

Static Correlators in $SU(2)$ gauge theory

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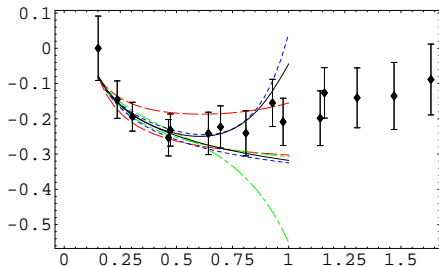
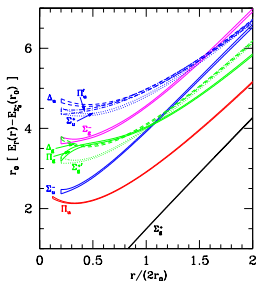
Introduction

- ▶ Study of quarkonium properties in the deconfined phase through meson correlators in Euclidean time is difficult, in particular due to the interplay between the bound states and scattering states in quarkonium spectral functions
- ▶ It is simpler to consider static mesons, where this problem is absent and the size of the system (static meson) can be controlled by hand
- ▶ Quark and anti-quark could be in color singlet or adjoint (octet/triplet) state and the interactions in these two channels are different, i.e. attractive and repulsive respectively.
- ▶ I will discuss the non-perturbative study of static quark anti-quark interaction in $SU(2)$ gauge theory using lattice

A. Bazavov, P. Petreczky, A.V.; Phys.Rev.D78:114026,2008

Review: A. Bazavov, P. Petreczky, A.V.; 0904.1748

Static quark anti-quark pairs at $T = 0$



Hybrids energy levels. $r_0[(V_{o,RS} - V_{s,RS})(r) + \Lambda_B^{RS}]$ (versus r/r_0) in RS scheme.

The excited energy levels (hybrids) can be related to the octet potential at short distances (multipole expansion): $E_H(r) = V_o(r) + \Lambda_H$.¹

¹C. Michael, hep-lat 9809211; Bali, Pineda; Phys.Rev.D69:094001,2004; N. Brambilla, A. Pineda, J. Soto, A. Vairo, NPB 566, 275 (2000)

Static quark correlators

Static meson operators:

$$O(x, y; t) = \bar{\psi}(x, t)U(x, y; t)\psi(y, t), \quad (1)$$

$$O^a(x, y; t) = \bar{\psi}(x, t)U(x, x_0; t)T^aU(x_0, y; t)\psi(y, t). \quad (2)$$

Correlators of these operators at $t = 1/T$ (integrating out the static fields):

$$G_1(r, T) = \frac{1}{2} \langle O(x, y; 0) \bar{O}(x, y; 1/T) \rangle \quad (3)$$

$$= \frac{1}{2} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle, \quad (4)$$

$$G_a(r, T) = \frac{1}{3} \sum_{a=1}^3 \langle O^a(x, y; 0) \bar{O}^a(x, y; 1/T) \rangle \quad (5)$$

$$= \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) U(x, y; 0) L(y) U^\dagger(x, y, 1/T) \rangle.$$

Wilson line $L(\vec{x}) = \prod_{t=0}^{N_\tau-1} U(\vec{x}, t), 0$.

Static quark correlators

These correlators depend on the choice of the spatial transporters:

- ▶ in the special gauge, where $U(x, y, z; t) = 1$ give "standard" definition of the singlet and triplet free energies

$$\exp(-F_1(r, T)/T) = \frac{1}{2} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \quad (6)$$

$$\begin{aligned} \exp(-F_3(r, T)/T) &= \frac{1}{3} \langle \text{Tr} L^\dagger(x) \text{Tr} L(y) \rangle - \frac{1}{6} \langle \text{Tr} L^\dagger(x) L(y) \rangle, \\ r &= |x - y|. \end{aligned}$$

