Global QCD Analysis of Fragmentation Functions and Possible Medium Modifications

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plan of the talk

I. quick introduction & reminder

- QCD framework for fragmentation
  definition of FFs, relevant processes, limitations

II. current status of fragmentation fcts.

- DSS global analysis
  prejudices, results, issues, comparison with other fits,
  uncertainties, future directions

III. medium modification

- "pragmatic approach"
  idea & ansatz, analysis of eN & dAu data, 1st results
precise knowledge of fragmentation functions crucial for interpretation & understanding of, e.g., RHIC & LHC results and QCD in general

unpolarized pp cross sections are an important baseline for

- studies of saturation effects in dAu and AuAu collisions
- understanding of spin asymmetries & extraction of, e.g., $\Delta g$

incl. hadron data put fundamental ideas to the test

- fragmentation as fundamental as nucleon structure
- factorization and universality of fragmentation functions
- possible modifications in a nuclear medium
I. introduction

QCD framework for fragmentation
some properties of $D_i^h(z,\mu)$

- non-perturbative \textit{universal} objects
  - scale $\mu$-dep. predicted by pQCD

- needed to consistently absorb final-state collinear singularities like, e.g., in $pp \to \pi^0X$ ("factorization")

- describe the \textit{collinear} transition of a parton "i" into a massless hadron "h" carrying fractional momentum $z$

- bi-local operator: $D(z) \simeq \int dy^- e^{iP^+/z y^-} \text{Tr} \gamma^+ \langle 0| \psi(y^-) |hX\rangle \langle hX| \overline{\psi}(0) |0\rangle$
  - Collins, Soper '81, '83
  - no inclusive final-state
  - \rightarrow no local OPE \rightarrow \text{no lattice formulation}
  - also: power corrections are much less developed and entwined with mass effects unlike pdfs
time-like evolution

\[
\frac{dD_i^h(z, \mu^2)}{d\ln \mu^2} = \int_0^1 \frac{dy}{y} P_{ji}^T(z, \alpha_s) D_j^h \left( \frac{z}{y}, \mu^2 \right)
\]

\[P_{ji}^T(z, \alpha_s) = \frac{\alpha_s}{4\pi} P_{ji}^{(0)T} + \left( \frac{\alpha_s}{4\pi} \right)^2 P_{ji}^{(1)T} + \left( \frac{\alpha_s}{4\pi} \right)^3 P_{ji}^{(2)T} + \ldots\]

same as space-like PDF evolution

Gribov-Lipatov relation

related to \(P_{ij}^{(1)S}\) by analytic continuation

Curci, Furmanski, Petronzio; Floratos et al.; MS, Vogelsang

\(P_{qq}^{(2)T}, P_{gg}^{(2)T}\) known

naive AC fails: \(\pi^2\) terms

Moch, Vogt

find

small-\(z\) behavior markedly different from space-like case: much more singular

\[z P_{gg}^{(n)T} \propto \alpha_s^n \ln^{2n} z\]

has impact already at \(z \sim 0.1\)

resummations?
limitations for the use of FFs

• however, small z region completely spoiled by “mass effects”
  [problem: hadron energy can be even smaller than its (neglected) mass]
  → need to introduce a cut on z, typically $z \gtrsim 0.05 \div 0.1$

• implies that sum rules are of limited practical use in fits of FFs
  “energy-momentum conservation”: $\sum_h \int_0^1 zD^h_i(z, \mu) = 1$
  (a parton fragments with 100% probability into something preserving its momentum)

• FFs are - by definition - inclusive quantities
  → can compute inclusive distributions of hadrons with momentum fractions z but not a cross section for a “leading hadron”
  (under certain kin. conditions it might be a good approximation though)

• fragmentation is assumed to be independent of other colored particles
  → need a hard scale to be a valid approximation
single-inclusive $e^+e^-$ annihilation (SIA)

relevant: “normalized distribution” \[
\frac{1}{\sigma_{\text{tot}}} \frac{d\sigma^h}{dz}
\]

\[
= \frac{1}{\sigma_{\text{tot}}} \sum_{i=q,\bar{q},g} \left[ \int \frac{dy}{y} \right] C_i \left( \frac{z}{y}, Q, \mu_r, \mu_f \right) D_i^h (y, \mu_f)
\]

