Universal few/many-body physics in mixed dimensions

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Introduction

Remarkable progress in ultracold atoms are owing to the tunability of various parameters

- Interaction strength by Feshbach resonances
  - Superfluid-Mott insulator transition in Bose gases
  - Collapse & phase separation in Bose-Fermi mixtures
  - BCS-BEC crossover in Fermi gases
  - Efimov effect

- Dimensionality of space by strong optical lattices
  - 3D : BCS-BEC crossover and etc.
  - 2D : Berezinsky–Kosterlitz–Thouless transition
  - 1D : Tomonaga-Luttinger liquids
  - ... and more?
Mixed dimensions

2-species mixture of A atom ⬤ & B atom ⬥
confine “only” A atoms in lower dimensions (2D or 1D)

Choose laser frequency close to the resonance of A atoms
but far from the resonance of B atoms with low intensity
Idea of **mixed dimensions** often appears in physics ...

**graphene in condensed matter**

Photons in **3D** induce 3D Coulomb int. between electrons confined in **2D**

**brane world in cosmology**

We live in **3D** brane but gravitons in **extra dimensions**

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*physicsworld.com (2007)*

Mixed D = new type of imbalance

Fermi gas with equal number of \( \uparrow \) and \( \downarrow \) fermions

**BCS-BEC crossover** (s-wave superfluid for any coupling)

But if we introduce an imbalance \( (n_{\uparrow} \neq n_{\downarrow}) \) ...

\[ \eta \]

\[ \text{imbalance} \]

\[ \text{BEC} \quad \kappa_0 \quad \text{BCS} \quad \kappa \]

\[ \text{proposed T=0 phase diagram} \]

by D.T.Son & M.A.Stephanov

PRA74 (2006)

Imbalance leads to new and rich physics

- density imbalance \( (n_{\uparrow} \neq n_{\downarrow}) \)
- mass imbalance \( (m_K \neq m_Li) \)
- dimensionality of space \( (d_K \neq d_Li) \)
Rich physics in mixed dimensions

Rich few-body & many-body physics can be realized in mixed dimensions

2D-3D mixture

1D-3D mixture

• Modification of scattering properties
  confinement-induced 2-body & 3-body resonances

• Interesting and rich many-body phase diagram
  induced s-wave & p-wave superfluidity, dimer BEC, (stable) trimer Fermi gas, ...
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4. Summary
Few-body physics in mixed D
2-body scattering
Effective scattering length

2-body scattering in 3D

\[ H_{3D} = -\frac{\nabla^2 x_A}{2m_A} - \frac{\nabla^2 x_B}{2m_B} + V_a(x_A, x_B) \]

\[ \Psi(x_A, x_B) \rightarrow \frac{1}{|x_A - x_B|} - \frac{1}{a} \]

3D scattering length “a”

tunable by Feshbach resonance

\[ A_{3D}(k) \propto \frac{1}{ik - \frac{1}{a} + \cdots} \]

2-body scattering in mixed D

\[ H_{\text{mixed D}} = H_{3D} + \frac{1}{2} m_A \omega^2 z_A^2 \]

\[ \Psi(x_A, x_B) \rightarrow \left[ \frac{1}{||x_A - x_B||} - \frac{1}{a_{\text{eff}}} \right] e^{-z_A^2/2l^2} \]

Effective scattering length “a_{\text{eff}}”
depending on “a” and “l = (m_A \omega)^{-1/2}”

\[ A_{\text{mixed D}}(k) \propto \frac{1}{ik - \frac{1}{a_{\text{eff}}} + \cdots} \]
Resonances in mixed D (\(m_A/m_B=0.15\))

2D-3D mixture

A=\(^6\text{Li}\) in 2D & B=\(^{40}\text{K}\) in 3D

1D-3D mixture

A=\(^6\text{Li}\) in 1D & B=\(^{40}\text{K}\) in 3D

\[a_{\text{eff}}/l\]

\[l/a\]

