Small-$x$ physics and QCD reggeon field theory

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• Introduction
• Theoretical issues
• Applications: DIS at small $x$, other items
• Outlook
Why this (theoretical) talk?

In doing high energy QCD calculations we are using different formalisms, e.g.

- Feynman diagrams in momentum space (configuration space)
- light cone perturbation theory
- dipole picture

Each formalism useful in its field of application, equivalence and connections mostly (but not fully: reggeization) understood.

QCD reggeon field (RFT) theory results from the first approach in momentum space, has direct application to cross sections.
Expect: RFT is a reformulation of QCD at high energies (Regge limit), i.e. high energy behavior of QCD scattering amplitudes can be cast into reggeon diagrams.

But: when doing a calculation, one may have to perform resummations (bootstrap, effective action).

This talk: discuss a few examples where this formulation is particularly useful.
Reggeon field theory: theoretical remarks

QCD reggeon field theory (Gribov):
Concept: reformulation of high energy QCD in terms of reggeized gluons (not: Pomeron fields) and interaction vertices = (nonlocal) field theory in 2+1 dimensions:

\((\vec{k}_t, E = 1 - j = -\omega) \leftrightarrow (\vec{b}, y = \ln s)\)

Natural/historical starting point are momentum space Feynman diagrams.
Bound state problem:

$n$ gluon state has rich spectrum of 'bound states' (BFKL, BKP), depends upon (color) quantum numbers.

Gluon plays very special role because of 'bootstrap': 'diagrams can be drawn in different ways'

\[ \sum_{g_A} \quad = \quad \text{reggeized gluon} = \text{bound state of two reggeized gluons.} \]

Holds in LO, NLO (and beyond?)
In pQCD: all elements can be computed in perturbation theory. How much has been done:

- gluon trajectory in NNLO (all orders from AdS/CFT)
- $2 \to 2$ interaction = BFKL kernel in NLO
- $n \to n$ interaction = BKP evolution
- $2 \to 4$ in NLO
- $2 \to 6$ in LO
- several impact factors (few in NLO)

What has been computed:

- spectrum of BFKL
- bound state of three gluons: Odderon
- bound states of $n$ gluons at large $N_c$: integrability
Physical picture: $s$-channel vs. $t$ channel

Reggeon field theory starts from $t$-channel:
'reggeon unitarity' (Gribov, Pomeranchuk), model independent

Main concept: $t$-channel states (symmetries).

In QCD: reggeon diagrams are derived from (momentum space) Feynman diagrams which can also be viewed from the $s$-channel and/or in configuration space: 'space-time picture'.
All contributions (reggeon diagrams) are frame independent. For physical picture: fix the Lorentz frame (role of time).

Two examples:
1) Convenient in DIS: target rest frame.
Use non-covariant (old-fashioned) perturbation theory: (Gribov; Weinberg)
Fourier transform and integration over energies → time-ordered diagrams:

Only one sequence in $t, z$, dominates:

$$t_1 < t_2 < \ldots < t_n < t'_n < t' < \ldots t'_2 < t'_1$$
2) Light cone picture (Balitsky):

\[ p_1 = \left( \frac{\sqrt{s}}{2}, 0, 0, \frac{\sqrt{s}}{2} \right), \quad p_2 = \left( \frac{\sqrt{s}}{2}, 0, 0, -\frac{\sqrt{s}}{2} \right) : \quad p_A = p_1 + \frac{p_A^2}{s} p_2 + p_{A\perp}, \quad p_B = p_2 + \frac{p_B^2}{s} p_1 + p_{B\perp} \]

Regge limit for

\[ < O(x_A) O(x_B) O(x_{A'}) O(x_{B'}) > : \]

\[ x_{A+}, x_{B-} \to -\infty \]
\[ x_{A' +}, x_{B' -} \to +\infty \]

Result: choose picture suitable for the problem.
AGK cutting rules (Abramovsky, Gribov, Kancheli)

Underlying question: decompose a reggeon diagram into intermediate states (unitarity) → cutting rules:

\[ \text{disc } T_{2 \to 2} = \sum \int T_{2 \to n} T_{2 \to n}^* \]