"coeff. fct." calculable in pQCD

\[
\text{LO: } C_q = \delta(1-y) \sigma_0; \quad C_g = 0
\]

\[
O(\alpha_s) \text{ NLO: } \text{Altarelli, Ellis, Martinelli, Pi '79; Furmanski, Petronzio '82}
\]

\[
O(\alpha_s^2) \text{ NNLO: } \text{Rijken, van Neerven '96,'97; Mitov, Moch '06}
\]

"scaling" variable \[
z \equiv \frac{2P^h \cdot q}{Q^2} = \frac{2E^h}{Q}
\]

where

\[
s = q^2 = Q^2
\]

\[
P_{e^\pm} = (Q/2, 0, 0, \pm Q/2)
\]

\[
q = P_{e^+} + P_{e^-}
\]
semi-inclusive DIS (SIDIS)

SIDIS = DIS plus one identified hadron with $x_F > 0$

$$\frac{d\sigma^h}{dx dQ^2 dz^h} \approx \sum_{f=q,\bar{q}} e_f^2 f(x, \mu_f) D_f^h(z^h, \mu'_f) + O(\alpha_s)$$

"scaling" variable $z^h \equiv \frac{p^h \cdot p^N}{p^N \cdot q}$

why important?

- charge separated data: $\pi^+, \pi^-, K^+, K^- \text{ HERMES}; h^+, h^- \text{ EMC}$
  → valuable handle on flavor separation

  LO analysis: $D_{d}^{\pi^+} \sim (1 - z) D_{u}^{\pi^+}$

Altarelli et al. '79; Furmanski, Petronzio '82; de Florian, MS, Vogelsang '98

Christova, Kretzer, Leader

pp $\rightarrow hX$

well-known framework: factorization, NLO corrections

$d\sigma \sim$

Aversa et al.; Jäger, Schäfer, MS, Vogelsang; de Florian

e.g., very nice data from RHIC:

plus STAR and BRAHMS
why important?

central rapidity (PHENIX data)

\[ pp \rightarrow gX \]
\[ pp \rightarrow qX \]

\[ p_T \text{ [GeV]} \]

→ low \( p_T \) data probe gluon fragmentation

forward rapidity (STAR data)

\[ \langle z \rangle = \langle x_2 \rangle \]

\[ \langle x_1 \rangle = \langle x_2 \rangle \]

\[ E_\pi \text{ [GeV]} \]

→ probe gluon and quark fragmentation at large \( z \)

BRAHMS \( \pi^\pm, K^\pm \) data (\( \eta \approx 3 \)) → flavor separation from pp data
II. current status of FFs

DSS global analysis, other fits, issues

with D. de Florian, R. Sassot,
**DSS analysis - overview**

**goal:** provide NLO (LO) sets for pions, kaons, protons, charged hadrons from a **global fit** to $e^+e^-$, ep, and pp data on 1-hadron production

- requires a flexible functional form

\[
D_i^h(z, 1 \text{ GeV}) = N_i z^{\alpha_i} (1 - z)^{\beta_i} \left[ 1 + \gamma_i (1 - z)^{\delta_i} \right]
\]

- try to avoid assumptions on parameter space if possible

SU(2), SU(3) breaking:

\[
D_{d+d}^{\pi^+} = N D_{u+\bar{u}}^{\pi^+} \quad D_{s}^{\pi^+} = D_{s}^{\pi^+} = N' D_{u}^{\pi^+}
\]

only normalization shifts can be fitted

but data do not discriminate between other unfavored FFs:

\[
D_{u}^{\pi^+} = D_{d}^{\pi^+} \quad D_{u}^{K^+} = D_{s}^{K^+} = D_{d}^{K^+} = D_{d}^{K^+}
\]

- like in PDF fits we allow for

  - relative normalizations/shifts of data sets
  - cuts: \( z > 0.05 \) pions, \( z > 0.1 \) otherwise
  - extra “TH errors”: scale uncertainty (pp); flavor tag; bin size, ...

