Boosting the saturation scale in nuclei

Boris Kopeliovich
UTFSM Valparaiso

In collaboration with Ivan Schmidt and Irina Potashnikova
Measuring the saturation scale

In the nuclear rest frame saturation looks like color filtering for a dipole ($\bar{q}q$, or $gg$) of transverse separation $r_T$ and energy $E$ propagating through a nuclear medium. The partial elastic dipole-nucleus amplitude at impact parameter $b$ reads,

$$f^{A\, dip}_d(b) = 1 - e^{-\frac{1}{2} \sigma^{N}_{dip}(r_T,E)} T_A(b) = 1 - e^{-\frac{1}{4} r_T^2} Q^2_A(b,E)$$

Calculation of $Q^2_A$ from the first principles looks pretty hopeless, but one can get it from phenomenology.

A parton propagating through the nucleus experiences broadening of transverse momentum, which turns out to be exactly the saturation scale

$$\Delta p^2_T = Q^2_A$$

Dolejsi, Hüfner, B.K. (1993)
Johnson, B.K., Tarasov (2000)
Measuring the saturation scale

One can predict broadening fitting the dipole cross section to photoproduction and DIS at small $Q^2$ (no fit to broadening!)

$$Q_A^2 = 2 C(E) T_A = \frac{1}{4} \sigma_{\text{tot}}^{\pi p}(E) \left[ Q_N^2(E) + \frac{3}{2} \langle r_{ch}^2 \rangle_{\pi} \right] T_A$$

$$Q_N(E) = 0.19 \text{ GeV} \times (E/\text{GeV})^{0.14}$$
Measuring the saturation scale

Broadening has been measured in:

- $pA$ (Drell-Yan; E772&E866)
  \[ \Delta (p_{T}^{\bar{u}u})^2 = z_{\bar{u}u}^2 \Delta p_{T}^2 \]  
  (quark broadening)
  \[ \langle z_{\bar{u}u} \rangle \approx 0.9 \]

- $J/\Psi$ and $\Upsilon$ in $pA$ (E772)
  \[ \Delta (p_{T}^{\bar{u}u})^2 \approx \Delta p_{T}^2 \times 9/4 \]  
  (gluon broadening)

- $eA$ (SIDIS; HERMES&CLAS)
  \[ \Delta (p_{T}^{h})^2 = z_{h}^2 \Delta p_{T}^2 \]  
  (quark broadening)
Measuring the saturation scale

Broadening in SIDIS originates mainly from the first stage of hadronization, before the leading quark color is neutralized. This brings a significant model dependence.

At higher energies of EIC a (pre)hadron is produced outside the nucleus and measurement of $Q_A$ becomes more certain. However, the region of small $x$ is dominated by dijet production. This makes the value of $z_h$ model dependent.

One does not need to know $z_h$ provided that the whole jet is reconstructed.

S.Domdey, D.Grünewald, B.K., H.J.Pirner, 2009
Measuring the saturation scale at small $x$

Experimentally known is $z_h = \frac{p_h^+}{q_{\gamma^*}}$

$$
\Delta (p_T^h)^2 = \frac{z_h^2 \Delta p_T^2}{\int d^2 r_T \int_0^1 d\alpha |\Psi_{\gamma^*}(r_T, \alpha)|^2 \sigma_{dip}(r_T, x)}
$$

$$
\times \int d^2 r_T \left\{ \int_{1-z_h}^{z_h} \frac{d\alpha}{\alpha^2} |\Psi_{\gamma^*}(r_T, \alpha, Q^2)|^2 \sigma_{dip}(r_T, x) D_{h/q} \left( \frac{z_h}{\alpha}, Q^2 \right) \\
+ \int_{0}^{1-z_h} \frac{d\alpha}{(1-\alpha)^2} |\Psi_{\gamma^*}(r_T, \alpha, Q^2)|^2 \sigma_{dip}(r_T, x) D_{h/q} \left( \frac{z_h}{1-\alpha}, Q^2 \right) \right\}
$$

$\Psi_{\gamma^*}$ is known from pQCD; $\sigma_{dip}$, $D_{h/q}$ from phenomenology.
Proton modification in pA

Due to broadening the nuclear target probes the parton distribution in the beam hadron with a higher resolution, so in a hard reaction the effective scale $Q^2$ for the beam PDF drifts to a higher value $Q^2 + Q_A^2$.

The projectile gluon distribution is suppressed at large $x \rightarrow 1$, but enhanced at small $x$. This breaks-down $k_T$-factorization, but is a higher twist effect (at fixed $x_1$).
Hadron production at forward rapidities is dominated by fragmentation of the projectile valence quarks. Since the energy is very high, at RHIC $\sim 10^4$ GeV, the saturation scale is rather large even for quarks, $Q_A^2 = 1.2$ GeV$^2$. This causes a significant nuclear suppression of the projectile valence quarks at large $x$.

