CGC in heavy-ion coll.

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Applications:
- KLN multiplicity
- Improvements of large-x sources and the eccentricity / ini.con. for hydro
- Improvements of small-x gluon distrib. from BKrc evolution (→ LHC)
- Particle correlations and JIMWLK
Glauber model for the initial transverse density profile

Wounded nucleon model: number of participants scaling

\[
\frac{dN}{d^2r_t dy} \sim \rho_{\text{part}},
\]

\[
\rho_{\text{part}}(\vec{r}_\perp, \vec{b}) = \rho^A_{\text{part}}(\vec{r}_\perp, \vec{b}) + \rho^B_{\text{part}}(\vec{r}_\perp, \vec{b})
= T_A(\vec{r}_\perp + \vec{b}/2) \left[ 1 - (1 - \sigma_{\text{NN}}^{\text{inel}} T_B(\vec{r}_\perp - \vec{b}/2)/B)^B \right] \\
+ T_B(\vec{r}_\perp - \vec{b}/2) \left[ 1 - (1 - \sigma_{\text{NN}}^{\text{inel}} T_A(\vec{r}_\perp + \vec{b}/2)/A)^A \right]
\]

\[
T_A(x_\perp) = \int dz \rho_A(x_\perp, z) \quad \rho_A(r) = \frac{\rho_0}{1 + \exp\left[\left(\frac{r-R_0}{a}\right)\right]}
\]
Inclusive gluon production

**$K_t$-factorization:**

$$\frac{dN}{d^2 r_t dy} \sim \int \frac{d^2 p_t}{p_t^2} \int d^2 k_t \alpha_s \phi_A(x_1, k_t^2) \phi_B(x_2, (p_t - k_t)^2)$$

Kharzeev, Levin, Nardi model:

$$\phi(x, k_t^2; \vec{r}_t) \sim \frac{1}{\alpha_s(Q_s^2)} \frac{Q_s^2(x, \vec{r}_t)}{\text{max}(Q_s^2, k_T^2)}$$

$$Q_s^2 \sim \rho_{\text{part}} x^{-\lambda} \quad (\lambda \approx 0.28)$$
Works better than original KLN at large $N_{\text{part}}$ if one does not approximate $Q_s(r_t, b, A) \rightarrow \langle Q_s \rangle (b, A)$

also see Kuhlmann, Heinz, Kovchegov: nucl-th/0604038
Improving on $\Phi_{KLN}$:

- Large $x$: modelling L.C. sources

✓ implemented
\[ Q_{sB}^2 = Q_{sp}^2 T_B \]
\[ T_B = \frac{\sum_{i \geq 1} p_i t_i}{\sum_{i \geq 1} p_i} = \frac{\sum_{i \geq 0} p_i t_i}{\sum_{i \geq 1} p_i} = \frac{\langle T_B \rangle}{p_B} \]
\[ \langle T_B \rangle(\vec{r}_\perp) = \int dz \rho_{WS}(z, \vec{r}_\perp) \]

\[ \Phi_B = \Phi(Q_{sp}^2 \langle T_B(r_t) \rangle) \]

\[ \Phi_B = p_B \Phi(Q_{sp}^2) \]

\[ \Phi_B(\langle T_B \rangle) \rightarrow p_B \Phi_B(\langle T_B \rangle/p_B) \]

\[ \frac{dN}{d^2r_t} \sim p_A \Phi_A \otimes p_B \Phi_B \]
Fluctuations of nucleons ("light-cone sources")

important for

- central / peripheral Au+Au (eccentricity !)
- smaller systems (Cu+Cu)
- fluctuations of $v_2$

\[ \epsilon_{\text{std}} = \frac{\sigma_y^2 - \sigma_x^2}{\sigma_y^2 + \sigma_x^2} \]

\[ \epsilon_{\text{part}} = \frac{\sqrt{(\sigma_y^2 - \sigma_x^2)^2 - 4\sigma_{xy}^2}}{\sigma_y^2 + \sigma_x^2} \]
Very good fit, both Au+Au and Cu+Cu pretty much down to pp (too good, actually...)
Pressure gradients and elliptic flow

Idea: Pressure gradients convert spatial anisotropy to momentum anisotropy

\[
\varepsilon = \frac{\langle y^2 - x^2 \rangle}{\langle x^2 + y^2 \rangle} \\
\nu_2 = \frac{\langle p_x^2 - p_y^2 \rangle}{\langle p_x^2 + p_y^2 \rangle}
\]
hydro best for central collisions
(if ini. cond. appropriate)

so, this is probably not best way to plot:

Hirano et al.
Voloshin/Poskanzer plot: scaled flow vs density

\[ \frac{v_2}{\varepsilon} \rightarrow \text{centrality} \]

fluctuations of $\varepsilon$ crucial!

\[ \sim \frac{1}{1 + \rho_0/\rho} \]

fit

(1/S)(dN/dy)[mb$^{-1}$]

centrality →
In conformal limit, $v_2/\varepsilon = \text{const (flat)}$

\[
\frac{v_2}{\varepsilon} = \frac{\varepsilon}{1 + K/K_0}
\]

\[
1/K = \frac{\sigma}{S} \frac{dN}{dy} c_s
\]

Glauber IC

nucl-th/0508009

CGC IC
extracted EoS, viscosity depend on IC!