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overall quality of the fits

• typically $\chi^2$/d.o.f. $\approx 2$
  mainly from a few isolated points, e.g., SIDIS $\pi^-$ and $K^-$
  some tension among data sets with flavor tagging

• $\chi^2$ grows $\approx 25\%$ for LO fits
  mainly from pp data (fits try to make up for large NLO corrections)

• predictive power

  NLO $p_T$ distribution of forward $\pi^0$
  crucially dependent on $D_g$
  Daleo, de Florian, Sassot

  many opportunities for an EIC to contribute - studies needed
some results

SIA data still work very well within a global fit

- pions
- kaons

similar for protons and charged hadrons and “flavor tagged data”

KRE: S. Kretzer
AKK: S. Albino et al.
AKK uses $z > 0.1$
good description of SIDIS multiplicities

Kretzer's assumption
\[ D_{d}^{\pi^+} \simeq (1 - z)D_{u}^{\pi^+} \]
works for \( \pi^+ \)
but not for \( \pi^- \)

\( x, Q^2 \) range can be significantly extended at an EIC

shaded bands: our estimate of "Q^2-binning effects"
“pp” data also well reproduced

- large scale uncertainties
- probe z values well below 0.1
  but x-sec mainly samples \( z \gtrsim 0.5 \)
- \( p_{T,\text{min}} \) cut has no impact on fit

![Graphs and plots showing data for different particles and rapidities.](image-url)
meet the $D_i(z)$’s: pion FF

\[ zD_1^\pi(z) \quad Q^2 = 10 \text{ GeV}^2 \]

\[ zD_2^\pi(z) \quad Q^2 = 10 \text{ GeV}^2 \]

- singlet fragmentation $D_\Sigma$ very similar (fixed by SIA at $M_Z$)
- u-frag. smaller than in AKK (due to SIDIS); compensated by larger $D_s$ in SIA
- find: SU(2) violation $< 10\%$; SU(3) violation $\sim 17\%$

\[ \langle z \rangle \gtrsim 0.6 \]
charge symmetry violation?

our fit prefers a $\approx 10\%$ breaking of SU(2) sym. driven by $\pi^{\pm}$ multiplicities

small, large, or just about right?

estimate magnitude in a simple model

define charge-symmetry breaking as

$$\delta D(z) = \frac{D_u^{\pi^+} - D_d^{\pi^-}}{D_u^{\pi^+}}$$

find $\delta D(z) < 0$ and $O(\text{few percent})$!

Londergan, Pang, Thomas
PRD54 (1996) 3154

http://lappweb.in2p3.fr/lapth/ffgenerator

same sign as in DSS
meet the $D_i(z)$’s: kaons

smaller $u$ & larger $s$-frag. required by SIDIS data

impact on pol. PDF fit: $\Delta s$

note: some issues with $K^-$ data (slope!)

await final HERMES data
## other recent NLO fits of FFs:

Table from arXiv:0804.2021 (ECT* FF workshop)

