Pion production in nucleon-nucleon collisions at low energies: status and perspectives

Vadim Baru

Forschungszentrum Jülich, Institut für Kenphysik (Theorie), D-52425 Jülich, Germany
ITEP, B. Cheremushkinskaya 25, 117218 Moscow, Russia

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in collaboration with E. Epelbaum, A. Filin, J. Haidenbauer, C. Hanhart, A. Kudryavtsev, V. Lensky and U.-G. Meißner

Motivation

Study of $\pi$ production in $NN$ collisions:

- test of ChPT in the process with large momentum transfer

- allows for determination of LECs ($(N\bar{N})^2\pi$ contact term)
  \[ \Rightarrow \text{direct connection to other low-energy processes} \]

- key to dispersive corrections to $\pi d$ scattering: $\pi d \to NN \to \pi d$ (our work 2007)
  \[ \Rightarrow \text{extraction of s-wave } \pi N \text{ scattering lengths from data on } \pi^- p \text{ and } \pi^- d \text{ atoms} \]
  \[ (\text{V.B., C.Hanhart, M.Hoferichter, B.Kubis, A.Nogga, D.Phillips (2010))} \]

- accurate data are available due to \textit{COSY; IUCF; TRIUMF, Uppsala}

Isospin Conserving $\pi$-production: necessary for studying isospin violation (IV)

$pn \to d\pi^0$: Opper et al. (2003), v.Kolck et al (2000), Bolton and Miller (2009), A. Filin et al. (2009)
$

dd \to \alpha\pi^0$: Stephenson et al.(2003), Gårdestig et al.(2004); Nogga et al.(2006), Fonseca et al. (2009)
CSB effects and neutron-proton mass difference

\[ \delta m_N = m_n - m_p = \delta m_{N}^{\text{str}} + \delta m_{N}^{\text{em}} = 1.29 \text{ MeV} \]

\(\delta m_{N}^{\text{str}}\) and \(\delta m_{N}^{\text{em}}\) contribute to different low-energy reactions

Possibilities to determine \(\delta m_{N}^{\text{str}}\) and \(\delta m_{N}^{\text{em}}\):

- Cottingham sum rule – provides an electromagnetic contribution to the nucleon self mass: \(\delta m_{N}^{\text{em}} = (-0.7 \pm 0.3) \text{ MeV} \implies \delta m_{N}^{\text{str}} = (2.0 \pm 0.3) \text{ MeV} \) (Gasser, Leutwyler (1982))

- Weinberg’s idea (1977): large CSB effects in \(\pi^0\) processes, e.g., \(a_{\pi^0p} - a_{\pi^0n}\)

\[ a_{\pi^0p} - a_{\pi^0n} = -\frac{1}{4\pi(1 + M_\pi/m_N)f_\pi^2} \delta m_{N}^{\text{str}} + O(q^4) \] (Meißner, Steininger (1998))

However, experimentally very difficult, if possible!

- Forward-backward asymmetry in \(pn \to d\pi^0\) can serve to pin down \(\delta m_{N}^{\text{str}}\) and \(\delta m_{N}^{\text{em}}\) (v.Kolck, Miller, Niskanen (2000))

- \(dd \to \alpha\pi^0\) (Stephenson et al., Gårdestig et al.; Nogga et al., Fonseca et al.)

- Lattice calculations (Beane et al. (2007))
CSB effects in $pn \rightarrow d\pi^0$

$$\frac{d\sigma}{d\Omega}(\theta) = C_0 + C_1 P_1(\cos \theta) + \ldots$$

$$\frac{d\sigma}{d\Omega}(\theta) \neq \frac{d\sigma}{d\Omega}(\pi - \theta)$$

$$A_{fb} = \frac{\int_0^{\pi/2} \left( \frac{d\sigma}{d\Omega}(\theta) - \frac{d\sigma}{d\Omega}(\pi - \theta) \right) \sin \theta d\theta}{\frac{\pi}{2} \left( \int_0^{\pi/2} \left( \frac{d\sigma}{d\Omega}(\theta) + \frac{d\sigma}{d\Omega}(\pi - \theta) \right) \sin \theta d\theta \right)} = \frac{C_1}{2C_0}$$