- Consistent with hydro limit: Saturation seen
- Viscosity estimate
  - $\eta/s \sim 0.1$ (CGC)
  - $\eta/s \sim 0.2$ (Glauber)
- Speed of sound
  - $c_s(CGC) / c_s(Gl.) \sim 0.7$
Scaling properties:

\[ \frac{dN_g}{d^2 r \, dy} = \frac{4 \pi N_c}{N_c^2 - 1} \int \frac{d^2 p_t}{p_t^2} \int d^2 k_t \alpha_s \phi(x_1, k_t^2) \phi(x_2, (p_t - k_t)^2) \]

\[ \sim Q_{s \text{ min}}^2 \log \frac{Q_{s \text{ max}}^2}{Q_{s \text{ min}}^2} \]

CGC: \[ \frac{dN}{dy} \sim \min \left( \rho^A_{\text{part}}, \rho^B_{\text{part}} \right) \]

Glauber: \[ \frac{dN}{dy} \sim \frac{\rho^A_{\text{part}} + \rho^B_{\text{part}}}{2} \]

\[ \epsilon_{\text{CGC}} > \epsilon_{\text{Glauber}} \]
fKLN approaches Glauber in peripheral collisions!

~ 30% effect, comparable to dissip. correction

fKLN approaches Glauber in peripheral collisions!
Simple fits are great but what about real hydro solutions?
rather steep (non-hydro like) $v_2/\varepsilon$ for Glauber I.C.

→ hydro + Glauber ain't good friends !?
scaled flow vs density

\[ v_2^{\text{data}/\varepsilon_{\text{MC-KLN}}} \] increases with centrality!
(Unlike Glauber-hydro)

At small \( b \), one should account for fluctuations though.
Improving on $\Phi_{KLN}$:

- large $x$:
  - Single-nucleon limit at surface
  - Fluctuations of L.C. sources

Important for hydro I.C.
Improving on $\Phi_{KLN}$:

- small $x$: BKrc evolution

  partially implemented (J. Albacete, ...)

  - impact param. dependence
  - tables for MC codes
Energy dependence of $dN/dy$ in central AA

The extrapolation to Pb-Pb collisions at the LHC is completely driven by the small-x evolution.

**Modifications**

a) $\varphi(x, k) \rightarrow h(x, k) = k^2 \nabla_k^2 \varphi$

b) $\alpha_{fr} = 0.5$

c) $m = 0$

d) No $(1 - x)^4$ corrections

- RHIC data does not discriminate power-law behavior from a logarithmic one
- Logarithmic behaviour seems to be dictated by lower energy data
exciting discovery by STAR:

long-range rapidity correlations at RHIC!
(on “near side”)

PYTHIA pp, $p_T^{\text{trig}} > 2.5$ GeV
EPOS string model

2 → 2 hard scattering (~ back to back)

K. Werner et al, arXiv:1004.0805

fragmentation (~ soft Lund)
Large amplitude of ridge in Au+Au largely due to hydro (radial boost):

\[
A = K_R \frac{\gamma_B - \gamma_B^{-1}}{\alpha_s(Q_s)}
\]

Gavin, McLerran, Moschelli: 0806.4718
production of two semi-hard partons

A.D., Gelis, McLerran, Venugopalan:
0804.3858

\[ C(p, q) = 16(2\pi)^2 \alpha_s^2 \, S_\perp \frac{N^2}{(N^2 - 1)^3} \frac{1}{p_\perp^2} \frac{1}{q_\perp^2} \]

\[ \int d^2 k_\perp \frac{\Phi_A(x_1, (p_\perp + k_\perp)^2)}{(p_\perp + k_\perp)^2} \frac{\Phi_A(x_1, (q_\perp - k_\perp)^2)}{(q_\perp - k_\perp)^2} \frac{\Phi_B^2(x_2, k_\perp^2)}{k_\perp^4} \]
long-range rapidity evolution and small-$x$ evolution