<table>
<thead>
<tr>
<th>Name</th>
<th>Ref.</th>
<th>Species</th>
<th>Error</th>
<th>$z_{min}$</th>
<th>$Q^2$ (GeV$^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>AKK</td>
<td>[4]</td>
<td>$\pi^\pm, K^{\pm}, K^0, p, \bar{p}, \Lambda, \bar{\Lambda}$</td>
<td>no</td>
<td>0.1</td>
<td>$2 - 4 \cdot 10^4$</td>
</tr>
<tr>
<td>AKK08</td>
<td>[5]</td>
<td>$\pi^\pm, K^{\pm}, K^0, p, \bar{p}, \Lambda, \bar{\Lambda}$</td>
<td>yes</td>
<td>0.05</td>
<td>$2 - 4 \cdot 10^4$</td>
</tr>
<tr>
<td>BKK</td>
<td>[6]</td>
<td>$\pi^+ + \pi^-, \pi^0, K^+ + K^-, K^0 + K^0, h^+ + h^-$</td>
<td>no</td>
<td>0.05</td>
<td>$2 - 200$</td>
</tr>
<tr>
<td>BFG</td>
<td>[7]</td>
<td>$\gamma$</td>
<td>no</td>
<td>$10^{-3}$</td>
<td>$2 - 1.2 \cdot 10^4$</td>
</tr>
<tr>
<td>BFGW</td>
<td>[8]</td>
<td>$h^\pm$</td>
<td>yes$^1$</td>
<td>$10^{-3}$</td>
<td>$2 - 1.2 \cdot 10^4$</td>
</tr>
<tr>
<td>CGRW</td>
<td>[9]</td>
<td>$\pi^0$</td>
<td>no</td>
<td>$10^{-3}$</td>
<td>$2 - 1.2 \cdot 10^4$</td>
</tr>
<tr>
<td>DSS</td>
<td>[10, 11]</td>
<td>$\pi^\pm, K^{\pm}, p, \bar{p}, h^\pm$</td>
<td>yes$^2$</td>
<td>0.05-0.1</td>
<td>$1 - 10^5$</td>
</tr>
<tr>
<td>DSV</td>
<td>[12]</td>
<td>polarized and unpolarized $\Lambda$</td>
<td>no</td>
<td>0.05</td>
<td>$1 - 10^4$</td>
</tr>
<tr>
<td>GRV</td>
<td>[13]</td>
<td>$\gamma$</td>
<td>no</td>
<td>0.05</td>
<td>$\geq 1$</td>
</tr>
<tr>
<td>HKNS</td>
<td>[14]</td>
<td>$\pi^\pm, \pi^0, K^{\pm}, K^0 + K^0, n, p + \bar{p}$</td>
<td>yes</td>
<td>0.01 - 1</td>
<td>$1 - 10^8$</td>
</tr>
<tr>
<td>KKP</td>
<td>[15]</td>
<td>$\pi^+ + \pi^-, \pi^0, K^+ + K^-, K^0 + K^0, p + \bar{p}, n + \bar{n}, h^+ + h^-$</td>
<td>no</td>
<td>0.1</td>
<td>$1 - 10^4$</td>
</tr>
<tr>
<td>Kretzer</td>
<td>[16]</td>
<td>$\pi^\pm, K^{\pm}, h^+ + h^-$</td>
<td>no</td>
<td>0.01</td>
<td>$0.8 - 10^6$</td>
</tr>
</tbody>
</table>

AKK08: Albino, Kniehl, Kramer || HKNS: Hirai, Kumano, Nagai, Sudoh

e$^+e^-$ & pp data
impose isospin sym. for pions
Hessian method for uncertainties
fit hadron masses
large-z resummations
mass corrections (looks ad hoc for pp)
e$^+e^-$ data
impose isospin sym. for pions
Hessian method for uncertainties
comparison for pion fragmentation

making use of nice online-plotting tool for fragmentation fcts
F. Arleo, J.Ph. Guillet, M. Werlen

faithful measure of uncertainties on FFs?
recall: DSS, AKK08, HKNS are based on different data sets and assumptions
comparison for kaon fragmentation

\[ \bar{s} \rightarrow K^+ \]

\[ u \rightarrow K^+ \]

\[ g \rightarrow K^\pm \]
estimating uncertainties in DSS

many methods - we choose “Lagrange multipliers”

idea: see how fit deteriorates when forced to give a different $O_i$

$$\Phi(\{\lambda_i\}, \{a_j\}) = \chi^2(\{a_j\}) + \sum_i \lambda_i O_i(\{a_j\})$$

- directly examines $\chi^2$ profile; no assumptions like in Hessian method
- role of each data set can be assessed
- easy to implement
- $z$-dependent errors on $D_i^h(z)$ less straightforward

for the time being, we study

$$O_i(\{a_j\}) = \eta_i(\{a_j\}, z_{\text{min}}) = \int_{z_{\text{min}}}^{1} dz \, z \, D_i^h(z, Q^2)$$

