The neutron-neutron scattering length

Anders Gårdestig

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Relevant publications

In collaboration with Daniel Phillips (supported by NSF and DOE):

- A.G. and D.R. Phillips
  arXiv.org/abs/nucl-th/0501049

- A.G. and D.R. Phillips
  arXiv.org/abs/nucl-th/0603045

- A.G.
  arXiv.org/abs/nucl-th/0604035

Review on $a_{nn}$:
Effective range expansion

At low energies $NN$ the $s$-wave phase shift can be written as

$$p \cot \delta = -\frac{1}{a} + \frac{1}{2} r_0 p^2$$

where $r_0$ is the effective range parameter.
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precursor to (pionless) effective field theory
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Similar, but different! Why?
QCD Lagrangian almost symmetric under $u \leftrightarrow d$ exchange (Charge Symmetry, CS), $P_{CS} = \exp(i\pi\tau_2/2)$ broken by $m_u \neq m_d$ (and EM effects)

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Experimental evidence:

$n-p$ mass difference

$\rho^0-\omega$ mixing ($e^+e^- \rightarrow \pi^+\pi^-$)

mirror nuclei (e.g. $^3\text{He}-^3\text{H}$) binding energy, N-S anomaly

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- $np \rightarrow np$: $A_n(\theta_n) \neq A_p(\theta_p)$ analyzing powers
- $A_{fb}(np \rightarrow d\pi^0)$ (TRIUMF) and $dd \rightarrow \alpha\pi^0$ (IUCF, soon COSY)
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CSB reviews:

[Miller, Nefkens, and Šlaus, PRt194, 1 (1990);
Miller and van Oers, nucl-th/9409013;
Miller, Opper, and Stephenson, ARNPS56, 293 (2006), nucl-ex/0602021]
CSB and scattering lengths

\[ \text{CSB} \Rightarrow a_{nn}^{\text{str}} \neq a_{pp}^{\text{str}} \]

\[ a_{nn}^{\text{str}} \neq a_{pp}^{\text{str}} \leftrightarrow \text{enhancement factor: } \frac{\Delta a_{\text{CSB}}}{a} = (10 - 15) \frac{\Delta V_{\text{CSB}}}{V} \]
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Calcs of \( B(\text{^3H}) - B(\text{^3He}) \) rely on \(|a_{pp}^{\text{str}}| < |a_{nn}^{\text{str}}|\), fails if \(|a_{pp}^{\text{str}}| > |a_{nn}^{\text{str}}|\)!

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Difficulties: EM corrections \((a_{NN}^{\text{str}})\), no free \(n\) target \((a_{nn}^{\text{str}})\)
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\[ |a_{pp}^{\text{str}}| > |a_{nn}^{\text{str}}|! \]

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WHAT TO DO?
Direct measurements:
Wild idea #1: Simultaneous underground nuclear explosions
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[Furman et al., JPG 28, 2627 (2002); Muzichka et al., NPA 789, 30 (2007)]
Solutions(?)

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Indirect $nn$ experiments:
Implemented idea: Reactions giving $nn$ with small rel. energy

$nn$: 3-body forces needed, expts differ:
$\alpha_{nn} = 16.004 \pm 0.009 \text{ fm}$ ($n;np$)
$\alpha_{nn} = 18.70 \pm 0.5$ ($n;p$)
$\alpha_{nn} = 18.59 \pm 0.42$ ($n;nnp$)

Standard value: $d_{nn} = 18.59 \pm 0.42$ ($n;nnp$)

Need accurate theoretical input for extraction!
Solutions

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Indirect $nn$ experiments:
Implemented idea: Reactions giving $nn$ with small rel. energy

$nd \rightarrow nnp$: 3-body forces needed, expts differ:
- $a_{nn} = -16.1 \pm 0.4 \text{ fm } (n, np)$ [Huhn et al., PRL85, 1190 (2000)] and
- $a_{nn} = -16.5 \pm 0.9 \text{ fm } (n, p)$ [von Witsch et al., PRC74, 014001 (2006)] VS
- $a_{nn} = -18.7 \pm 0.7 \text{ fm } (n, nnp)$ [González Trotter et al., PRC73, 034001 ('06)]