- finite width predicted
- amplitude works out, after factoring in flow + fragmentation peak
Two-particle production diagrams are $N_c$-suppressed:

\[
\frac{1}{N_c^2 - 1}
\]
however, we should rather compute THIS diagram:

\[
C(p_\perp, q_\perp) = \frac{g^{12}}{64(2\pi)^6} \left( f_{abc} f_{a'\bar{b}\bar{c}} f_{a\bar{b}\bar{c}} f_{a'\bar{b}\bar{c}} \right) \int \prod_{i=1}^{4} \frac{d^2 k_{i\perp}}{(2\pi)^2 k_{i\perp}^2} \\
\times \frac{L_\mu(p_\perp, k_{1\perp}) L_\mu(p_\perp, k_{2\perp})}{(p_\perp - k_{1\perp})^2 (p_\perp - k_{2\perp})^2} \frac{L_\nu(q_\perp, k_{3\perp}) L_\nu(q_\perp, k_{4\perp})}{(q_\perp - k_{3\perp})^2 (q_\perp - k_{4\perp})^2} \\
\times \left\langle \rho^*_1(k_{2\perp}) \rho^*_1(k_{4\perp}) \rho_1^b(k_{1\perp}) \rho_1^\bar{b}(k_{3\perp}) \right\rangle \\
\times \left\langle \rho^*_2(p_\perp - k_{2\perp}) \rho^*_2(q_\perp - k_{4\perp}) \rho_2^c(p_\perp - k_{1\perp}) \rho_2^\bar{c}(q_\perp - k_{3\perp}) \right\rangle
\]
Large-Nc approximation
(factorization of $\langle \rho^4 \rangle \sim \langle \rho^2 \rangle^2$)

$$
\langle \rho^a \rho^b \rho^c \rho^d \rangle = \delta^{ab} \delta^{cd} \langle \rho^2 \rangle^2 + \delta^{ac} \delta^{bd} \langle \rho^2 \rangle^2 + \delta^{ad} \delta^{bc} \langle \rho^2 \rangle^2
+ \mathcal{O} \left( \frac{1}{N_c} \right)
$$

$\langle \rho^2 \rangle$ can be obtained from BFKL or BK eqn.
(standard unintegrated gluon distrib.)

$$
\partial_Y \langle \rho^a \rho^b \rho^c \rho^d \rangle = \delta^{ab} \delta^{cd} \mathcal{Z} + \delta^{ac} \delta^{bd} \mathcal{Z} + \delta^{ad} \delta^{bc} \mathcal{Z}
$$

$$
\mathcal{Z} = \langle \rho^2 \rangle \mathcal{K} \otimes \langle \rho^2 \rangle
$$

BFKL kernel
\[ \frac{d}{dY} \langle \alpha^a_r \alpha^b_r \alpha^c_s \alpha^d_s \rangle = \]
\[ \frac{g^2 N_c}{(2\pi)^3} \int d^2 z \left( \frac{\alpha^a_z \alpha^b_r \alpha^c_s \alpha^d_s}{(r - z)^2} + \frac{\alpha^a_r \alpha^b_z \alpha^c_s \alpha^d_s}{(s - z)^2} + \frac{\alpha^a_r \alpha^b_z \alpha^c_s \alpha^d_s}{(\bar{s} - z)^2} - 4 \frac{\alpha^a_r \alpha^b_r \alpha^c_s \alpha^d_s}{z^2} \right) \]
\[ + \frac{g^2}{\pi} \int \frac{d^2 z}{(2\pi)^2} \left( f^{eka} f^{fkb} \frac{(r - z) \cdot (\bar{r} - z)}{(r - z)^2 (\bar{r} - z)^2} \left[ \alpha^a_r \alpha^f_r - \alpha^e_r \alpha^f_r - \alpha^e_r \alpha^f_s + \alpha^e_r \alpha^f_r \right] \alpha^c_s \alpha^d_s \right) \]
\[ + f^{eka} f^{fkc} \frac{(r - z) \cdot (s - z)}{(r - z)^2 (s - z)^2} \left[ \alpha^e_r \alpha^f_s - \alpha^c_r \alpha^f_s - \alpha^c_r \alpha^f_r + \alpha^e_r \alpha^f_s \right] \alpha^b_s \alpha^d_s \]
\[ + f^{eka} f^{fkd} \frac{(r - z) \cdot (\bar{s} - z)}{(r - z)^2 (\bar{s} - z)^2} \left[ \alpha^e_r \alpha^f_s - \alpha^c_r \alpha^f_s - \alpha^c_r \alpha^f_r + \alpha^e_r \alpha^f_s \right] \alpha^b_s \alpha^c_s \]
\[ + f^{ekb} f^{fkc} \frac{(\bar{r} - z) \cdot (s - z)}{(\bar{r} - z)^2 (s - z)^2} \left[ \alpha^e_s \alpha^f_s - \alpha^c_s \alpha^f_s - \alpha^c_s \alpha^f_r + \alpha^e_s \alpha^f_s \right] \alpha^a_r \alpha^d_s \]
\[ + f^{ekb} f^{fkd} \frac{(\bar{r} - z) \cdot (\bar{s} - z)}{(\bar{r} - z)^2 (\bar{s} - z)^2} \left[ \alpha^e_s \alpha^f_s - \alpha^c_s \alpha^f_s - \alpha^c_s \alpha^f_r + \alpha^e_s \alpha^f_s \right] \alpha^a_r \alpha^c_s \]
\[ + f^{ekc} f^{fkd} \frac{(s - z) \cdot (\bar{s} - z)}{(s - z)^2 (\bar{s} - z)^2} \left[ \alpha^e_s \alpha^f_s - \alpha^e_s \alpha^f_s - \alpha^e_s \alpha^f_r + \alpha^e_s \alpha^f_s \right] \alpha^a_r \alpha^b_r \right) . \]