“truncated energy fractions”
uncertainties from Lagrange multipliers: pions

recall:

\[ O_i(\{a_j\}) = \eta_i(\{a_j\}, z_{\text{min}}) \]
\[ = \int_{z_{\text{min}}}^{1} dz \, z \, D_i^h(z, Q^2) \]

here:

\[ z_{\text{min}} = 0.2, \, Q = 5 \, \text{GeV} \]

\[ \eta_0: \text{best fit value} \]

next:

generate the \( \chi^2 \) profiles

choose the \( \Delta \chi^2 \) you want to tolerate, e.g., \( \Delta \chi^2 = 15 \)

and read off uncertainties

\[ \delta\eta_{u+\bar{u}}^{\pi^+} \leq 3\% \]
\[ \delta\eta_{u+\bar{u}}^{\pi^+} \leq 5\% \]
\[ \delta\eta_{s+\bar{s}}^{\pi^+} \approx 10\% \]

\[ \delta\eta_{g}^{\pi^+} \leq 10\% \]
\[ \delta\eta_{c+c}^{\pi^+} \geq 10\% \]
\[ \delta\eta_{b+b}^{\pi^+} \leq 10\% \]

**kaons:** uncertainties at least twice as large as for pions
future avenues for $D(z, \mu)$

- HERMES (COMPASS?)  more/final hadron multiplicities

- RHIC/RHIC-II  more luminosity (high $p_T$ hadrons) will yield better bounds

- $e^+e^- (BELLE)$  analyze precision measurements of identified hadrons
  - gluon fragmentation $D_g$ from scaling violations in $e^+e^-$

- EIC  possible precision studies of (polarized) semi-inclusive DIS

\[
\frac{d\sigma^h}{dxdQ^2dz^h} \approx \sum_{f=q,\bar{q}} e_f^2 f(x, \mu_f) D_f^h(z^h, \mu'_f) + \mathcal{O}(\alpha_s)
\]

charge separated data for $\pi^+, \pi^-, K^+, K^-, ...$

- flavor separation at higher (safer) $Q^2$
what can be expected from BELLE

Belle MC

<1% of data sample  
→ work in progress

precision at high z!

R. Seidl, M. Grosse Perdekamp, ...
III. medium modifications

idea & ansatz, some prel. results

with R. Sassot, P. Zurita
motivation

Q: how does hadronization proceed in a nuclear environment?

- increasing number of data available to address this
  - SIDIS off nuclei (HERMES)
  - dAu data (BRAHMS, PHENIX, STAR)
    → clear indications of non-trivial A dependence
      "hadron attenuation", "Cronin effect", ...

- many different models [recent review: A. Accardi et al., arXiv:0907.3534]
  "(pre)hadron absorption"; "hadron attenuation"
  "interaction of the seed parton (energy loss)"
  modified DGLAP evolution; ...

reproduce main/some features of data in spite of very different approaches and underlying ideas
examples: hadron attenuation

**HERMES**

$\pi^+$ multiplicity ratios
(also for $\pi^-$, $K^\pm$, p, pbar)

\[
R(x, z, Q^2) = \frac{\left(\frac{N_{SIDIS}}{N_{DIS}}\right)_A}{\left(\frac{N_{SIDIS}}{N_{DIS}}\right)_D}
\]

**find:** $R < 1$

not included in any fit of nPDFs: nDS, EPS, HKN

**analysis requires modified fragmentation**
examples: dAu data

\[
R > 1
\]

\[\begin{array}{c}
\text{PHENIX 2007 } \pi^0 \\
\text{STAR 2006 } \pi^+ + \pi^-
\end{array}\]

\[\begin{array}{c}
\text{EPS09NLO} \\
\text{HKN07 (NLO)}
\end{array}\]

**EPS**: analyzed only with nPDFs but no modification in fragmentation

need some normalization shift: \[N_{\text{PHENIX}}=1.03, \ N_{\text{STAR}}=0.9\]

data used to constrain nuclear gluon PDF
our approach/goal

based on standard pQCD framework: hard scattering, factorization, universality of nPDFs and nFFs

\[ f_{i/p}(x, Q^2) \rightarrow f_{i/A}(x, Q^2) \]

vacuum/nucleon \rightarrow nuclei

\[ D_{i/p}^H(z, Q^2) \rightarrow D_{i/A}^H(z, Q^2) \]

find out how far we can push factorization & universality possible to reconcile R>1 in dAu and R<1 in eN ?