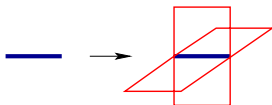
- ▶ in Coulomb gauge one can simply define

$$O(x, y; t) = \bar{\psi}(x, t) \psi(y, t), \quad (7)$$

$$O^a(x, y; t) = \bar{\psi}(x, t) T^a \psi(y, t). \quad (8)$$

Static quark correlators: APE smearing

We consider APE smeared links for the spatial transporters $U(x, y; t)$,
APE smearing



$$U_{x,\mu} \rightarrow U'_{x,\mu} = (1 - 6c)U_{x,\mu} + c \sum_{v \neq \mu} U_{x,\mu} U_{x+\hat{v},\mu} U_{x+\hat{\mu},v}^\dagger. \quad (9)$$

- ▶ Without the smearing it is not possible to get a useful signal for Wilson loops even at $T = 0$.
- ▶ The UV fluctuations of the U-fields - lattice perturbation theory².

²Lepage and Mackenzie, Phys.Rev.D48:2250-2264,1993.

Static meson correlators at finite T

The physical free energy of a static quark anti-quark pair is given by the thermal average of the singlet and triplet free energy

$$\begin{aligned} \exp(-F(r, T)/T) &= \\ \frac{1}{4} \exp(-F_1(r, T)/T) + \frac{3}{4} \exp(-F_3(r, T)/T) & \\ = \frac{1}{4} \langle \text{Tr}L(r) \text{Tr}L(0) \rangle. & \end{aligned} \quad (10)$$

At high temperature in the leading order HTL approximation the singlet and triplet free energies are

$$F_1(r, T) = -\frac{3}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (11)$$

$$F_3(r, T) = +\frac{1}{4} \frac{\alpha_s}{r} \exp(-m_D r) - \frac{3}{4} \alpha_s m_D \quad (12)$$

$$F(r, T) = -\frac{3}{32} \frac{\alpha_s^2}{r^2 T} \exp(-2m_D r) - \frac{3}{4} \alpha_s m_D \quad (13)$$

Static meson correlators at finite T

One can formally take the limit $rm_D \ll 1$:

$$F_1(r, T) \approx -\frac{3}{4} \frac{\alpha_s}{r} \quad (14)$$

$$F_3(r, T) \approx \frac{1}{4} \frac{\alpha_s}{r} - \alpha_s m_D \quad (15)$$

$$(16)$$

Static quark-antiquark correlators

Using the transfer matrix formalism one can show that³

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r, T)/T}, \quad (17)$$

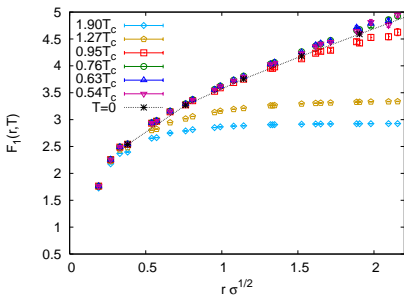
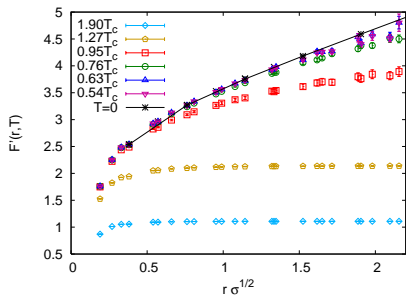
$$G(r, T) = \langle \text{Tr}L(r)\text{Tr}L(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n(r, T)/T}, \quad (18)$$

where E_n are the energy levels of static quark and anti-quark pair. The coefficients $c_n(r)$ depend on the choice of the transporters U .

If $c_1 = 1$ the dominant contribution to G_3 would be the 1st excited state E_2 , thus justifying the name singlet and triplet free energy.

³Jahn and Philipsen, Phys.Rev.D70:074504,2004

Color averaged and singlet free energies at $\beta = 2.5$



When $r \ll 1/T$ the color singlet and color averaged free energy are related $F(r, T) = F_1(r, T) + T \ln 4$.

