High Twist Effects in e+A Collisions

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Outline

- Why study high twist effects?
  
  To probe nonperturbative QCD dynamics beyond PDFs
  – beyond the probability distributions

- High twist effect and power correction:

  Calculable power corrections are in general “small”

- Observables – large high twist effects:

  Resummation to all powers

- Observables – leading twist does not contribute:

  Single spin asymmetry, transverse momentum broadening, …

- Summary and outlook
High twist matrix elements

- **Matrix elements of parton operators:**
  \[ \langle p_h, S_h | O(\psi, F^{+\alpha}, D^\mu) | p_h, S_h \rangle \]
  
  Twist = dimension of the operator – its spin

- **Parton distributions and helicity distributions:**

  Matrix elements of twist-2 operators:
  \[ \langle p_h, S_h | \overline{\psi} \gamma^\mu \psi | p_h, S_h \rangle, \quad \langle p_h, S_h | F^{+\alpha} F^{+\beta} | p_h, S_h \rangle (-g_{\alpha\beta}) \]
  
  Probability interpretation

- **Multi-parton correlation functions:**

  Matrix elements of high twist operators:
  \[ \langle p_h, S_h | \overline{\psi} \gamma^\mu F^{+\alpha} F^{+\beta} \psi | p_h, S_h \rangle (-g_{\alpha\beta}), \quad \langle p_h, S_h | \overline{\psi} \gamma^\mu F^{+\alpha} \psi | p_h, S_h \rangle (\epsilon_{\alpha\beta} S_h^\beta) \]
  
  NO simple probability interpretation!
  
  More interesting QCD dynamics!
Cross section and power corrections

- **QCD confinement:**
  Experiments measure hadrons and leptons, not partons!

- **Cross section with a large momentum transfer:**
  Power expansion:
  \[
  \sigma_h(Q, p_h, S_h) = \sum_n \sigma_h^{(n)}(p_h, S_h) \left(\frac{1}{Q}\right)^n
  \]

- **Factorization – connecting partons to hadrons:**
  \[
  \sigma_{\text{DIS}}(Q, M) \approx C_a^{(0)}(Q/\mu, \alpha_s(\mu)) \otimes f_{a/h}^{(2)} + \frac{1}{Q^2} C_b^{(2)}(Q/\mu, \alpha_s(\mu)) \otimes f_{b/h}^{(4)} + \ldots
  \]
  \[
  \sigma_{\text{DY}}^{h_1 h_2}(Q, M) \approx C_{ab}^{(0)}(Q/\mu, \alpha_s(\mu)) \otimes f_{a/h_1}^{(2)} \otimes f_{b/h_2}^{(2)}
  + \frac{1}{Q^2} C_{ab}^{(2)}(Q/\mu, \alpha_s(\mu)) \otimes f_{a/h_1}^{(2)} \otimes f_{b/h_2}^{(4)} + (h_1 \leftrightarrow h_2) + \frac{1}{Q^4} C_{h_1 h_2}^{(4)} + \ldots
  \]

  **Twist-n parton distribution/correlation:**
  \[
  f_{a/h}^{(n)}
  \]

  **High twist effects = power corrections**
“Enhance” the power corrections

- Calculable high twist effects are in general “small”:
  \[ \sigma_h^{\text{DIS}}(Q, M) \approx C_a^{(0)}(Q/\mu, \alpha_s(\mu)) \otimes f_{a/h}^{(2)} + \frac{1}{Q^2} C_b^{(2)}(Q/\mu, \alpha_s(\mu)) \otimes f_{b/h}^{(4)} + \ldots \]

  If the 1st power correction is large, immediate question is what is the size of the next power corrections

  High twist effects are small for fully inclusive cross section
  \[ \left( \frac{M_h}{Q} \right)^n \ll 1 \quad \text{when} \quad n > 0, \quad \text{and} \quad Q \gg M_h \]

- Observables – large power corrections – resummation:
  \[ \sum_n C^{(n)} (A^{1/3}/Q^2)^n, \quad \sum_n C^{(n)} (1/Q^2(1-x))^n, \quad \sum_n C^{(n)} (1/x^\alpha/Q^2)^n, \ldots \]

- Observables – leading power term vanishes:
  Single transverse spin asymmetry: \[ A_N \propto \sigma(S_T) - \sigma(-S_T) \]
  Transverse momentum broadening: \[ \Delta p_T^2 = \langle p_T^2 \rangle_{eA} - \langle p_T^2 \rangle_{eN} \]
EMC effect, Shadowing and Saturation:

- Saturation in $F_2 = R_{F_2}$ decreases until saturation in $F_2(D)$

- Need $x_B$ as small as $10^{-3}$ at $Q^2 = 2\text{GeV}^2$ to probe the saturation

Inclusive $e + A$ collisions
Negative gluon distribution at low $x$, $Q^2$?

