Some Perspectives on Hard-Scattering at an EIC

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- Comments and observations in some arenas where pQCD methods would meet an EIC:

I. Some comments on an EIC
II. From leading-twist “vacuum” pQCD to power corrections (some examples)
III. Time-development of jet structure
I. Comments on an EIC. (viz. 08 NSAC long-range plan.) My observations in red; open to correction.

1. Low-$x$ at high luminosity: evolution, shadowing and saturation; $F_L$. Especially high-luminosity for study of final states.

2. Polarization studies. Unprecedented window into evolution even at c.m. energy around 50.

3. Generalized parton distributions and elastic scattering. Difficult, but important opportunities given the how fundamental these are.
4. Diffraction studies. Study the “$A - Q^2$ plane” to disentangle effects we know are there: $A^{4/3}$, $A(A - 1)$ …

5. Propagation of scattered partons in nuclear medium. Complement AA; complete fixed-target.

6. Quark-hadron duality studies. Follow the histories of scattered quanta; confinement in action.
• Quite a list – at a sort of geometric mean of RHIC and CEBAF, but opening new windows.

• For a “staged” half-portion of energy: still get new coverage for all of these.

• The goal is qualitatively new insight by varying energy and luminosity, not simply an incremental improvement of an already-established understanding. The case, of course, needs to be made.
II. From leading-twist “vacuum” pQCD to power corrections (some examples)

How we use asymptotic freedom

– Infrared safety & asymptotic freedom:

\[ Q^2 \hat{\sigma}_{SD}(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n \left( \frac{Q^2}{\mu^2} \right) \alpha_s^n(\mu) + \mathcal{O}\left( \frac{1}{Q^p} \right) \]

\[ = \sum_n c_n(1) \alpha_s^n(Q) + \mathcal{O}\left( \frac{1}{Q^p} \right) \]

– e^+e^- total; jets: a sum over collinear rearrangements and soft emission organizes all long-time transitions, which must sum to \( \leq 1 \) by unitarity.
What we’re *really* looking at here (with local source $J$)

\[
\sigma[f] = \lim_{R \to \infty} \int d^4x e^{-i q \cdot y} \int d\hat{n} f(\hat{n}) \times \langle 0 | J(0) T[\hat{n}_i T_{0i}(x_0, R\hat{n}) J(y)] | 0 \rangle
\]

(Sveshnikov & Tkachov 95, Korchemsky, Oderda & GS 96, Bauer, Fleming, Lee & GS 08, Hofman & Maldacena 08)

With $T_{0i}$ the energy momentum tensor

“Weight” $f(\hat{n})$ introduces no new dimensional scale

Short-distance dominated if all $d^k f / d\hat{n}^k$ bounded

We have to ask only very “smooth” questions! (come back to this)
− Generalization: factorization

\[ Q^2 \sigma_{\text{phys}}(Q, m, f) = \omega_{\text{SD}}(Q/\mu, \alpha_s(\mu), f) \otimes f_{\text{LD}}(\mu, m) + \mathcal{O}\left(\frac{1}{Q^p}\right) \]

\[ \mu = \text{factorization scale}; \]
\[ m = \text{IR scale (} m \text{ may be perturbative)} \]

− “New physics” in \( \omega_{\text{SD}} \); \( f_{\text{LD}} \) “universal” – for a given target or observed particle

− Almost all collider applications. Enables us to compute the Energy-transfer-dependence in \( |\langle Q, \text{out}|A + B, \text{in}\rangle|^2 \).

− But again, requires a smooth weight for final states!
Resummation?

– Whenever there is factorization, there is evolution

\[ 0 = \mu \frac{d}{d\mu} \ln \sigma_{\text{phys}}(Q, m) \]

\[ \mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \omega}{d\mu} \]

– Wherever there is evolution there is resummation,

\[ \sigma_{\text{phys}}(Q, m) = \sigma_{\text{phys}}(q, m) \exp \left\{ \int_q^Q \frac{d\mu'}{\mu'} P(\alpha_s(\mu')) \right\} \]

– For example: \( \sigma_{\text{phys}} = \tilde{F}_2(Q^2, N) \), DIS moment.
- & then we know $\tilde{P}(N, \alpha_s) = \gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots$, and we get

$$\tilde{F}_2(N, \mu) = \tilde{F}_2(N, \mu_0) \exp \left[ -\frac{1}{2} \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'^2} \gamma(N, \alpha_s(\mu')) \right]$$

- and with $\alpha_s(\mu) = 4\pi/b_0 \ln(\mu^2/\Lambda^2_{\text{QCD}})$, this is

$$\tilde{F}_2(N, Q) = \tilde{F}_2 q/H(N, Q_0) \left( \frac{\ln(Q^2/\Lambda^2_{\text{QCD}})}{\ln(Q_0^2/\Lambda^2_{\text{QCD}})} \right)^{-2\gamma_N^{(1)}/b_0}$$
– It works pretty well. *Approximate scaling at moderate $x$, pronounced evolution for smaller $x$:*

![Graphs showing scaling and evolution of $F_2(x,Q^2)$ with $x$ and $Q^2$.](image-url)
With these methods can describe both particles and jets in pp at 200 GeV ...

Especially for the single-particle inclusive cross sections, the range of agreement was a surprise. A great impetus for polarization, AA, pA and eA studies. In ratios, at least we understand the denominator!
• What about those corrections?

\[ Q^2 \sigma_{\text{phys}}(Q, m, f) = \omega_{\text{SD}}(Q/\mu, \alpha_s(\mu), f) \otimes f_{\text{LD}}(\mu, m) + \mathcal{O}\left(\frac{1}{Q^p}\right) \]

• In general they are quite complicated, but for \( F_2(x, Q^2) \) DIS with \( Q^2 \gg \Lambda^2 \), they are a series of factorized expressions, corresponding to the scattering of multiple partons.

