Elliptic flow in ideal hydrodynamics with fluctuating initial profiles

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Quantifying the Properties of Hot QCD Matter
INT, Seattle 9.6.2010
1. Introduction
Why fluctuations?

This is what we usually plot. It seems that hydro does quite well here.
Why fluctuations?

If we show these plots, we see that there is still some work to do.
2. Our model
Monte Carlo Glauber

- Nucleons are distributed into nuclei using Woods-Saxon
- Nucleons collide if

\[(x_i - x_j)^2 + (y_i - y_j)^2 \leq \frac{\sigma_{NN}}{\pi}\]

- We do not use any hard core radius so we do not need to tune our W-S parameters

How big effects hard core radius can cause? Remember that $d$ effects W-S distribution. Here W-S parameters are tuned so that all cases give same W-S distribution.

Broniowski and Rybczynski, arXiv:1003.1088
Centrality classes

Number of participants is related to number of charged hadrons so let's use $N_{\text{part}}$ to define centrality classes.

<table>
<thead>
<tr>
<th>Centrality</th>
<th>$N_{\text{part}}$ range</th>
<th>$\langle N_{\text{part}} \rangle$</th>
<th>$\langle b \rangle$ [fm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-5 %</td>
<td>325-394</td>
<td>352</td>
<td>2.25</td>
</tr>
<tr>
<td>5-10 %</td>
<td>276-324</td>
<td>299</td>
<td>4.07</td>
</tr>
<tr>
<td>10-20 %</td>
<td>197-275</td>
<td>234</td>
<td>5.72</td>
</tr>
<tr>
<td>20-30 %</td>
<td>138-196</td>
<td>166</td>
<td>7.40</td>
</tr>
<tr>
<td>30-40 %</td>
<td>93-137</td>
<td>114</td>
<td>8.76</td>
</tr>
</tbody>
</table>
Initial profiles from MCG

In our model energy density is distributed around centers of wounded nucleons with 2D Gaussian

\[ \epsilon(x, y) = \text{const.} \sum_{wn} \frac{1}{2\pi \sigma^2} \exp \left[ -\frac{(x - x_i)^2 + (y - y_i)^2}{2\sigma^2} \right]. \]

Now \( \sigma \) is a free parameter in our model.

Some events with \( b = 0 \) fm. Our initial states are bumpy but smooth enough for hydro.
Hydrodynamical evolution

Our hydro setup:

- Ideal hydrodynamics
- No baryon number density
- Bjorken flow in beam direction
- Initial time $\tau_0 = 0.17$ fm and overall energy density normalization from EKRT model, Eskola et al., Nucl. Phys. B570:379, 2000
- Freeze-out temperature $T_f = 160$ MeV

With this setup we get a rough fit to pion spectra at different centralities. We want to study the effects of fluctuations, not to fit the data as good as possible.
Hydrodynamical evolution

Animation of hydro evolution in a central collision.

Freezeout surface
Freeze-out

Thermal spectrum is calculated using Cooper-Frye formula. First we find isothermal freeze-out surface and then we integrate over it

$$\frac{dN}{d^2p_Tdy} = \int_{\sigma} f(x, p)p^\mu d\sigma_\mu.$$
One note about freeze-out surfaces. Currently we are not able to find all possible surfaces, but contribution from neglected parts is small when system size and $\sigma$ are large enough.
Resonance decays

MC

PYTHIA

"Traditional way"

Take average over many events

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Calculating $v_2$

We always know where reaction plane is, but in experiments this is not true. So we calculate $v_2$ using event plane method.

Event plane is determined by event flow vector

$$Q_x = \sum_i p_T \cos(n\phi_i)$$
$$Q_y = \sum_i p_T \sin(n\phi_i)$$

Then we can calculate elliptic flow

$$v_2 = \frac{\langle \cos(2(\phi - \psi_{EP})) \rangle}{R},$$

where $R$ is correction from statistical fluctuations.
Calculating $v_2$

Because of the fluctuations in the initial state, we do not know where our true event plane should be. Because we have only finite number of particles in one event, event plane fluctuates.

For Gaussian fluctuations

$$\mathcal{R} = \langle \cos(\mathbf{k} \cdot (\psi_{EP}^{\text{true}} - \psi_{EP})) \rangle$$

$$= \frac{\sqrt{\pi}}{2 \sqrt{2}} \chi \exp(-\chi^2/4) \left[ I_{k-1} (\chi^2/4) + I_{k+1} (\chi^2/4) \right],$$


When we use 2 sub-events we can calculate $\mathcal{R}$ and $\chi$ for sub-events and then using equation above we can calculate $\mathcal{R}$ for full event.
3. Results
Monte Carlo vs. optical

Averaged initial profiles from 5000 Monte Carlo events and from optical Glauber. Fixed impact parameter here.

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Monte Carlo vs. optical

This difference between MCG and optical Glauber translates to surprisingly different values for $v_2$! This is actually good thing for high-$p_T$ $v_2$ behavior (if we like to think that viscosity is small).
500 hydro runs in each centrality class. 20 events from each hydro run.

Fluctuations can increase number of high-$p_T$ particles. This is however quite sensitive to our free parameter $\sigma$.

If we use Gaussian smoothing for energy density and $s \sim \epsilon^{3/4}$, our entropy goes as $\sqrt{\sigma}$.

Multiplicity depends on $\sigma$, but we have better control of profile that goes into hydro.
Elliptic flow

Here we see that fluctuations have large effect in integrated $v_2$. eWN profiles gives very nice results indeed!
Elliptic flow

Very promising results also on $v_2(p_T)$ at different centralities! But how does our $\sigma$ parameter effect these results?
Elliptic flow

Order of 10% effects. This is small compared to what fluctuations did.
Clear peak at $\psi_{EP} \approx \psi_{PP}$. Big difference in the peak shape between centralities.

At least some of the difference can be explained with statistical fluctuations in event plane determination.

Should we worry about the width of the peak even in 20-30% centrality? At least in 0-5% event plane is not the same as participant plane.
Need for computational resources

With current setup one hydro run and Cooper-Frye takes about an hour.

If we make $\sigma$ very small, we must use denser grid in hydro and running time increases significantly.

If we make 500 hydro runs and 50 000 events from them, time consumed in making particles and their decays is negligible.

If we have super computer with decent number of cores for running serial jobs, results for centrality class can be done over night.

We thank CSC — IT Center for Science for CPU-time.
Conclusions

- One must be careful when constructing initial state. Should one fix entropy or energy when we use Gaussian smoothing?
- Monte Carlo Glauber and optical Glauber are surprisingly different.
- Fluctuations in initial density profile can significantly increase elliptic flow in the most central collisions.
- Fluctuations change $v_2$ more in the central collisions than in the semi-peripheral collisions.
- We need only decent number of hydro events to calculate spectra and $v_2$. 