- NLO global fitting leads to negative gluon distribution at low $x$ and $Q^2$

MRST, CTEQ PDF’s have the same features

Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

No!
Recombination prevents negative gluon

- Small-x gluons are not localized in a Lorentz contracted nucleon
  - Gluon recombination

- Recombination slows down $Q^2$-evolution
  - Prevents the distribution to be negative

Gribov, Levin, Ryskin, 83

Mueller, Qiu, 86, McLerran, Venugopalan, 94, …
Eskola, et al. 03
Hard probe at low $x$

- Hard probe – process with a large momentum transfer:
  \[ q^\mu \text{ with } Q = \sqrt{|q^2|} \gg \Lambda_{\text{QCD}} \]

- Size of a hard probe is very localized and much smaller than a typical hadron at rest:
  \[ \frac{1}{Q} \ll 2R \sim \text{fm} \]

- But, it might be larger than a Lorentz contracted hadron:
  \[ \frac{1}{Q} \sim \frac{1}{xp} \gg 2R \left( \frac{m}{p} \right) \text{ or equivalently } x \ll x_c \equiv \frac{1}{2mR} \sim 0.1 \]

If an active parton $x$ is small enough
the hard probe could cover several nucleons
in a Lorentz contracted large nucleus!
In target rest frame:

- Lifetime of the $q\bar{q}$ state:
  \[
  \Delta E_{q\bar{q}} \sim \nu - E_{q\bar{q}} \sim \frac{Q^2}{2\nu} \left[ 1 + \mathcal{O}\left( \frac{m_{q\bar{q}}^2}{Q^2} \right) \right]
  \]
  \[
  \Delta z_{q\bar{q}} \sim \frac{1}{\Delta E_{q\bar{q}}} \sim \frac{2\nu}{Q^2} = \frac{1}{mx_B}
  \]
  \[
  \Delta z_{q\bar{q}} \gg 2 \text{ fm, inter-nuclear distance, if } x_B \ll 0.1
  \]

- If $x_B \ll 0.1$, the $q$-$q\bar{q}$ state of the virtual photon can interact with whole hadron/nucleus coherently.

The conclusion is frame independent
Saturation: Radiation = Recombination

Estimate:

\[ 1 \approx \frac{8\pi \alpha_s}{3Q^2 R^2} xG(x, Q^2) \]

Saturation scale:

\[ Q_s^2(x) \approx \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda} \]

\( Q^2 \gg Q_s^2(x) \): Dilute regime (rapid growth: BFKL, DGLAP)

\( Q^2 \lesssim Q_s^2(x) \): Saturation: \( n \sim 1/\alpha_s \) (large but constant)

Proton is dilute enough

Use nuclear target!

✧ How to approach the saturation region?

✧ How to treat the saturation in QCD?
A-dependence to structure function

- Factorization – OPE:
  \[ F(x_B, Q^2) = \sum_f c_f^{(2)}(x_B/x, Q^2/\mu^2) \otimes \phi_f(x, \mu^2) + \frac{1}{Q^2} c_f^{(4)}(x_B/x, Q^2/\mu_F^2) \otimes \phi_f^{(4)}(x, \mu^2) + \ldots \]

- Leading twist contribution:
  - \( c_f^{(2)}, c_f^{(4)}, \ldots \) insensitive to long-distance hadron state
  - \( \phi_f(x, \mu^2) \rightarrow \phi_f^A(x, \mu^2) \) with all A-dependence given in \( \phi_f^A(x, \mu_0^2) \)

  Suppression in small-x region – Leading twist shadowing

- High twist or power corrections:
  - Calculate \( c_f^{(n)} \)'s corresponding to A-enhanced \( \phi_f^{(n)A}(x, \mu^2) \)
  - Resum all large power contributions if possible