$\pi^- d \rightarrow nn\gamma$: $-18.59 \pm 0.40 \text{ fm } (\pi^-, n\gamma) \Rightarrow \text{standard value}$
(PSI and LAMPF) [Machleidt and Slaus, JPG:NPP27, R69 (2001)]

Need accurate theoretical input for extraction!
Bonn $nd \rightarrow nnp$ set-up

neutron beam

41.15°

proton arm

55.5°

neutron detector

transmission foil detector

CD$_2$ target

proton recoil telescope

shielding

collimator

gas target with beam stopper

deuteron beam

INT, Seattle, WA, 3/25/20 – p.6/27
TUNL $nd \rightarrow nnp$ set-up
Stopped pions captured on $d$ in atomic $s$-wave orbitals

![Diagram of LAMPF set-up]

Fig. 1. Schematic of the mid-level cut-away view of the experimental layout.
\[ \pi^- d \rightarrow nn\gamma \text{ data (LAMPF)}. \]

\[ p_1 = k + \theta_3 \]

\[ p_2 = \theta_3 \]

\[ \gamma \text{ and } n_1 \text{ detected at } 0.05 < \theta_3 < 0.1 \text{ (rad)} \] [Howell et al., PLB444, 252 (1998)]

\[ \text{Unnormalized, but shape fitted to give } a_{nn}! \]
Old theory for $\pi^- d \rightarrow nn\gamma$

Gibbs, Gibson, and Stephenson (GGS) [PRC11, 90 (1975)]:
- $\pi^- p \rightarrow \gamma n$, rel corr up to $O(p/M)$
- estimated pion rescattering
- tried different wave functions
- theoretical error (mainly SD): $\Delta a_{nn} = \pm 0.3$ fm
- Only accurate under the FSI peak!

de Téramond et al., [PRC16, 1976 (1977); 36, 691 (1987)]
similar error
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Can chiral perturbation theory ($\chi$PT) do better?
Advantages of an effective field theory like $\chi$PT:

- Consistent amplitudes and wave functions
- Recipe to estimate theoretical error
- Systematic improvement possible
- $\chi$PT = low-energy limit of QCD, retains chiral symmetry of QCD
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At low $E$: expansion in $\alpha_S \sim 1$ not possible. Instead:

Power counting gives hierarchy of amplitudes. Here:

- $Q \sim m_\pi$ small momentum/energy of problem
- $\Lambda_\chi \sim M \sim 4\pi f_\pi \sim 1$ GeV energy scale where $\chi$PT breaks down

Expand in $Q/\Lambda_\chi$
\[ \chiPT \text{ for } \pi^-d \rightarrow nn\gamma. \]

For \( \pi^-d \rightarrow nn\gamma \) we get

- \( O(Q^3) = \text{GGS} + \pi \text{ loops} + 2\text{-body} \)
- \( O(Q^3) \pi N \rightarrow \gamma N \) fitted to data \( \Rightarrow \) no free parameters

For capture on \( d: q\pi = 0 \), only one CGLN amplitude (\( F_1 \)) survives
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- 2B \( O(Q^4): \pi \) exchanges and contact term
- 1B \( O(Q^4): \) under investigation
$\chi$PT for $\pi^- d \rightarrow nn\gamma$.

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2B $O(Q^4)$: $\pi$ exchanges and contact term

1B $O(Q^4)$: under investigation

$\Rightarrow$ High precision possible
Quasifree (QF)

Final State Interaction (FSI)

Two-body effects (2)

\[ \Gamma \propto |\mathcal{M}_{QF} + \mathcal{M}_{FSI} + \mathcal{M}_2|^2 \]
Chirally inspired wave functions.

Start from asymptotic wave functions SE integrated in from $r = \infty$ with chiral OPEP and TPEP [Phillips & Cohen, NPA668, 45 (2000)]:

- Coupled integral equations for $d \,(^3S_1 - ^3D_1)$
- Uncoupled integral equations for $nn\,(^1S_0, ^3P_J, ^1D_2, \text{no} \,^3F_2)$
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Match with spherical well solution at $r = R = 1.4$ to $3.0$ fm
(Regulates unknown short-distance physics)

Calc indep of $R$?
One-body amplitudes.