Infinite series of confinement-induced resonances

Resonance is shifted to “\(a<0\)” side
Resonances in mixed D ($m_A/m_B=0.15$)

2D-3D mixture

$A=^6\text{Li}$ in 2D & $B=^{40}\text{K}$ in 3D

\[
\frac{a_{\text{eff}}}{l} \quad \frac{l}{a}
\]

\[
\left(2n + \frac{1}{2}\right)\hbar\omega_M - \frac{\hbar^2}{ma^2} = \frac{1}{2}\hbar\omega_A
\]

See also 0D-3D case; P. Massignan & Y. Castin, PRA (2006)

Infinite series of confinement-induced resonances

Resonance is shifted to “a<0” side
Scattering in mixed dimensions with ultracold gases

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(Dated: January 31, 2010)
First experiment @ Florence

3D scattering length $a/a_0$ between $A=^{41}\text{K}$ & $B=^{87}\text{Rb}$

$A=^{41}\text{K}$ in 2D & $B=^{87}\text{Rb}$ in 3D

G. Lamporesi et al, arXiv:1002.0114
Universality

When \( \text{a}_{\text{eff}} \gg l \)”, the confinement length “\( l \)” can be neglected

Then the system is universal, being characterized only by \( \text{a}_{\text{eff}} \)

E.g., binding energy of AB dimer

\[
E_{\text{dimer}} = -\frac{1}{2m_{AB}a_{\text{eff}}^2} \quad \text{for} \quad \text{a}_{\text{eff}} > 0
\]

(Cf. universality in 3D required “\( a \gg r_0 \)”)

\( \bullet \) \( a_{\text{eff}} < 0 \) : weak attraction (“BCS”) side

\( \bullet \) \( |a_{\text{eff}}| \to \infty \) : resonant (unitarity) limit with scale inv.

\( \bullet \) \( a_{\text{eff}} > 0 \) : strong attraction (“BEC”) side

\( E_{\text{dimer}} = -\frac{1}{2m_{AB}a_{\text{eff}}^2} \)
Universal limit

At finite density & temperature, the universality requires

\[ a_{\text{eff}} \text{ & any other scales} \gg l \]

- confinement length / typical scale
- mixed D
- experiment?
- pure 3D

We will work in the "universal limit" \( l \to 0 \)

\[
H_{\text{mixed D}} \Rightarrow - \sum_{i=1}^{N_A} \frac{\nabla^2 \tilde{x}_A}{2m_A} - \sum_{j=1}^{N_B} \frac{\nabla^2 x_B}{2m_B} + V_{a_{\text{eff}}}
\]

\[
\tilde{x}_A = \begin{cases} (x, y) & \text{for 2D} \\ (z) & \text{for 1D} \end{cases}
\]

\[ x_B = (x, y, z) \]

( \( \tilde{x}_A \) is 2D or 1D coordinate while \( x_B \) is 3D coordinate)
Field theoretical description

\[ H = \int d\mathbf{x} \psi_A^\dagger(\mathbf{x}) \left( -\frac{\nabla^2}{2m_A} - \mu_A \right) \psi_A(\mathbf{x}) \]
\[ + \int d\mathbf{x} dz \psi_B^\dagger(\mathbf{x}, z) \left( -\frac{\nabla^2 + \nabla^2_z}{2m_B} - \mu_B \right) \psi_B(\mathbf{x}, z) \]
\[ + g_0(a_{\text{eff}}) \int d\mathbf{x} \psi_A^\dagger(\mathbf{x}) \psi_B^\dagger(\mathbf{x}, 0) \psi_B(\mathbf{x}, 0) \psi_A(\mathbf{x}) \]

scattering of A and B atoms occurring in 2D plane (z=0)

• \( \mu_A \) and \( \mu_B \) control the densities of A and B atoms
Few-body physics in mixed D

3-body scattering
Efimov effect in 3D

When 2 atoms resonantly interact, 3 atoms form Efimov trimers with a geometric spectrum \( E_n/E_{n+1} \big|_{a \to \infty} = \lambda^2 \)

3 bosons

The Efimov effect is a quantum mechanical phenomenon that occurs when three particles interact in a way that is not possible with only two particles. It is characterized by the existence of a series of discrete energy levels in the spectrum of the system, which are not present in the spectrum of the system with only two particles. The Efimov effect is a consequence of the non-local nature of quantum mechanics and the attraction between particles.

