equation of state below $T_c$

Bazavov et al arXiv:0903.4379

$(\varepsilon - 3p)/T^4$

asqtad: $N_\tau = 8$
- 6

p4: $N_\tau = 8$
- 6

• Lattice EoS ≠ Hadron Resonance Gas EoS
Hadron Resonance Gas model

- **EoS of interacting** hadron gas well approximated by **non-interacting** gas of hadrons and resonances

\[ P(T) = \sum_i \int d^3p \, \frac{p^2}{3E} f(p, T) \]

- valid when
  - interactions mediated by resonances

- Prakash & Venugopalan, NPA546, 718 (1992): experimental phase shifts

→ HRG good approximation at low temperatures
→ lattice should reproduce HRG at \( T \leq 120 - 140 \) MeV

- **practical problem**: how to convert fluid to particles?
- energy conservation iff EoS is the same before and after freeze-out
Hadrons on lattice

• Hadron masses depend on lattice cutoff
  ⇒ i.e. on temperature:
  E.g. for pseudoscalar mesons

\[ m_{ps}^2 = m_{ps0}^2 + \frac{1}{r_1^2} \frac{a_{ps}^i x + b_{ps}^i x^2}{(1 + c_{ps}^i x)^{\beta_i}} \]

\[ x = \left( \frac{a}{r_1} \right)^2 \]

\[ a = \frac{1}{N_T T} \]

+ 16 pseudoscalar mesons on lattice

• HRG with lattice mass spectrum?
Hadronic fluctuations

i.e. baryon number, strangeness and charge susceptibilities

\[ \chi_2^x = \frac{1}{VT^3} \frac{\partial^2 \ln Z}{\partial (\mu_x/T)^2} = \frac{1}{T^2} \frac{\partial^2 P}{\partial \mu_x^2}, \]

where \( \mu_x = \mu_B, \mu_S \) or \( \mu_Q \)

- Lattice masses \( \rightarrow \) fluctuations in resonance gas and lattice similar
very little room for modifications in hadron gas

BUT, what is physical mass spectrum?

conservative estimate: free particle masses
Phenomenological EoS

- $T < T_{sw}$: HRG interaction measure (black)
- $T > T_{sw}$: Lattice interaction measure (red)


- $\epsilon$ and $P$ overshoot Stefan-Boltzmann limit!

- Interaction measure too large, but where?
Interaction measure

**Cheng et al (’08)**

\[
\frac{(\varepsilon - 3p)}{T^4} \quad Tr_0
\]

\[T \text{ [MeV]}\]

**Bazavov et al (’09)**

\[
\frac{(\varepsilon - 3p)}{T^4} \quad Tr_0
\]

asqtad: \(N_\tau=8\)

p4: \(N_\tau=4\), 6, 8

\[T \text{ [MeV]}\]

- peak region sensitive to \(N_\tau\)
Procedure for EoS

- HRG below $T \approx 180 - 190$ MeV
- Parametrize lattice using:

$$\frac{\epsilon - 3P}{T^4} = \frac{d_2}{T^2} + \frac{d_4}{T^4} + \frac{c_1}{T^{n_1}} + \frac{c_2}{T^{n_2}}$$

- Require that:

$$\frac{\epsilon - 3P}{T^4} \bigg|_{T_0}, \quad \frac{d}{dT} \frac{\epsilon - 3P}{T^4} \bigg|_{T_0}, \quad \frac{d^2}{dT^2} \frac{\epsilon - 3P}{T^4} \bigg|_{T_0}$$

are continuous

$$\frac{s}{T^3} \bigg|_{T=800\text{MeV}} = (90\% - 95\%) \frac{s_{SB}}{T^4}$$

$\implies T_0, d_4, c_1, c_2$ fixed

- $\chi^2$ fit to lattice above $T = 250$ MeV
For the 95% SSB limit we get

\( T_0 = 171.8 \text{ MeV}, \quad d_2 = 0.2654, \quad d_4 = 6.563 \times 10^{-3}, \quad c_1 = -4.370 \times 10^{-5}, \quad c_2 = 5.774 \times 10^{-6}, \quad n_1 = 8, \quad n_2 = 9 \)
Interaction measure II

- add estimated peak to the fit
Phenomenological EoS

- obtain pressure via

\[
\frac{P}{T^4} - \frac{P_0}{T_0^4} = \int_{T_0}^{T} dT' \frac{\epsilon - 3P}{T'^5}
\]
Speed of sound