\[ A^\mu (x^+, r) \equiv \delta^{\mu-} \alpha (x^+, r) = -g \delta^{\mu-} \delta (x^+) \frac{1}{\sqrt{2}} \rho (x^+, r) \quad k^2 \alpha (k) = g \rho (k) \]
however, “subleading-Nc” piece contributes at the same order to $C(p,q)$

Complete Balitsky/JIMWLK four-point function:
(in Gaussian approximation)

$$\langle \rho^a \rho^b \rho^c \rho^d \rangle = \delta^{ab} \delta^{cd} \langle \rho^2 \rangle^2 + \frac{1}{N_c} f^{abe} f^{cde} \mathcal{F}(k_i) \langle \rho^2 \rangle^2 + \cdots$$

\[
\begin{align*}
    &\frac{1}{N_c} f^{abk} f^{cdk} \delta^{ac} \delta^{bd} \delta^{a'c'} \delta^{b'd'} = N_c^2 (N_c^2 - 1) \\
    &\frac{1}{N_c} f^{abk} f^{cdk} \delta^{ac} \delta^{bd} \delta^{a'c'} \delta^{b'd'} = N_c^2 (N_c^2 - 1)
\end{align*}
\]

Projectile \hspace{2cm} Target

[Note: independent/uncorrel. production]

\[
\frac{1}{N_c} f^{abk} f^{cdk} \delta^{ac} \delta^{bd} \delta^{a'c'} \delta^{b'd'} = N_c^2 (N_c^2 - 1)^2
\]
estimates for pp @ LHC (w/o JIMWLK)

\[ y_p = y_q = 0 \ , \ p_t = q_t = 2 \text{ GeV} \ , \ \text{reBK} \]

\[ \Delta \phi \]

\[ C_2(p,q) \]

\[ q_t = p_t, \ \varphi = \pi/8; \ y_q = 2 \]

\[ p_t = q_t \ (\text{GeV}) \]

\[ C_2(p,q) \]

\[ p_t = q_t = 2 \text{ GeV} \]

\[ \Delta y \]
Summary:

- non-trivial QCD dynamics determines initial conditions for hydro:
  - realistic modeling of L.C. sources is very important for $v_2/\varepsilon$ vs centrality

- controlled extrapolation to LHC requires recent improvements in small-x evolution (BKrc), needs to be interfaced with MC

- CGC is more than saturation of two-point function (unintegr. gluon distrib.)! multi-particle correlations!
Backup Slides
Ridge in pp @ LHC:

- Kinematic regime:
  \[ p, q \sim Q_s \] (say, 1-3 GeV for pp @ LHC, AA @ RHIC)
- Effect disappears for small \( Q_s \) / large \( p, q \)
- \( \Phi \ll \pi \)

- Particle correlations probe complete B-JIMWLK evolution equation incl. "\( N_c \) corrections"
  (unlike single-inclusive cross-section !)
- Should be interesting for pp @ LHC
Eccentricity fluctuations:

\[ v_2 = C(c_s, \eta/s) \varepsilon \]

\[ \frac{\delta v_2}{v_2} = \frac{\delta \varepsilon}{\varepsilon} \]

true or not?

non-flow ???
fluctuations from finite bin widths have not been removed yet likely to reduce ratio below the model!

systematic uncertainties are still large and under investigation

STAR Preliminary

\[
\frac{\sigma_{v_2}}{\langle v_2 \rangle}
\]