These rules were derived before the advent of QCD. They do not only apply to the Pomeron:

\[ \text{diffractive : absorptive : double } = 1: (-4): 2 \]
Also: cuts across a reggeized gluon, relevant for diffraction in DIS (see below)

\[ \frac{\text{diffRACTiVe}}{\text{absorPTiVe}} : \text{double} = 1 : O(g^4) : O(g^4) \]

Inclusive cross section: rescattering cancels (factorization theorem)
Application: DIS at small $x$

A. Total cross section (all inclusive):

DIS on proton and nuclei:

Large-$N_c$ limit: fan diagrams, $2 \rightarrow 4$ gluon vertex $\rightarrow$ BK-kernel, BK equation, saturation,.. Connection between different languages well understood.
Define states at 'time' rapidity $x$:

$$|p(y)\rangle = e^{yH}|p(y = 0)\rangle, \quad H = H_{2\rightarrow2} + H_{2\rightarrow4} + H_{4\rightarrow2} + \ldots$$

$n$-gluon wave functions $\Psi_n = < n | p(y) \rangle$ satisfies set of coupled equations (JIMWLK):

$$\frac{\partial}{\partial y} \Psi_n = \sum_{n'} < n | H | n' \rangle \otimes \Psi_{n'}$$

Mean field approximation: $\Psi_4 = \Psi_2 \Psi_2$, obtain closed BK equation

$$\frac{\partial}{\partial y} \Psi_2 = < 2 | H | 2 \rangle \otimes \Psi_2 + < 2 | H_{4\rightarrow2} | 4 \rangle \otimes \Psi_2 \otimes \Psi_2$$

Expect: near saturation region need to include 'Pomeron loops': include $H_{4\rightarrow2}$ and $H_{2\rightarrow4} \rightarrow$ infinite set of equations.
Fan diagrams vs. higher twist:
at large $N_c$, triple Pomeron vertex decouples for all higher twist (JB, Wuesthoff, Kutak)

Consequence:
description in terms of fan diagrams has no twist expansion
(decoupling of mixing of non-quasipartonic operators).
B. Diffraction

Maybe not obvious: fan diagrams contain elastic rescattering.

Bootstrap identities allow to rewrite diagrams.

= + + + ...
Consequences for diffraction in DIS: be careful with AGK counting.
What we might want to do:

\[ 1 : (-4) : 2 \]

diffractive cross section

full screening corrections

But: 'diffractive cross section' contains NLO contribution to BFKL (some models not correct):

\[ 1 \]

\[ 8 \]
C. Inclusive jet cross section

First step beyond the total cross section: one-jet inclusive jet cross section. (JB, Salvadore, Vacca) Result:

Consequence: evolution equations below the jet involve higher correlators ($3$ gluon,...), i.e. they are no longer of the BK type.
Potential disagreement with other calculations (Kovchegov, Tuchin, Braun, Kovner et al, Blaizot et al, Levin et al,...).
Double inclusive cross section: signal important for saturation!
D. Baryons (JB, Motyka):

Beyond color dipoles: replace, in DIS, photon by a baryonic current (no large $N_c$):

Extra color configuration, evolution the same as for the QCD Odderon. Can decay into two dipoles.

'Heavy baryon' states as a new testing laboratory?
E. Inclusive jets in nucleus-nucleus scattering

'Braun equation’, inclusive cross section: (Braun)

Attempts to solve 'Braun equation’ (Motyka, Bondarenko).
1-jet inclusive: is the evolution more complicated?
F. Multiple interaction, underlying event:

Theoretical understanding unsatisfactory, reggeon diagrams could be helpful.
G. Survival probability in diffractive Higgs production

Attractive channel for Higgs production at the LHC: double diffractive final states

Hard rescattering: large corrections (JB, Motyka; Miller)

Needs complete summation.
Conclusion

Results:

- scattering amplitudes, cross sections can be formulated in terms of reggeon diagrams (leading log, generalized leading log, NLO, NNLO, Lipatov’s effective action)
- in general, comparison of different approaches useful
- examples where RFT seems particularly useful

To be addressed:

- diffraction in DIS, single and double inclusive cross section in $eA$
- nucleus-nucleus scattering
- multiple interactions at LHC
- survival probabilities (diffractive Higgs production)

Theoretical task: find RFT solutions beyond BK equation (near saturation region)