experiment: (Opper et al., TRIUMF (2003))

measurement at $T_{lab} = 279.5$ MeV (threshold $T_{lab} = 275.1$ MeV), $\eta = k_\pi / M_\pi = 0.17$

$$A_{fb} = (17.2 \pm 8 \pm 5.5) \times 10^{-4}$$
CSB effects in $pn \rightarrow d\pi^0$. Theory

$$A_{fb} = \frac{C_1}{2C_0}$$

Near threshold regime ($\eta = 0.17$) $\Longrightarrow$ s- and p-wave pion productions only:

<table>
<thead>
<tr>
<th>IC</th>
<th>IV</th>
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<tr>
<td>$^3P_1 \rightarrow^3 S_1s$</td>
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<td>$^1S_0 \rightarrow^3 S_1p$</td>
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$$\mathcal{M}_{pn\rightarrow d\pi^0} = M_{IC,s}^{IC}[\vec{S} \times \vec{n}]\vec{\varepsilon}_d + M_{1S_0}^{IC,p}(\hat{k}_\pi \vec{\varepsilon}_d) + M_{1D_2}^{IC,p}\left[(\vec{n}\hat{k}_\pi)(\vec{n}\vec{\varepsilon}_d) - \frac{1}{3}(\hat{k}_\pi \vec{\varepsilon}_d)\right] + M_{s}^{IV}(\vec{n}\vec{\varepsilon}_d) + \cdots$$

theory:

$$C_1 \sim \text{Re} M_{s-\text{wave}}^{IV} M_{p-\text{wave}}^{*IC} + \text{Re} M_{p-\text{wave}}^{IV} M_{s-\text{wave}}^{*IC}$$

LO...

$$N^2LO...$$

$C_0$ – total cross section, basically $^3P_1 \rightarrow^3 S_1s$
Pion reactions on few-nucleon systems

ChPT treatment (Weinberg 1992)

- expand the transition operator using ChPT. Include irreducible graphs only.
  - ChPT natural expansion parameter $\chi \sim \frac{q}{\Lambda_{\text{ChPT}}} \sim \frac{M_\pi}{m_N}$;
  - each graph gets its chiral order according to the counting rules;
  - $A = C_0 + C_1 \chi + C_2 \chi^2 + \cdots +$ non-analytic terms

- convolute with the (non-perturbative) wave functions

$A \Psi_f \Psi_i$

$A$ is perturbative
$
\Psi_i/f$ are treated non–perturbatively

- successful application to many low-momentum transfer reactions, for instance:
  - $\pi d \rightarrow \pi d$ Weinberg, Gasser et al, Beane et al, Meißner et al, our works
  - $\pi^3 \text{He} \rightarrow \pi^3 \text{He}, \pi^4 \text{He} \rightarrow \pi^4 \text{He}$ our works
  - $\gamma d \rightarrow \pi NN$ our works
  - $\pi d \rightarrow \gamma NN$ Gårdestig and Phillips
  - $\gamma d \rightarrow \pi^0 d$ Beane et al, Krebs et al.
Power Counting, $NN \rightarrow NN\pi$

Naive application of the Weinberg’s P.C. (with $q \sim M_\pi$) to $NN\pi \rightarrow$ disaster! (Park et al. (1996), Hanhart et al. (1998))

- NLO corrections increase discrepancy with the data
- $N^2$LO terms are larger than those at NLO.