EFT to $O(Q^3)$

[Fearing et al., PRC 62, 054006 (2000)]:

\[ A_1 \]

+pion loops at $O(Q^3)$ \Rightarrow \mathcal{A}_1

all parameters fitted to data
Two-body amplitudes $O(Q^3)$

First diagram has a Coulomb-like propagator, $1/\bar{q}^2$
Second diagram has $1/\bar{q}^2$ and also an off-shell pion prop
Third diagram (2 off-shell props) vanishes in Coulomb gauge

$\Rightarrow A_2$
$R$-dep error at $\mathcal{O}(Q^3)$

\[ \Delta a_{nn}^{(\text{theory})} = \pm 0.2 \text{ fm (FSI only)} \]

\[ \Delta a_{nn}^{(\text{theory})} = \pm 1 \text{ fm (full spectrum)} \]
Chiral relations

Chiral Lagrangian:

$$\mathcal{L} = N^\dagger (i v \cdot D + g_A S \cdot u) N$$

$$- 2d_1 N^\dagger S \cdot u N N^\dagger N + 2d_2 \epsilon^{abc} \epsilon_{\kappa \lambda \mu \nu} u^\kappa u^{\lambda, a} N^\dagger S^{\mu} \tau^b N N^\dagger S^\nu \tau^c N \ldots$$

where $f_\pi u_\mu = -\tau^a \partial_\mu \pi^a - \epsilon^{3ba} V_\mu \pi^b \tau^a + f_\pi A_\mu + \mathcal{O}(\pi^3)$

$1N (g_A)$: Goldberger-Treiman and Kroll-Ruderman

$$\frac{g_A}{f_\pi} = \frac{g_\pi NN}{M} \quad |A_{KR}| = \frac{e g_A}{f_\pi}$$

relate axial coupling to $\pi N$ coupling and $\gamma \pi N$ coupling
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\( 2N (d_i) \): Axial isovector coupling to \( NN (^3S_1 \leftrightarrow ^1S_0) \)

Connects \( \pi \) (photo)prod to EW reactions
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2\( N (d_i) \): Axial isovector coupling to \( NN (^3S_1 \leftrightarrow ^1S_0) \)

Connects \( \pi \) (photo)prod to EW reactions and chiral 3NF!
$O(Q^4)$ axial isovector contact term

For $^3S_1 \leftrightarrow ^1S_0$ one single LEC:

$$\hat{d} \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{\hat{c}_3}{3} + \frac{2\hat{c}_4}{3} + \frac{1}{6}$$
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Relates SD physics of $pp$ fusion, $^3H \rightarrow ^3He e^- \bar{\nu}_e$ (not EFT):

[Schiavilla et al., PRC58, 1263 (1998)]

$p$-wave $\pi$ prod+3NF:

[Hanhart, van Kolck, Miller, PRL85, 2905 (2000)]

$\mu^- d \rightarrow nn\nu_\mu$:

[Ando et al., PLB533, 25 (2002)]

$\nu(\bar{\nu})d$ breakup:

[Ando et al., PLB555, 49 (2003)]

$pp$ fusion, hep, $^3H \rightarrow ^3He e^- \bar{\nu}_e$:

[Park et al., PRC67, 055206 (2003)]

$pp$ fusion, $\pi^- d \rightarrow nn\gamma$, $\gamma d \rightarrow nn\pi^+$:

[AG+DRP, PRL96, 232301 (2006); AG, PRC74, 017001 (2006)]

$pp$ fusion, $\nu(\bar{\nu})d, \mu^- d \rightarrow nn\nu_\mu$:

[Butler et al., PLB520, 97 (2001); Chen et al., PRC72, 061001(R) (2005)]
Two-body amplitudes $O(Q^4)$

(a) 

(b) 

(c) 

(d) 

(e) 

(f) 

(g) 

(h) 

(i) 

(j) 

(k) 

(l) 

(m)
Constraining contact term

For axial isovector $^3S_1 \leftrightarrow ^1S_0$ (Gamow-Teller) transitions common in $NN$ systems only one LEC combination:

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constrained by tritium beta decay
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Let’s do a numerical experiment!