  Both are important if power correction is not small!
Leading order $\alpha_s$ power correction

- **Leading power** $c_q^{(2)}$:
  \[
  \delta(x - x_B)
  \]

- **NLP** $c_q^{(4)}$:
  \[
  \frac{g^2}{Q^2} \left( \frac{1}{2N_c} \right) \left[ 2\pi^2 \tilde{F}^2(0) \right] x_B \lim_{x_1 \to x} \left[ \frac{1}{x - x_1} \delta(x_1 - x_B) + \frac{1}{x_1 - x} \delta(x - x_B) \right]
  \]

- **Nth power** $c_q^{(N=2n)}$:
  \[
  \int \frac{dy_2^- dy_1^-}{(2\pi)^2} \left[ F^{+\alpha}(y_2^-) F^{\alpha+}(y_1^-) \right] \theta(y_2^-) x_B \left[ -\frac{d}{dx} \delta(x - x_B) \right]
  \]

- Infrared safe!
Transverse structure function:

\[ F_T(x_B, Q^2) = \sum_{n=0}^{N} \frac{1}{n!} \left[ \frac{\xi^2}{Q^2} (A^{1/3} - 1) \right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B, Q^2) \]

\[ \approx F_T^{(0)}(x_B(1 + \Delta), Q^2) \]

\[ \Delta \equiv \frac{\xi^2}{Q^2} (A^{1/3} - 1) \]

\[ \xi^2 = \frac{3\pi \alpha_s}{8R^2} \langle F^{+\alpha}F_{\alpha}^+ \rangle \]

Single parameter for power correction

Longitudinal structure function:

\[ F_L^A(x, Q^2) = A F_L^{(LT)}(x, Q^2) + \sum_{n=0}^{N} \frac{A}{n!} \left( \frac{4\xi^2}{Q^2} \right)^n \left[ \frac{\xi^2 (A^{1/3} - 1)}{Q^2} \right]^n x^n \frac{d^n F_T^{(LT)}(x, Q^2)}{d^n x} \]

\[ \approx A F_L^{(LT)}(x, Q^2) + \frac{4\xi^2}{Q^2} F_T^A(x, Q^2) \]
Neglect LT shadowing (upper limit)

\[ \xi^2 \sim 0.09 - 0.12 \text{ GeV}^2 \]
Gross-Llewellyn Smith sum rule

\[ S_{GLS} = \int_0^1 dx \frac{1}{2x} \left( x F_{3}^{\nu N}(x, Q^2) + x F_{3}^{\bar{\nu} N}(x, Q^2) \right) \]
\[ \approx \# U + \# D = 3 \]


\[ \Delta_{GLS} = \frac{1}{3} \left( 3 - S_{GLS} \right) = \frac{\alpha_s(Q)}{\pi} + \frac{\kappa}{Q^2} + O\left( \frac{1}{Q^4} \right) \]

Fully coherent final-state power corrections to the sum rule almost cancel due to the unitarity:

\[ \int_{-\infty}^{+\infty} dx \varphi(x + \Delta x) = \int_{-\infty}^{+\infty} dx \varphi(x) \]

But, nuclear enhanced power corrections only for a limited values of \( x \in (0, 0.1) \)


Prediction is compatible with the trend in the current data
Power corrections to SIDIS

- **SIDIS:**

- **Low $P_{hT}$ – TMD factorization:**

  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q) \otimes \Phi_f \otimes D_{f \rightarrow h} \otimes S + \mathcal{O}\left(\frac{P_{h\perp}}{Q}\right)
  \]

- **High $P_{hT}$ – Collinear factorization:**

  \[
  \sigma_{\text{SIDIS}}(Q, P_{h\perp}, x_B, z_h) = \hat{H}(Q, \alpha_s) \otimes \phi_f \otimes D_{f \rightarrow h} + \mathcal{O}\left(\frac{1}{P_{h\perp}}, \frac{1}{Q}\right)
  \]