Modified power counting Cohen et al. (1996); Hanhart et al. (2000)

new small scale in the production operator: $p \simeq \sqrt{M_\pi m_N}$ — initial NN momentum in c.m.s

s-wave pion:

$$\chi \sim \frac{p}{m_N} \sim \sqrt{\frac{M_\pi}{m_N}}$$

p-wave pion: $k_\pi \leq M_\pi$

$$\chi \sim \frac{k_\pi}{p} \sim \frac{p}{m_N} \sim \sqrt{\frac{M_\pi}{m_N}}$$
s-wave pion production up to NLO

NLO contribution Hanhart, Kaiser (2002)

\[ pp \to d\pi^+: \quad A_{d\pi^+}^{a+b+c} = \frac{g_A^3 |q|}{256 f_\pi^5} (\hat{\sigma}_1 + \hat{\sigma}_2) \cdot \frac{\vec{q}}{2}, \quad \vec{q} = \vec{p} - \vec{p}' \]

\[ pp \to pp\pi^0: \quad A_{pp\pi^0}^{a+b+c} = 0 \]

growing operator $\implies$ large sensitivity to the $NN$ w. f. Gårdestig, Phillips, Elster (2005)
\( pp \to d\pi^+ \), s-wave pion production (our work 2006)

\[
A_{d\pi^+}^{a+b+c+d1(\text{irr})+d2(\text{irr})} = \frac{g_A^3 |\vec{q}|}{256 f_\pi^5} (\vec{\sigma}_1 + \vec{\sigma}_2) \cdot \frac{\vec{q}}{2} \left( -2 + 3 + 0 - \frac{1}{4} - \frac{3}{4} \right) = 0
\]

Theoretical uncertainty is \( \mathcal{O} \left( \frac{m_\pi}{m_N} \right) \sim 30\% \).

\( \to N^2\text{LO} \) calculation is necessary to reduce the uncertainty – important for IV.

\( \to N^2\text{LO} \ pp \to pp\pi^0 \)
in progress (our group, Kim et. al (2009))
p-wave pion production in $NN \rightarrow NN\pi$ (our work (2009))

first calculation: $pp \rightarrow pn\pi^+$ channel (Hanhart, Miller, v.Kolck (2000))

Our goals: to study different channels: $pp \rightarrow d\pi^+$, $pp \rightarrow pn\pi^+$ and $pn \rightarrow pp\pi^-$

- to see if it is possible to describe them simultaneously with only one unknown $(N\bar{N})^2\pi$ LEC $d$
- to investigate convergence of the chiral expansion
- to obtain accurate p-wave amplitudes necessary for CSB studies

\[
\mathcal{L}^{(0)} = N^\dagger \left[ \frac{g_A}{2f_\pi} \tau \cdot \vec{\sigma} \cdot \vec{\nabla} \pi \right] N + \frac{h_A}{2f_\pi} \left[ N^\dagger (T \cdot \vec{S} \cdot \vec{\nabla} \pi) \psi_\Delta + h.c. \right] + \cdots ,
\]

\[
\mathcal{L}^{(1)} = \frac{1}{8m_N f_\pi^2} (iN^\dagger \tau \cdot (\pi \times \vec{\nabla} \pi) \cdot \vec{\nabla} N + h.c.) - \frac{1}{f_\pi^2} N^\dagger \left[ c_3 (\vec{\nabla} \pi)^2 \right] + \frac{1}{2} \left( c_4 + \frac{1}{4M_N} \right) \varepsilon_{ijk} \varepsilon_{abc} \sigma_k \tau_c \partial_i \pi_a \partial_j \pi_b \right] N - \frac{d}{f_\pi} N^\dagger (\tau \cdot \vec{\sigma} \cdot \vec{\nabla} \pi) N N^\dagger N + \cdots .
\]
p-wave pion production and $(N\bar{N})^2\pi$ LEC

Low-momentum transfer: $NN \rightarrow de\nu, \mu d \rightarrow \nu_\mu NN, \pi d \rightarrow \gamma NN, \gamma d \rightarrow \pi NN, pd \rightarrow pd, ...$