In the diagram, the Efimov effect is illustrated with a series of energy levels labeled \( T^{(0)}, T^{(1)}, T^{(2)} \) and \( a_1^{(0)}, a_2^{(0)}, a_3^{(0)} \). The energy levels are plotted against the inverse scattering length \( 1/a \), which is a measure of the strength of the interaction between the particles.

The Efimov effect exists only in 3D (2.3<d<3.8) but not in 2D or 1D. It is a consequence of the dimensionality of the space in which the particles are moving. In higher dimensions, there are more degrees of freedom, which makes it possible for the Efimov effect to occur.


Florence group, PRL (2009)

\( \lambda_{RbRbK} = 131 \)

\( \lambda_{KKRb} = 3.48 \times 10^5 \)

How about mixed dimensions?
3-body problems in mixed D

\[ \left[ -\sum_{i=1}^{N_A} \frac{\nabla^2 \tilde{x}_{A_i}}{2m_A} - \sum_{j=1}^{N_B} \frac{\nabla^2 x_{B_j}}{2m_B} + V_{a_{\text{eff}}} \right] \Psi = E \Psi \quad \text{with} \quad N_A + N_B = 3 \]

4 types of 3-body problems...

\( N_A=2 \) in 2D/1D & \( N_B=1 \) in 3D

\( N_A=1 \) in 2D/1D & \( N_B=2 \) in 3D

Are there Efimov trimer states in mixed dimensions? Yes!
Efimov effect in mixed D

- If majority atoms are bosons, Efimov effect occurs for any $m_A/m_B$.
- If majority atoms are fermions, Efimov effect occurs for ...

2D-3D mixture

$m_A/m_B > 6.35$

$m_A/m_B < 0.0351$

1D-3D mixture

$m_A/m_B > 2.06$

$m_A/m_B < 0.00646$

Compare those critical mass ratios with 3D values:

$m_A/m_B > 13.6$

$m_A/m_B < 0.0735$
Implication for $^{40}$K-$^6$Li mixture

Interesting possibility in Fermi-Fermi mixture of $A=^{40}$K and $B=^6$Li when $^{40}$K is confined in lower dimensions ...

Confinement induces the Efimov effect !!!

Such trimers are long lived: $\tau^{-1} \sim \frac{1}{m r_0^2} \left(\frac{r_0}{l}\right)^{4.39} \ll \varepsilon_{\text{trimer}} \sim \frac{1}{m l^2}$
3-body recombination rate

3-body recombination ($A+A+B \rightarrow A+AB$) results in atom losses

$$\dot{n}_A \approx -2 \alpha n_A^2 n_B$$

Its rate constant $\alpha$ has the characteristic log-periodic behaviors with the scaling factor $\lambda = 22.0$ for $A=^{40}\text{K}$ in 1D & $B=^6\text{Li}$ in 3D

- **induced Efimov resonances** for $a_{\text{eff}}<0$
- **deconstructive interferences** for $a_{\text{eff}}>0$

Efimov parameter $\kappa_* \approx 1.91/l$ & width parameter $\eta_* \sim (r_0/l)^{2.39} \ll 1$

If observed, the first evidence of the Efimov effect in fermions !!!
Scaling factor for fermions

Scaling factor is expressed by \( \lambda = e^{\pi/s_0} \) with \( E_n/E_{n+1} = \lambda^2 \)

For \( A=^6\text{Li} \) & \( B=^{40}\text{K} \)
- pure 3D : no Efimov
- 2D-3D : no Efimov
- 1D-3D : no Efimov

For \( A=^{40}\text{K} \) & \( B=^6\text{Li} \)
- pure 3D : no Efimov
- 2D-3D : \( \lambda = 1.78 \times 10^5 \)
- 1D-3D : \( \lambda = 22.0 \)