• If partons are spread out over the surface of the “target”, multiple scatterings are unlikely, but if the density is large:

\[ \frac{G(x, Q_s^2)}{\pi R_{\text{target}}^2 Q_s^2} \sim 1 \]

multiple interactions are the rule: saturation.

• These “higher-twist” corrections should match onto the up-turns in HERA structure function data \ldots saturation, CGC. The “Initial state”.

• Higher twist and the final state:

• **Bloom-Gilman duality**: Higher twist acts to ‘redistribute probability around a smooth extrapolation of leading twist.'
• And rescattering in nuclear matter:

\[ \text{\( y^*_i \rightarrow y^{-}_i \rightarrow y^{-}_j \rightarrow y^*_u \rightarrow 0^- \)} \]

\[ \text{\( y^*_T \rightarrow x_0 \rightarrow \bar{x}_0 \rightarrow y^*_L \rightarrow x_0 \rightarrow \bar{x}_0 \)} \]

\[ \text{\( y^*_i \rightarrow y^{-}_i \rightarrow y^{-}_{i-1} \rightarrow \bar{x}_i \rightarrow x_i \rightarrow y^*_i \)} \]

\[ \text{\( \sum \text{leading powers in the } A^{1/3}/Q^2 \text{-dependence (Qiu, Vitev).} \)} \]
And what about the requirement of “smooth” sum over states?

Example: diffractive final states: gaps with no particles?

For $Q^2 \ll W^2$ in DIS:

Breaking the top rung of target ladder does not require radiation; combining the broken rung with the projectile requires radiation, but only in a small range of rapidity. (Gribov-Regge models of $A$-dependence). Evolution with $Q$ from the top even in presence of gap. The pomeron as a process, not a particle.
III. Time-development picture of jet structure

- **Get started**: First – find a jet. Then assign an axis $\hat{n}_J$: by minimizing $\sum_i E_i \cos \theta(i, \hat{n}_J)$ for particles $i$ in jet $J$.

- **Thrust**:

  $$\tau \equiv (1 - T) \equiv \frac{1}{Q_J} \sum_{i \text{ in } N} pT_i e^{-|\eta_i|}$$

- $pT_i$, $\eta_i$ measured relative to jet axis (minimizes $1 - T$) (can be chosen jet-by-jet).

- For multijet final states, define $\eta_i$ relative to closest jet.
Three-way factorization $\Rightarrow$ CO/IR (Sudakov) resummation.

Two logarithmic integrals exponentiate:

$$\sigma(\nu) = \int_0^\infty d\tau \alpha e^{-\nu \tau} \frac{d\sigma}{d\tau} = e^{\frac{1}{2}E(\nu, Q)}$$

$$E(\nu, Q) = 2 \int_0^1 \frac{du}{u} \int \frac{uQ^2}{u^2Q^2} \frac{dp_T^2}{p_T^2} A(\alpha_s(p_T)) \left(e^{-u} - 1\right) + \ldots$$

Expansion in $\alpha_s(Q)$ finite at all orders. The “cusp” function $A(\alpha_s)$ depends on color representation of the parent parton, only.
Convolution with non-perturbative but universal “shape function,” $f_{\text{NP}}$

$$\frac{d\sigma}{d\tau} = \int d\xi f_{\text{NP}}(\xi) \frac{d\sigma}{d\tau}$$

$e^+e^-$: fit at $Q = M_Z \Rightarrow$ predictions for all $Q$, any (quark) jet.

How general? – the cusp function $A(\alpha_s)$ is universal and can even be studied at strong coupling in SYM . . . although its nonperturbative power corrections are purely “nonconformal”, i.e. depend essentially on the running of the QCD coupling. Which is good, not bad.
Shape function phenomenology for thrust at LEP.

Note the range in $Q$.
Can this be portable to jets in nuclear matter?

- Well, in

\[ E(\nu, Q) \sim 2 \int_0^{1/du} \frac{uQ^2}{u^2Q^2} \int_0^{dp_T^2} A(\alpha_s(p_T)) \left( e^{-u\nu(p_T/Q)} - 1 \right) \]

\( u \) is conjugate to \( 1/tQ \), with \( t \) the “formation time” for gluon emission. So in a sense, \( E \) “tells a series of stories”, of all possible emissions that take time \( t \):

\[ E(\nu, Q) = 2 \int_0^{\infty} dt \int_0^{1/t^2} \frac{dp_T^2}{Q/t} A(\alpha_s(p_T)) \left( e^{-u\nu(p_T/Q)} - 1 \right) \]

\[ + \frac{1}{2} B(\alpha_s(\sqrt{uQ})) \left( e^{-u(\nu/2)} - 1 \right) \]

- All these stories (like the power corrections) are additive in \( E(\nu, Q) \).
• In principle, an analysis of shapes in ep, pp, eA and pA for thrust or other cleverly-chosen event shapes could provide the transition between the vacuum cusp function $A$ and the quantum history of fast partons in a nuclear medium.

• The additive nature of the shape function, and its kinematic linkage with fragmentation functions for $z \to 1$ suggest that a duality-based analysis, given sufficient data.
• Conclusion

Every high energy accelerator has led to new discoveries in QCD.

Both those where QCD “piggybacked” on searches for extensions to the Standard Model (Tevatron, LEP, HERA (!)) and those built for the purpose (RHIC, CEBAF).

QCD is the origin of the mass of almost all the “bright matter” in the universe. The case is to be made that EIC has unmatched potential to elucidate how this comes about, and how gluonic and quark degrees of freedom conspire to form the coherent structures of nucleon polarization and even nuclear structure.