This effect is quite relevant to the suppression pions at forward rapidities observed by BRAHMS. This is not the whole story, but an essential part of it.
Proton modification in pA

There is an asymmetry in the properties of colliding nucleons in pA collisions:

- The PDF of the beam proton at small $x$ is modified to a state with the multiplicity of constituents higher than in $NN$ collisions, while the properties of the target bound nucleons remain undisturbed.

The nuclear saturation scale, $Q_A^2 = 2C(E)T_A(b)$, where factor $C(E) = \frac{\partial \sigma_{dip}}{\partial r_T^2} \bigg|_{r_T=0}$ is related to the dipole cross section fitted to DIS on a free proton target.
In nuclear collisions the PDFs of bound nucleons in both nuclei are drifting towards a higher scales.

This in turn enhances broadening compared to $pA$, since the properties of the target nucleons change.

\[ \sigma_{\text{dip}}(\bullet) > \sigma_{\text{dip}}(\bigcirc) \]

Therefore, the broadening coefficient increases:

\[ C(E) \Rightarrow \tilde{C}(E, Q_A^2) \]
Reciprocity of saturation scales

As far as the properties of bound nucleons in nuclear collisions are modified compared to $NN$ collision, the saturations scales in the colliding nuclei should be revised. The relation $Q_A^2 = \frac{9}{2} C(E) T_A$ relevant to gluon saturation scale in $pA$, in the case of collision of two nuclei $A$ and $B$ is replaced by the system of reciprocity equations,

\[
\tilde{Q}_B^2(x_B) = \frac{9}{2} \tilde{C}(E_A^g, \tilde{Q}_A^2) T_B
\]

\[
\tilde{Q}_A^2(x_A) = \frac{9}{2} \tilde{C}(E_B^g, \tilde{Q}_B^2) T_A
\]

Here $E_{A,B}^g = s x_{A,B} / 2m_N$, where $x_{A,B}$ are the fractional light-cone momenta of the radiated gluon relative to the colliding nuclei, $x_A x_B = k_T^2 / s$. 

\[
\frac{9}{2} \tilde{C}(E_A^g, \tilde{Q}_A^2) T_B
\]

\[
\frac{9}{2} \tilde{C}(E_B^g, \tilde{Q}_B^2) T_A
\]
Reciprocity of saturation scales

To evaluate the magnitude of boosting the saturation scales we solve the equations for central collision of identical nuclei

$$\tilde{Q}_A^2(x_A) = \frac{9}{2} \tilde{C}(\sqrt{s}\langle k_T \rangle/2m_N, \tilde{Q}_A^2) T_A$$

relying on the small $r_T$ form of the dipole cross section

$$C(E, Q^2) = \frac{\pi^2}{3} \alpha_s(Q^2) x g(x, Q^2).$$

with $x = 2E/\sqrt{s}$.

In order to reproduce correctly the soft limit of $pA$ collision we replace $Q^2 \Rightarrow Q^2 + Q_0^2$, so that $C(E, Q_0^2) = C(E)$. The value of $Q_0$ depends on the PDF parametrization. For MSTW2008 $Q_0^2 = 1.7$ GeV$^2$. 
Saturation scale in AA vs e(p)A

The saturation scale for gluons in central collisions of heavy nuclei is quite large even at RHIC. At the energy of LHC it may reach very high values $\tilde{Q}_A^2 \sim 10 \text{ GeV}^2$.

Compared to the saturation scale which has been and can be measured in $e(p)A$ collisions, the saturation scale in $AA$ collisions is boosted to significantly higher values. It increases by about 50% at the energies of RHIC and up to factor three at the energy of LHC.
Measurements of the saturation scale for quarks and gluons performed in \( eA \) and \( pA \) collisions well agree with the phenomenological predictions for broadening.
Summary

- Measurements of the saturation scale for quarks and gluons performed in $eA$ and $pA$ collisions well agree with the phenomenological predictions for broadening.
- Multiple interactions in the nuclear target significantly modify the PDF of the projectile proton in $pA$ collisions. They lead to a suppression of the PDF at large and enhancement at small $x$. At the same time, the properties of the target nucleons remain unchanged.
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• Mutual multiple interactions in nuclear collisions affect the properties of both nuclei. They boost their saturation scales up to values significantly higher than what is known for $e(p)A$ collisions. The effect is described by a system of reciprocity equations, which should be solved numerically.
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- This is a trigger driven effect!