The T -dependence for $F_1(r, T)$ and $F(r, T)$ is quite different at high T , $F(r, T)$ has a very strong T -dependence even at very small distances.⁴

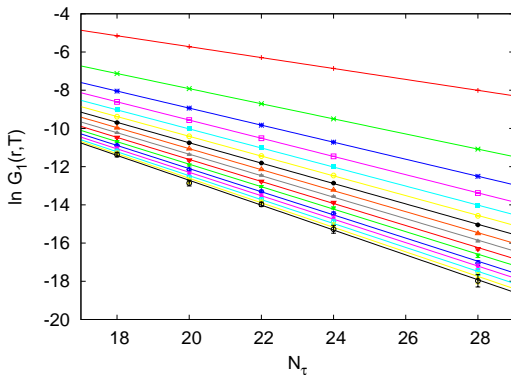
⁴Lüscher-Weisz; $F'(r, T) = F(r, T) - T \ln 4$

Extracting the overlap factor c_1

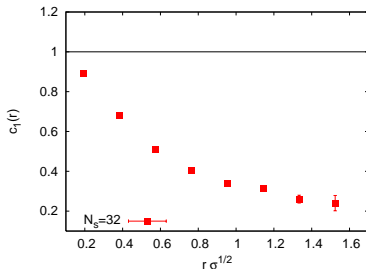
Truncate and fit at fixed r :

$$G_1(r, T) = \sum_{n=1}^{\infty} c_n(r) e^{-E_n(r, T)/T}$$

$$G_1(r, T) = c_1(r) e^{-a(r) N_\tau}$$



Coefficient $c_1(r)$ at $\beta = 2.5$



In perturbation theory $c_1 = 1$ up to $\mathcal{O}(\alpha_s^3)$ corrections ⁵

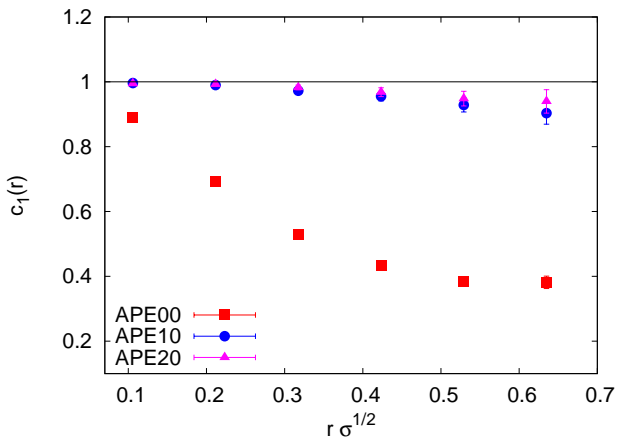
$$c_1(r) \equiv (1 + \delta Z_s(r)), \quad \delta Z_s(r) = \frac{C_F C_A^2}{2} \frac{\alpha_s^3}{\pi} \ln r \mu,$$

where $C_{F/A}$ is the Casimir of the fundamental/adjoint representation.

Also at small distances, $c_2(r) \sim (r \Lambda_{QCD})^4$.

⁵N. Brambilla, A. Pineda, J. Soto, A. Vairo, NPB 566, 275 (2000)

Coefficient c_1 at $\beta = 2.7$



Singlet free energy

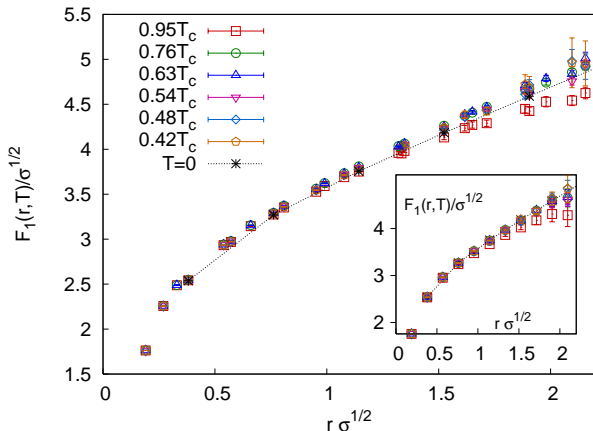


Figure: The color singlet free energy in $SU(2)$ gauge theory below the deconfinement temperature at $\beta = 2.5$ calculated on $32^3 \times N_\tau$ lattices. Also shown is the $T = 0$ potential. The inset shows the color singlet free energy from which the contribution from the matrix element $T \ln c_1$ has been subtracted.