Large-momentum transfer: $NN \rightarrow NN\pi$

Nakamura (2008): $pp \rightarrow de^+\nu, pp \rightarrow pn\pi^+$

Conclusion: failure of simultaneous description
$$(N\bar{N})^{2\pi} \text{ LEC } d \text{ in } NN \rightarrow NN{\pi}$$

$\text{pp} \rightarrow d{\pi}^{+}, \text{pp} \rightarrow \text{pn}{\pi}^{+}$

$\text{pn} \rightarrow \text{pp}{\pi}^{-}$

Description with the same LEC $d$ – non-trivial test of consistency

Why do we expect this to work?

$$\Psi_{q}(r = 0) = \left(1 + m_{N} \int_{0}^{\infty} d^{3}p \frac{T(p, q, q)}{q^{2} - p^{2} + i0}\right) = C(\Lambda) \exp \left\{ \frac{1}{\pi} \int_{0}^{\infty} ds' \frac{\delta_{NN}(s')}{s' - s(q) + i0} \right\}$$

- energy dependence of $\Psi_{q}(0) – \text{model independent}$
- $C(\Lambda) – \text{model dependent}$
- $C_{1S_{0}}(\Lambda)$ and $C_{3S_{1}}(\Lambda)$ are absorbed in $d$
p-wave pion production mechanism

\[ d: \ ^1S_0 \to ^3S_1 p \text{ in } pp \to pn\pi^+/d\pi^+ \]
\[ d: \ (^3D_1 - ^3S_1) \to ^1S_0 p \text{ in } pn \to pp\pi^- \]

\[ \bullet \ d = d(\Lambda) – \text{depends on the regularization scheme and type of NN interaction} \]
\[ \bullet \ d \text{ absorbs the short-range part of the production operator} \]

\[ A_{c_i} \sim \left( \frac{c_3}{2} + c_4 + \frac{1}{4m_N} \right) \frac{(\vec{p} - \vec{p}')^2}{(\vec{p} - \vec{p}')^2 + M_\pi^2} \to const + O(N^4LO) \]

LEC d absorbs \( A_{c_i} \)
\( \text{NN} \rightarrow \text{NN}\pi, \text{Results} \) (our work (2009))

\[ \text{pp} \rightarrow d\pi^+ \]

Positive \( d \simeq 3 \) is clearly preferred


\[ NN \rightarrow NN\pi, \text{ Results} \quad \text{(our work (2009))} \]

\[ pp \rightarrow d\pi^+ \]

\[ pn \rightarrow pp\pi^- \quad (\eta = 0.6), \quad M_{pp} \leq 1.5 \text{ MeV} \quad \left( ^3S_1 - ^3D_1 \right) \rightarrow ^1S_0p \]

New measurement of \( pn \rightarrow pp\pi^- \) at different energies (ANKE at COSY 2010) IMPORTANT!

Positive \( d \simeq 3 \) is clearly preferred
$pn \rightarrow pp\pi^-$, Measurement at COSY

Analyzing power at $T_{lab} \sim 340$-380 MeV (ANKE (2010), S. Dymov et al.: PRELIMINARY)

Positive $d \approx 3$ is clearly preferred

Prediction for double polarization observables

Measurement: $d\sigma/d\Omega(1 - A_{xx}) \sim |C_1 - C_2/3|^2 \cdot \sin^2(\theta)$ – direct access to $p$-wave amplitudes $C_1(d)$ and $C_2(d)$. (V.B., S. Dymov, C. Hanhart, A. Kacharava, Yu. Uzikov, C. Wilkin)
\[ \frac{d\sigma}{d\Omega} = C_0 + C_2 P_2(\cos \theta) + \ldots \]

\[
\begin{align*}
C_0 &= \frac{|a_0|^2 + |a_1|^2 + |a_2|^2}{4} + C_0^{I=1}, \\
C_2 &= \frac{|a_2|^2}{4} - \frac{1}{\sqrt{2}} \text{Re}[a_0 a_2^*],
\end{align*}
\]

\[
A_y(90^\circ) \left( C_0 - \frac{C_2}{2} \right) = \frac{1}{4} (\sqrt{2} \text{Im}[a_1 a_0^*] + \text{Im}[a_1 a_2^*]).
\]

\( \eta \)

\( \eta \)

Influence of \( Pp \) states needs to be understood.