Remember:

Tjon line: $B(^4\text{He}) \text{ vs } B(^3\text{H})$

Phillips line: $^2a_{nd} \text{ vs } B(^3\text{H})$
Tjon line [Nogga, Kamada, Glöckle, PRL85, 944 (2000)]
FIG. 4. The results for $^2a_{nd}$ and $E_3^H$ from Table I: $np-nn$ forces alone (pluses), $np-pp$ forces alone (squares), and $np-nn$ and $np-pp$ forces plus electromagnetic interactions (stars and circles, respectively). The four straight lines (Phillips lines) are $\chi^2$ fits ($np-nn$, solid; $np-pp$, dashed-dotted; $np-nn$ with EMI’s, dashed; $np-pp$ with EMI’s, dotted). The lines with EMI’s miss the experimental error bar for $^2a_{nd}$ [33]. The physically interesting domain around the experimental values is shown in the inset.
Gamow-Teller vs FSI

\[ M_{GT} \text{ of } pp \text{ fusion vs FSI peak height} \]

\[ \Gamma_{FSI} \text{ (arbr. units)} \]

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Gårdestig-Phillips line(s)

$M_{GT}$ of $pp$ fusion vs FSI peak height

- $1.4 \text{ fm}$
- $1.6 \text{ fm}$
- $1.8 \text{ fm}$
- $2 \text{ fm}$
- $2.2 \text{ fm}$
- $2.4 \text{ fm}$
- $2.6 \text{ fm}$
- $2.8 \text{ fm}$
- $3 \text{ fm}$

$\Gamma_{FSI}$ (arbr. units)

- OPE, N2LO
- TPE, N3LO, no CT
- TPE, N3LO with CT
$\Delta a_{nn}^{(\text{theory})} = \pm 0.2$ fm (FSI only)

$\Delta a_{nn}^{(\text{theory})} = \pm 1$ fm (full spectrum)
R-dep error at $O(Q^4)$

\[ \Delta a_{nn}(\text{theory}) = \pm 0.05 \text{ fm (FSI only)} \]
\[ \Delta a_{nn}(\text{theory}) = \pm 0.3 \text{ fm (full spectrum)} \]
χS relates SD physics of 2B EW reactions to $(\gamma)_{\pi}NN$!
Summary and Conclusions I

- $\chi_S$ relates SD physics of 2B EW reactions to $\langle \gamma \rangle_{\pi NN}$!
- Chiral 3NF constrained by EW 2B obs!
- $\chi$PT reduces theory error for $a_{nn}$, $\Delta a_{nn} = \pm 0.05$ fm, a factor $>3$ better than previous calcs!

[AG+DRP, PRL 96, 232301 (2006)]
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- Better input possible from $\gamma d \rightarrow nn\pi^+$ or $\mu^- d \rightarrow nn\nu\mu$?
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- Chiral 3NF constrained by EW 2B obs!
- $\chi$PT reduces theory error for $a_{nn}$, $\Delta a_{nn} = \pm 0.05$ fm, a factor $>3$ better than previous calcs!
  
  [AG+DRP, PRL 96, 232301 (2006)]

- What remains:
  Higher order EM corrections in wfs?
  $N^3$LO 1B
  Orthonormalization

- Better input possible from $\gamma d \rightarrow nn\pi^+$ or $\mu^- d \rightarrow nn\nu_\mu$?