Problems with extracting the triplet

$$e^{-F(r)/T} = G(r, t) = e^{-E_1(r)/T} + e^{-E_2(r)/T},$$

$$e^{-F_1(r)/T} = G_1(r, T) = c_1(r) e^{-E_1(r)/T} + c_2(r) e^{-E_2(r)/T},$$

$$e^{-F_3(r)/T} = (1 - c_1(r)) e^{-E_1(r)/T} + (1 - c_2(r)) e^{-E_2(r)/T}$$

$$F_3(r, T) = E_2(r) - T \ln \left(1 - c_2(r) + (1 - c_1(r)) e^{\Delta E_{12}(r)/T} \right) - T \ln 3,$$

If $c_1 \neq 1$ then $F_3(r, T)$ has a contribution from the **singlet!**

Adjoint free energy

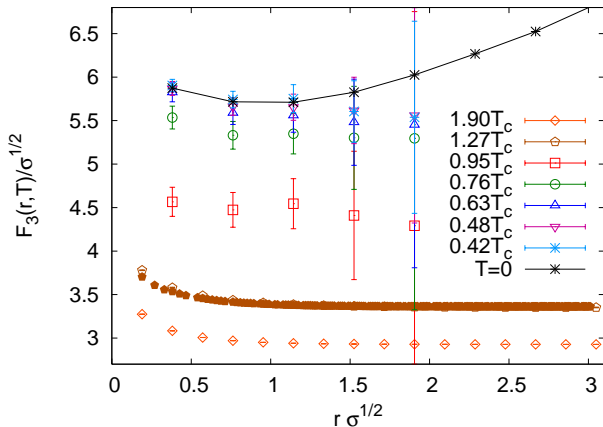


Figure: The triplet free energy at different temperatures calculated at $\beta = 2.5$. The filled symbols correspond to calculations in Coulomb gauge.

Comparison to C. Michael, S.J. Perantonis, J. Phys. G 18, 1725 (1992)

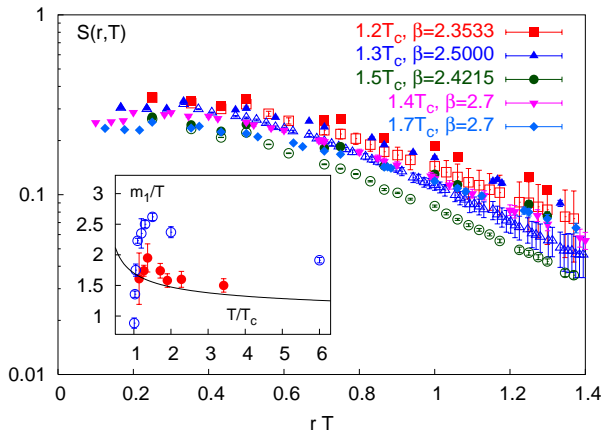
Screening function

$$S(r, T) = (F_\infty(T) - F_1(r, T)) r = -(r T) \ln \frac{\langle \text{Tr} L^\dagger(x) L(y) \rangle}{|\langle L \rangle|^2}$$

Its exponential fall-off is governed by the Debye screening mass:

$$\ln S(r, T) \sim -m_D r$$

Screening function



Conclusion

- ▶ The singlet, triplet and color averaged static meson correlators calculated using different levels of APE smearing
- ▶ APE smearing procedure allows to strongly reduce distance dependence from the matrix elements c_1 in the singlet channel thus providing the correct interpretation of the triplet correlator
- ▶ Compared to fixing Coulomb gauge APE smearing procedure offers improvement in assessing triplet free energy contribution of static quark anti-quark pair
- ▶ The screening function shows correct exponential fall-off behaviour