Confinement induces the Efimov effect!
Scaling factor for bosons

Scaling factor is expressed by \( \lambda = e^{\pi/s_0} \) with \( E_n/E_{n+1} = \lambda^2 \)

For \( A=^{41}\text{K} \) & \( B=^{87}\text{Rb} \)

- pure 3D : \( \lambda = 3.48 \times 10^5 \)
- 2D-3D : \( \lambda = 59.3 \)
- 1D-3D : \( \lambda = 67.6 \)

For \( A=^{87}\text{Rb} \) & \( B=^{41}\text{K} \)

- pure 3D : \( \lambda = 131 \)
- 2D-3D : \( \lambda = 27.7 \)
- 1D-3D : \( \lambda = 34.6 \)

Confinement greatly reduces the scaling factor!
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   - single-layer Fermi gas
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4. Summary
Many-body physics in mixed D single-layer Fermi gas
Fermi gas in 2D-3D mixture

Fermi-Fermi mixture (e.g. $^{40}\text{K}$ and $^{6}\text{Li}$) in 2D-3D mixed dimensions

parameters of the system
- $a_{\text{eff}}$
- $k_{\text{FA}} \equiv (4\pi n_{A})^{1/2}$
- $k_{\text{FB}} \equiv (6\pi^{2}n_{B})^{1/3}$
- $m_{A}/m_{B} < 6.35$
- $T=0$

Investigate the phase diagram in terms of $(a_{\text{eff}}k_{F})^{-1}$ $[k_{F} \sim k_{\text{FA}} \sim k_{\text{FB}}]$

- $a_{\text{eff}}k_{F} \to -0$: weak attraction ("BCS") limit
- $|a_{\text{eff}}k_{F}| \to \infty$: resonant (unitarity) limit with scale inv.
- $a_{\text{eff}}k_{F} \to +0$: strong attraction ("BEC") limit with AB dimer

$$E_{\text{dimer}} = -\frac{1}{2m_{AB}a_{\text{eff}}^{2}}$$
Weak attraction (BCS) limit

- AB pairing does not take place due to the mismatch of 2D and 3D Fermi surfaces
- Instead B atoms in 3D induce an effective attraction between A atoms in 2D

\[ V_{\text{ind}}(r) = - \frac{a_{\text{eff}}^2}{m_{AB}} \frac{\sin(2k_{FB}r) - 2k_{FB}r \cos(2k_{FB}r)}{4\pi r^4} + O(a_{\text{eff}}^3) \]

2D p-wave pairing between A atoms

\[ \frac{\Delta(p)}{\varepsilon_{FA}} \propto (\hat{p}_x + i\hat{p}_y) e^{-\#/(a_{\text{eff}}k_{FB})^2} \]

(Cf. If B=bosons, \[ V_{\text{ind}}(r) = - \frac{4\pi n_B a_{\text{eff}}^2}{m_{AB}} \frac{e^{-\sqrt{2}r/\xi_B}}{r} \] with \( \xi_B = \frac{1}{\sqrt{8\pi a_B n_B}} \))
Strong attraction (BEC) limit

- A atoms in 2D capture
- B atoms from 3D to form dimers

- When $a_{\text{eff}}k_F < O(1)$, dimer size < mean distance

Dimer BEC in 2D

$$T_{\text{BKT}} \rightarrow \frac{2\pi n_d}{M} \ln^{-1} \left( -\frac{380}{4\pi} \ln n_d a_{\text{eff}}^2 \right)$$

N. Prokof’ev, et al., PRL (01), PRA (02)
Possible phase diagram

If $p_x + ip_y$ pairing extends to the unitarity limit: $a_{\text{eff}} k_F \to \infty$

$$\frac{\Delta(p)}{\varepsilon_{FA}} \propto (\hat{p}_x + i\hat{p}_y) e^{-\#/(a_{\text{eff}} k_{FB})^2} \to (\hat{p}_x + i\hat{p}_y) \times O(1)$$

- Majorana fermions @ vortices
- Non-Abelian statistics
- Topological quantum computation ...