We can describe all channels of \( NN \rightarrow NN\pi \) with the same LEC \( d \)!
Drawbacks of PWA

- old \( pp \rightarrow pp\pi^0 \) data were used to extract \( C_0^{l=1} \). New data (COSY 2003) are 50% larger!
- \( C_0^{l=1} \) is not corrected for the difference between \( pp \) and \( pn \) interactions at low energies:
  \[ a_{pp} \simeq 7.8 \text{ fm} \ll a_{pn} \simeq 23.7 \text{ fm} \]

Integrated ratio of the Jost functions:

\[
R = \frac{\int d\tau_3 |F_{pn}(p)|^2}{\int d\tau_3 |F_{pp}(p)|^2} \simeq 1.5 \quad \text{for} \quad \eta = 0.22
\]

- No \( Pp \) states

Nakamura’s result: failure of bridging program between \( pp \) fusion and \( NN\pi \)

Conclusion is based on wrong PWA!
s- and p-wave isospin conserving pion production is under control.

Next goal: Charge Symmetry Breaking effects
CSB effects in $pn \rightarrow d\pi^0$. Power counting

**CSB operators for s-wave pion up to $N^2$LO**

\[\mathcal{L}_{IC}^{(0)} = N^\dagger \left[ \frac{1}{4F^2_\pi} \tau \cdot (\hat{\pi} \times \pi) + \frac{g_A}{2F_\pi} \tau \cdot \vec{\sigma} \cdot \vec{\nabla} \pi \right] N + \cdots,\]

\[\mathcal{L}_{IV}^{(0)} = \frac{\delta m_N}{2} N^\dagger \tau_3 N - \frac{\delta m_N^{\text{str}}}{4F^2_\pi} N^\dagger \tau \cdot \pi_3 N - \frac{\delta m_N^{\text{em}}}{4F^2_\pi} N^\dagger (\tau_3 \pi^2 - \tau \cdot \pi_3) N + \cdots\]

No terms at NLO $\Rightarrow$ theoretical uncertainty of LO calculation is $\sim \chi^2 \sim \frac{M_\pi}{m_N} \sim 15%$

IV p-wave pion starts from $N^2$LO.
CSB effects in $pn \rightarrow d\pi^0$. Additional IV operator at LO.

New contribution from diagram (b) at LO (our work (2009))

$\pi N$ Lagrangians relevant at LO:

$$\mathcal{L}^{(0)}_{iv} = -\frac{\delta M^\text{str}_N}{4F^2_\pi} N^\dagger \tau \cdot \pi \pi_3 N - \frac{\delta M^\text{em}_N}{4F^2_\pi} N^\dagger (\tau_3 \pi^2 - \tau \cdot \pi \pi_3) N$$

$$\mathcal{L}^{(0)}_{WT} = \frac{1}{4F^2_\pi} N^\dagger \tau \cdot (\dot{\pi} \times \pi) N$$

WT Lagrangian is time-dependent $\implies$ vertex $\sim q_0 + M_\pi$ depends on $\delta m_N$ through $q_0$

$$V_{WT} = \frac{i\sqrt{2}}{4f^2_\pi} \left( \frac{-3m_\pi}{2} - \delta m_N \right)$$

Terms $\sim \delta m_N$ add up!
CSB effects in $pn \rightarrow d\pi^0$. Our results

Filin et al. (2009) Strategy:

To minimize the theoretical uncertainty of IC amplitudes as much as possible using data!