- **Multiple-scattering in eA:**
Suppression of production rate at large $z_h$

- Sum over all possible number of scatterings: 

\[
\frac{dW_{\mu\nu}}{dz} \approx \frac{1}{2} e_{\mu\nu}^T \sum_f Q_f^2 A\phi_f(x, Q^2) \sum_{n=0}^N \frac{1}{n!} \left[ \frac{z\kappa^2(A^{1/3} - 1)}{Q^2} \right]^n \frac{d^n D_f(z, Q^2)}{d^n z} 
\]

\[
\approx A \frac{1}{2} e_{\mu\nu}^T \sum_f Q_f^2 \phi_f(x, Q^2) D_f \left( z + \frac{z\kappa^2(A^{1/3} - 1)}{Q^2}, Q^2 \right),
\]

Net effect of coherent multiple scattering of a propagating quark (not a pre-hadron state) is equivalent to a shift of $z$ in the fragmentation function – “energy loss”

- The amount of the shift:

\[
\Delta z = z \frac{\kappa^2(A^{1/3} - 1)}{Q^2} 
\]

\[
\kappa^2 = \frac{3\pi\alpha_s(Q^2)}{4\gamma_0^2} \langle p | \hat{F}^2(y_i) | p \rangle
\]

Depends on the same universal matrix element responsible for the leading contribution to nuclear shadowing of inclusive DIS

Qiu, Vitev, 04

Guo & Li, 2006
Life time of the virtual parton state

- Fragmentation is not necessary taken place outside the medium:

- Interaction between the color neutral pre-hadron state and the colored partons are strongly suppressed – neglected in this calculation.

- Probability of coherent multiple scattering is proportional to the lifetime of the virtual parton state.

\[ \Delta z \propto (1 - z) \]
Comparison with HERMES data

Ratio of multiplicity with target A and target D: $R_A^A/R_D^D$

- $\Delta z^* = \Delta z (1 - z)/(1 - z_c)$ with fitted $z_c = 0.6$

- Complement to the effect from medium modification of fragmentation function

Guo & Li, 2006
Radiation induced by multiple scattering

Short-distance hard scattering

\[ p_c^2 \ll O(Q^2) \]

First scattering determines the parton production rate

\[ \ln\left(\frac{Q^2}{p_c^2}\right) \]

Induced radiation modifies the fragmentation

Modifies DGLAP evolution of the Fragmentation function – “energy loss”

Guo & Wang PRL 2000, …
Wang & Wang, PRL 2002, …
Energy loss in cold nuclear matter

Radiation energy loss:

\[ \Delta E = \nu \langle \Delta z \rangle = \left( E - E' \right) \langle \Delta z \rangle \]

\( \langle \Delta z \rangle \) - radiative energy loss fraction

\[
x_B = \frac{Q^2}{2 p \cdot q}, \quad x_A = \frac{1}{m_N R_A}
\]

\[
\langle \Delta z \rangle = \tilde{C}(Q^2) \frac{C_A \alpha_s^2(Q^2)}{N_c Q^2 x_A^2} \ln \frac{1}{2 x_B}
\]

\( \langle -dE/dL \rangle_{\text{cold}} \approx 0.5 - 0.6 \text{ GeV/fm} \)

\( \propto L^2 \)


Transverse momentum broadening

- Low $p_T$ distribution is ill-defined in the fixed order calculation
  - All order resummation (CSS formalism)
- Multiple scattering in medium:
  - Each scattering is too soft to calculate perturbatively
  - Resummation + multiple scattering (not yet achieved)
- Moment of $p_T$-distribution is perturbative calculable:
  \[
  \langle p_T^2(Q^2) \rangle = \int dp_T^2 \frac{d\sigma}{dQ^2 dp_T^2} \bigg/ \int dp_T^2 \frac{d\sigma}{dQ^2 dp_T^2}
  \]
  - observed particles only
- Momentum broadening:
  \[
  \Delta \langle p_T^2(Q^2) \rangle = \langle p_T^2(Q^2) \rangle_{eA} - \langle p_T^2(Q^2) \rangle_{ep}
  \]
  - Easy to measure, sensitive to the medium properties
  - Perturbatively calculable

Luo, Qiu, Sterman, 1994
$\Delta p_T^2 = \langle p_T^2 \rangle_A - \langle p_T^2 \rangle_D$
High twist effect is a very important part of QCD dynamics – appears in every hadronic observables

Provides rich information of nonperturbative QCD beyond PDFs – probability distributions

Necessary for understanding the transition region between a dilute parton system to a saturated glass state

High twist effects are in general smaller than the leading twist contribution – need to look for

Thank you!
Backup transparencies