N.Read & D.Green, PRB (2000)
Many-body physics in mixed D double-layer Fermi gas
Bilayer Fermi gas

Optical lattice creates many layers ...

parameters of the system

- $a_{\text{eff}}$
- $k_{FA} \equiv (4 \pi n_A)^{1/2}$
- $k_{FB} \equiv (6 \pi^2 n_B)^{1/3}$
- $m_A/m_B < 6.35$
- $T=0$
- $d$

The phase diagram becomes 2-dimensional:

$(a_{\text{eff}} k_F)^{-1}$ vs. $(k_F d)$

$[k_F \sim k_{FA} \sim k_{FB}]$

Interlayer correlation induced by B atoms would lead to rich physics (even without tunneling)
Bilayer Fermi gas

Optical lattice creates many layers ...

parameters of the system

- $a_{\text{eff}}$
- $k_{FA} \equiv (4\pi n_A)^{1/2}$
- $k_{FB} \equiv (6\pi^2 n_B)^{1/3}$
- $m_A/m_B < 6.35$
- $T=0$
- $d$

Interlayer correlation induced by B atoms would lead to rich physics (even without tunneling)
Weak attraction (BCS) limit

- B atoms in 3D induce $V_{\text{ind}}(r)$ between A atoms in 2D
  - **P-wave pairing of A atoms in the same layer for large d**
  - **S-wave pairing of A atoms in different layers for small d**
Strong attraction (BEC) limit

- A atoms in 2D capture B atoms from 3D to form dimers
- When $a_{\text{eff}} k_F < O(1)$ & $a_{\text{eff}} < d$, dimer size < mean distance

**Dimer BEC in each 2D layer**

- 2 layered BECs are coupled via the residual B atoms in 3D
Interlayer trimer formation

- When 2 dimers in different layers overlap \(a_{\text{eff}} > d\), they form a trimer!

- Interlayer AAB trimer is formed for \(O(-1) < d/a_{\text{eff}} < O(1)\)

- A single trimer state is stable!
  (as far as tunneling is negligible)

- RF spectroscopy will be possible

- Efimov spectrum
Phases of bilayer Fermi gas

Very rich but “minimal” phase diagram!!!
Possible many-body approaches

- mean-field approximation
- $\varepsilon$ expansion & large-N expansion
- Monte-Carlo simulation
- experiment using $^{40}\text{K}-^{6}\text{Li}$ mixture
Mixed dimensions = New arena of universal physics!

- confinement-induced 2-body resonances (observed)
- 3-body (Efimov) resonances, critical mass ratio for fermions
- rich phase diagram including dimer BEC, trimer Fermi gas, intralayer p-wave & interlayer s-wave superfluidity, ...

Idea of mixed dimensions can be extended to Bose-Fermi mixtures, Bose-Bose mixtures, multilayer geometry, multiwire geometry

New interesting research direction!

References
Efimov effect : Y.N. & S.Tan, PRA 79, 060701(R) (2009)
Backup slides
Resonances in mixed D ($m_A/m_B=6.67$)

2D-3D mixture

$A=^{40}\text{K}$ in 2D & $B=^6\text{Li}$ in 3D

1D-3D mixture

$A=^{40}\text{K}$ in 1D & $B=^6\text{Li}$ in 3D

$a_{\text{eff}}/l$

Infinite series of confinement-induced resonances

Resonance is shifted to “$a<0$” side
Weak attraction (BCS) limit

- B atoms in 3D induce $V_{\text{ind}}(r)$ between A atoms in 2D

**P-wave pairing of A atoms in the same layer for large $d$**

or

**S-wave pairing of A atoms in different layers for small $d$**

If B = bosons, ...

**intralayer p-wave**

**interlayer s-wave**
Other bilayer systems

- bilayers in condensed matter
- bilayer semiconductors
- bilayer quantum hall systems
- bilayer graphenesis...