- IC p-wave ($N^2$LO) calculation – very good description of data
- take $C_0$ directly from data on pionic deuterium atom
  $\sigma(nn \rightarrow d\pi^-) = 252^{+5}_{-11} \cdot \eta \ [\mu b]$
- perform complete LO calculation of CSB amplitudes

\[
\delta M^\text{str}_N \sim \left( \frac{\delta m^\text{em}_N}{2} - \delta m^\text{str}_N \right) + \frac{\delta m^\text{str}_N + \delta m^\text{em}_N}{2} \rightarrow \frac{3}{2} \delta m^\text{str}_N
\]

Our LO result:

\[
A_{fb} = (11.5 \pm 3.5) \times 10^{-4} \delta M^\text{str}_N
\]

\[
\begin{align*}
\delta m^\text{str}_N &= (1.5 \pm 0.8 \ (\text{exp.}) \pm 0.5 \ (\text{th.})) \ \text{MeV} \\
\delta m^\text{str}_N &= (2.0 \pm 0.3) \ \text{MeV} \\
\delta m^\text{str}_N &= (2.26 \pm 0.57 \pm 0.42 \pm 0.10) \ \text{MeV}
\end{align*}
\]

$pn \rightarrow d\pi^0$ at LO

Gasser and Leutwyler

lattice data Beane et al. (2007)
CSB effects in $pn \rightarrow d\pi^0$. Comparison to other works

CSB at LO (Bolton and Miller): main origins of discrepancies with our results

- $s$- and $p$-wave IC amplitudes are calculated at NLO
  $\Rightarrow 30\% +15\%$ uncertainties in $A_{fb}$

- $\Delta$-isobar contribution at NLO to the $p$-wave amplitudes is too large
  $\Rightarrow$ phenom. formfactors with $\Lambda \approx 400$ MeV are introduced.
Still $A_y$ in $pp \rightarrow d\pi^+$ is overestimated by about 30%.
$NN \to NN\pi$. Summary and Outlook

**s-wave production, NLO calculation**

- quantitative understanding of $pp \to d\pi^+$ with 15% uncertainty in the amplitude

**p-wave production, N$^2$LO calculation**

- studying different channels of $NN\pi$ simultaneously – good test for LEC $(N\bar{N})^2\pi$ in different kinematic regimes
- good overall description of all channels with the same LEC!
- chiral expansion indeed converges in accordance to power counting

**CSB effects in $pn \to d\pi^0$, LO calculation**

- unique opportunity to extract $\delta m_N^{\text{str}}$ from a dynamical process:
  $\delta m_N^{\text{str}} = (1.5 \pm 0.8 \text{ (exp.)} \pm 0.5 \text{ (th.)})$ MeV
- Non-trivial consistency with the Cottingham sum rule and lattice results
**NN → NNπ. Outlook**

We are on the way to control pion dynamics in $NN \rightarrow NN\pi$

Further steps and future plans

- $pp \rightarrow pp\pi^0$ still needs to be understood. $N^2$LO is in progress (Kim et al. (2008), our group)
- $N^2$LO calculation for $pp \rightarrow d\pi^+$ (in progress) and CSB effects for $pn \rightarrow d\pi^0$ together with $dd \rightarrow \alpha\pi^0$
- $NN \rightarrow NN\pi + pp \rightarrow de^+\nu + \cdots$ within the same framework is interesting!
- results from ANKE are highly awaited
  - $pn \rightarrow pp\pi^-$ at low energies is the best chance to fix $4N\pi$ CT in $NN \rightarrow NN\pi$
  - planned measurements of spin-correlation coefficients, $A_y$ and $d\sigma/d\Omega$ for $pn \rightarrow pp\pi^-$ and $pp \rightarrow pp\pi^0$ would allow for a partial wave analysis
- extension of ChPT $NN$ interaction above pion threshold