- Complete the circle:
  $\mu^- d \rightarrow nn\nu_\mu$ (1%) at PSI; expt and calc under way!
  $\nu(\bar{\nu})d$ breakup (SNO) with chiral wfs
  $^3H \rightarrow ^3Hee^-\bar{\nu}_e$ with ($r$-space) chiral wfs?
π⁻d → nnγ under good experimental and theoretical control
Summary and Conclusions II

- $\pi^- d \rightarrow nn\gamma$ under good experimental and theoretical control
- Bonn-TUNL discrepancy needs to be resolved, work under way
Summary and Conclusions II

- $\pi^- d \to nn\gamma$ under good experimental and theoretical control
- Bonn-TUNL discrepancy needs to be resolved, work under way
- direct measurement of $a_{nn}$ finally possible?
Intro to Phase Shifts

Free wave (radial part of 3D):

\[ u_l \sim \sin(pr - \frac{l\pi}{2}) \]
Scattered wave:

\[ u_l \sim \sin\left(pr - \frac{l\pi}{2} + \delta_l\right) \]
Intro to Phase Shifts

Scattered wave:

\[ u_l \sim \sin(pr - \frac{l\pi}{2} + \delta_i) \]

Interaction \( V \Rightarrow \) phase shift:

\( \delta > 0 \) attractive potential

\( \delta < 0 \) repulsive potential
Definition of Scattering Length

The low energy cross section

\[ \frac{d\sigma}{d\Omega} = \frac{1}{p^2} \sin^2 \delta_0 \rightarrow a^2 \]

defining the (S-wave) scattering length

\[ a \equiv - \lim_{p \to 0} \frac{\delta_0}{p} \]

where the sign is conventional.

For impenetrable sphere: \( a > 0 \) and \( \sigma = 4\pi a^2 \)
Scattering length and wfs

At zero energy the asymptotic $NN$ wave function behaves as $1 - r/a$

a) $a < 0$, attractive potential, scattering state
b) $a > 0$, repulsive potential
c) $a > 0$, attractive potential, bound state
CGLN amplitudes

Spin decomposition

\[ A_1(\gamma N \rightarrow \pi N) = F_1(E_\pi, x)i\sigma \cdot \epsilon_\gamma + F_2(E_\pi, x)\sigma \cdot \hat{q} \sigma \cdot (\hat{k} \times \epsilon_\gamma) \]
\[ + F_3(E_\pi, x)i\sigma \cdot \hat{k} \hat{q} \cdot \epsilon_\gamma + F_4(E_\pi, x)i\sigma \cdot \hat{q} \hat{q} \cdot \epsilon_\gamma \]

Isospin

\[ F_i^a(E_\pi, x) = F_i^{(-)}(E_\pi, x)i\epsilon^{a3b}\tau^b + F_i^{(0)}(E_\pi, x)\tau^a + F_i^{(+)}(E_\pi, x)\delta^{a3} \]

and for \( \gamma n \rightarrow \pi^- p \)

\[ F_i(\gamma n \rightarrow \pi^- p) = \sqrt{2}[F_i^{(0)} - F_i^{(-)}] \]

\( q = 0 \Rightarrow \) only \( F_1 \), dominated by KR for charged pions
Deuteron wave functions (OPE).

![Graph](https://example.com/graph.png)

- **u(r)** (fm$^{-1/2}$) and **w(r)** (fm$^{-1/2}$) functions for different radii (R).
- **Asymptotic** behavior highlighted.
- **Nijm93** model shown.

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Deuteron wave functions (TPE).

- $u(r)$ (fm$^{-1/2}$)
- $w(r)$ (fm$^{-1/2}$)

Asymptotic:
- $R_d = 0$ fm, OPE
- $R_d = 0$ fm, TPE

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$nn$ scattering wfs, GGS
$nn$ scattering wfs, GGS vs GP

$p = 10$ MeV/c

$\eta = 0$
$\eta = -5p \sim \psi$
$\eta = -0.5$

Forbidden state
Zero Range
$nn$ scattering wfs, GGS
$nn$ scattering wfs, GGS vs GP

GGS use $P_5(r)$

RSC

GP use sph well varying

OPEP+TPEP

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TPE wfs

\[ p = 10 \text{ MeV}/c \]

comparison of OPE and TPE

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\( \hat{d} \) can only be established if new FR derived:

\[
\left( c_4 + \frac{1}{4M} \right) \frac{2ie}{f_\pi^2} \left[ \left( \delta^{ab} \tau^3 - \delta^{a3} \tau^b \right) \left[ S \cdot q_1, S \cdot \epsilon_\gamma \right] \\
- \left( \delta^{ab} \tau^3 - \delta^{b3} \tau^a \right) \left[ S \cdot q_2, S \cdot \epsilon_\gamma \right] \right]
\]