- no softening below the HRG!
Effect on flow I

- ideal fluid, \( b = 7 \, \text{fm} \)
- keep everything fixed:
  - \( \tau_0 = 0.6 \, \text{fm/c}, \ T_{\text{dec}} = 125 \, \text{MeV} \)

\[
\frac{1}{2\pi p_T} \frac{dN}{dp_T} \quad \text{versus} \quad p_T \ (\text{GeV/c})
\]

\[
V_2 \ (\%) \quad \text{versus} \quad p_T \ (\text{GeV/c})
\]

\( \rightarrow \) harder EoS, flatter spectra
Effect on flow II

- ideal hydro, Au+Au at $\sqrt{s_{NN}} = 200$ GeV
- chemical equilibrium

\[ \begin{align*}
\text{s95p: } T_{dec} &= 140 \text{ MeV} \\
\text{EoS Q: first order phase transition at } T_c &= 170 \text{ MeV, } T_{dec} = 125 \text{ MeV}
\end{align*} \]
Chemical non-equilibrium

- ideal fluid, $b = 7 \text{ fm}$
- keep everything fixed:
- $\tau_0 = 0.2 \text{ fm/c}$, $T_{chem} = 150 \text{ MeV}$, $T_{dec} = 120 \text{ MeV}$

$\Rightarrow$ harder EoS, flatter spectra
Effect on flow III

- ideal hydro, \( Au+Au \) at \( \sqrt{s_{NN}} = 200 \ \text{GeV} \)
- \( T_{\text{chem}} = 150 \ \text{MeV} \)

**Graph:**

- **EoS Q:** \( T_{\text{dec}} = 120 \ \text{MeV}, \ s_{\text{ini}} \propto N_{\text{bin}}, \ \tau_0 = 0.2 \ \text{fm/c} \)
- **s95p, \ \tau_0 = 0.8:** \( T_{\text{dec}} = 120 \ \text{MeV}, \ s_{\text{ini}} \propto N_{\text{bin}}, \ \tau_0 = 0.8 \ \text{fm/c} \)
- **s95p, \ \tau_0 = 0.2:** \( T_{\text{dec}} = 120 \ \text{MeV}, \ s_{\text{ini}} \propto N_{\text{bin}} + N_{\text{part}}, \ \tau_0 = 0.2 \ \text{fm/c} \)
Conclusions

- below $T_c$ lattice and HRG differ because of hadron mass spectrum

⇒ HRG good description below $T_c$

- some uncertainty in the parametrization of the EoS

⇒ but it doesn’t matter

- proton $v_2(p_T)$ may or may not be sensitive to EoS — details matter!

- EoS tables available at
  and
Budapest-Wuppertal EoS

Aoki et al. hep-lat/0510084

- \( P(T) \) from expectation values of
  - gauge action \( \langle S_g \rangle \)
  - chiral condensates \( \langle \bar{\Psi} \Psi \rangle \)

- temperature scale?
and its “interpretation”

M. Chojnacki \textit{et al.}, arXiv:0712.0947

- HRG speed of sound \textbf{below} \( T_c \)
- lattice speed of sound \textbf{above} \( T_c \) (parametrization by Wuppertal group)
- fix the scale by \( T_c = 167 \) MeV

\[ s(T) = s(T_0) \exp \left[ \int_{T_0}^{T} \frac{dT'}{T' c_s^2} \right] \]
Equations of State

- **FB-Bielefeld**: Frankfurt-BNL interpretation of hotQCD EoS, $s95p$
- **K-Wuppertal**: Krakow interpretation of Wuppertal EoS
Speed of sound

\[ c_s^2 \] vs. \( T \) (MeV)

- FB-Bielefeld
- K-Wuppertal

\begin{itemize}
  \item no softening below the HRG!
\end{itemize}
Flow anisotropy

- Au+Au collision at RHIC, $\sqrt{s} = 200$ GeV, $b=7$ fm

\[ \epsilon_p = \frac{\langle T_{xx} - T_{yy} \rangle}{\langle T_{xx} + T_{yy} \rangle} \]

\[
\begin{align*}
\tau (\text{fm}) & \quad 0 & 2 & 4 & 6 & 8 & 10 & 12 \\
\epsilon_p & \quad 0 & 0.02 & 0.04 & 0.06 & 0.08 & 0.1 & 0.12
\end{align*}
\]

FB-Bielefeld
K-Wuppertal
$v_2(p_T)$ at RHIC

- Au+Au collision at RHIC, $\sqrt{s} = 200$ GeV, $b=7$ fm

FB-Bielefeld: $T_c = 140$ MeV,  
K-Wuppertal: $T_c = 145$ MeV
Other observables?

- HBT?

- not promising

- $\gamma$ and $l^+ l^-$?

- what are the rates?
Conclusions

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