- no straightforward probe for nFFs - always entangled with PDF effects
- use nDS (EPS) nPDFs to isolate medium effects in the initial state
- experience from global fits: NLO corrections are essential
- an approximation, of course, with many limitations but worth a try
ansatz for \( D_{i/A} \)

choose a convolutional approach

\[
D_{i/A}^{H}(z, Q_{0}^{2}) \equiv \int_{z}^{1} dy \ W_{i}(y, A) \ D_{i/p}^{H}(\frac{z}{y}, Q_{0}^{2})
\]

"weight fct" to be fitted from DSS global analysis

simple examples:

\[
W_{i}(y, A) = \delta(1-y) \quad \text{no medium effect}
\]

\[
W_{i}(y, A) = \delta(1-\epsilon_{i}-y) \quad \text{shift in } z \text{ ("energy loss")}
\]

\[
W_{i}(y, A) = n_{i} \ y^{\alpha_{i}} (1-y)^{\beta_{i}} \quad \text{shape (enhancement/suppression)}
\]

assume smooth \( A \) dependence of coefficients \( \epsilon_{i}, n_{i}, \alpha_{i}, \text{and} \beta_{i} \)

\[
e.g. \quad n_{i} = \lambda_{n_{i}} + \gamma_{n_{i}} A^{\delta_{n_{i}}}
\]
how well do we know nPDFs?

3 fits available at NLO: nDS, HKN, EPS

- comparison by EPS

• below $x \approx 0.01$ based only on extrapolation
• DIS fixed target data do not constrain gluon → large uncertainties

a high-energy EIC can contribute significantly to nPDF and nFF analyses

no DIS data below $x \approx 0.01$
some prel. results: pion production

optimized weight function for nFF analysis is a combination:

$$W_i(y, A) = n_i \delta (1 - \epsilon_i - y) + n_i' y^{\alpha_i} (1 - y)^{\beta_i}$$

with 2 different functions: one for quarks (=antiquarks), one for gluons

$\rightarrow \sim 30$ parameters for general $A$ dependence $n_i = \lambda_n + \gamma_n A^{\delta_n}$

A dependence can be simplified further:

it should vanish as $A \rightarrow 1$ and often $\delta_n = 1$ works very well

17 free parameters

$\chi^2 = 350.5$ for 368 data points ✔

[works for kaons as well but data only weakly constrain parameters]
comparison with HERMES data

\[ R(x, z, Q^2) = \left( \frac{N_{SIDIS}}{N_{DIS}} \right)_A \left( \frac{N'_{SIDIS}}{N'_{DIS}} \right)_D \]

- **medium effect in initial-state only:**
  nDS PDF & DSS FF
  (dashed lines)
  does not work at all

- **medium effect in initial & final-state:**
  nDS PDF & modified FF
  (solid lines)
  works very well
available dAu pion data from RHIC

PHENIX $\pi^0$

- medium-effect is fairly small
- well reproduced by new nFFs (much better than with std. FFs)
- data constrain mainly gluon fragmentation at large $z$ (as expected from pp fits)
... similar for STAR dAu $\pi^0,\pm$ data

$\pi^0$ data taken from PhD thesis of O. Grebenyuk, arXiv:0909.3006

for the time being, it works well - more data, LHC?
resulting pion fragmentation functions

\[ Q^2 = 10 \text{ GeV}^2 \]

suppression increasing with \( A \)

enhancement around \( z \approx 0.5 \) increasing with \( A \)
conclusions

- important progress in recent years
  NLO global analyses available, 1st estimates of uncertainties

- more data on the horizon
  SIDIS multiplicities, LHC, RHIC, \( e^+e^- \) precision data from BELLE

- theoretical issues
  treatment of charm & bottom fragmentation, medium effects, ...

- stay tuned for finalized analysis of medium modified FFs, updates of DSS, and FFs for other species (\( \eta, \Lambda \))