Not published before \( \text{(not in [Bernard, Kaiser, Mei\ss{}ner, IJMPE 4, 193 (1995)])} \)

\( \text{[AG, PRC 74, 017001 (2006)]} \)
\[
\frac{\Gamma_{QF}}{\Gamma_{FSI}} = 2.422(1 - 0.0035 + 0.0003 + 0.013 - 0.035)
\]
Role of higher partial waves

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Sensitivity to $a_{nn}$

Neutron TOF spectrum at $\theta_3 = 0.075$ rad $\Rightarrow \frac{\Delta a_{nn}}{a_{nn}} = 0.83 \frac{\Delta \Gamma}{\Gamma}$
Boost corrections

Corrections to CGLN

\[
\Delta F_1^{(0)}(E_\pi) = \frac{e g_A}{2f_\pi} \frac{-(E_\pi p_n \cdot \hat{k} + E_\pi^2)}{2M^2}(\mu_p + \mu_n)
\]

\[
\Delta F_1^{(-)}(E_\pi) = \frac{e g_A}{2f_\pi} \frac{E_\pi p_n \cdot \hat{k} + E_\pi^2}{M^2}
\]

New spin-momentum structures

\[
G^{(0)}(E_\pi) = \frac{e g_A}{2f_\pi} \frac{iE_\pi p_n \cdot \epsilon_\gamma \sigma \cdot \hat{k}}{2M^2}(\mu_p + \mu_n - 1)
\]

\[
G^{(-)}(E_\pi) = \frac{e g_A}{2f_\pi} \left( \frac{E_\pi p_n \cdot (\hat{k} \times \epsilon_\gamma)}{2M^2}(\mu_p - \mu_n + \frac{1}{2}) - \frac{ip_n \cdot \epsilon_\gamma \sigma \cdot (2p_n + E_\pi \hat{k})}{M^2} \right)
\]

\[
\mu_p - \mu_n + \frac{1}{2} = 5.2, \text{ but } p_n \cdot (\hat{k} \times \epsilon_\gamma) \approx E_\pi^2 \sin \theta_3 \text{ with } \theta_3 = 0.075 \text{ rad}
\]

Similarly \( p_n \cdot \epsilon_\gamma \approx E_\pi \sin \theta_3 \)

Thus only CGLN corr’s important, \( O(\mu^2/2M^2) \sim 1\% \)
Both peaks scale the same way $\Rightarrow 0.10\%$ for $a_{nn}$
‘Off-shellness’

Off-shell nucleon transformed into 2B and on-shell 1B
New 2B \( O(Q^5) \Leftrightarrow p^2/M^2 \sim \mu^2/M^2 \sim 2\% \) of \( O(Q^3) \) 2B

\[ \Rightarrow \Delta a_{nn} = 0.02 \text{ fm} \]

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Subthreshold extrapolation

![Graph showing decay rate against neutron TOF (channels)]

- Red line: subthreshold
- Blue line: no subthreshold

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Error from $d$ wfs

![Graph showing neutron TOF (channels) vs. decay rate (arbr. units) with three curves for different $R_d$ values: $R_d = 2.0$ fm, $R_d = 1.5$ fm, and Bonn B.](image)
Error from $nn$ wfs

![Graph showing decay rate versus neutron TOF (channels)]

- **R = 2 fm, ann = -17.15 fm**
- **R = 1.5 fm, ann = -17.15 fm**
- Nijm I
- Nijm II

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FSI only

\[ \text{decay rate (arbr.units)} \]

- \( R = 2 \text{ fm}, \text{ann} = -17.15 \text{ fm} \)
- \( R = 1.5 \text{ fm}, \text{ann} = -17.15 \text{ fm} \)
- \( \text{Nijm I} \)
- \( \text{Nijm II} \)

Neutron TOF (channels)

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