Quarkonium correlators on the lattice

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Contents of this talk

- Introduction
  -- Quark Gluon Plasma & J/ψ suppression
  -- Lattice studies on J/ψ suppression
- Our approach to study charmonium dissociation
- Charmonium wave functions at T>0
- Discussion & Summary

from the Phenix group web-site
$J/\psi$ suppression as a signal of QGP

Confined phase:
- linear raising potential
  $\rightarrow$ bound state of $c - \bar{c}$

De-confined phase:
- Debye screening
  $\rightarrow$ scattering state of $c - \bar{c}$

T.Hashimoto et al.('86), Matsui&Satz('86)

Lattice QCD calculations:
- Spectral function by MEM: T.Umeda et al.('02), S.Datta et al.('04), Asakawa&Hatsuda('04), A.Jakovac et al.('07), G.Aatz et al.('06)
- Wave func.: T.Umeda et al.('00)
- B. C. dep.: H.Iida et al. ('06)

$\rightarrow$ all calculations conclude that $J/\psi$ survives till $1.5T_c$ or higher
Sequential \( J/\psi \) suppression scenario

\[
\begin{align*}
J/\psi \ (1S) & : \ JPC = 1^{--} \quad M=3097\text{MeV} \quad (\text{Vector}) \\
\psi \ (2S) & : \ JPC = 1^{--} \quad M=3686\text{MeV} \quad (\text{Vector}) \\
\chi_{c0} \ (1P) & : \ JPC = 0^{++} \quad M=3415\text{MeV} \quad (\text{Scalar}) \\
\chi_{c1} \ (1P) & : \ JPC = 1^{++} \quad M=3511\text{MeV} \quad (\text{Axial Vector})
\end{align*}
\]

It is important to study dissociation temperatures for not only \( J/\psi \) but also \( \psi (2S), \chi_c \)'s

E705 Collab.('93)
Hot QCD on the lattice

Lattice QCD enables us to perform nonperturbative calculations of QCD

\[
\langle X \rangle = \frac{1}{Z_{QCD}} \int Dq(x) D\bar{q}(x) DA_\mu(x) X(q, \bar{q}, A_\mu) e^{-S_{QCD}}
\]

Path integral by Monte Carlo integration

Finite T Field Theory on the lattice
- 4dim. Euclidean lattice
- gauge field \( U_\mu(x) \rightarrow \) periodic B.C.
- quark field \( q(x) \rightarrow \) anti-periodic B.C.
- Temperature \( T = 1/(N_t a) \)
Spectral function on the lattice

Thermal hadron (charmonium) correlation functions $C_H(\tau, T)$

\[
C_H(\tau, T) = \sum_{\vec{r}} \langle J_H(\tau, \vec{r}) J_H^\dagger(0, \vec{0}) \rangle 
= \int_0^\infty d\omega \sigma_H(\omega, T) \frac{\cosh(\omega(\tau - 1/2T))}{\sinh(\omega/2T)}
\]

Spectral function $\sigma_H(\omega, T)$

- discrete spectra
  - bound states
  - charmonium states
- continuum spectra
  - 2-particle states
  - $c \bar{c}$ scattering states
  - melted charmonium
Spectral functions in a finite volume

Moments are discretized in finite (V=L^3) volume
\[ p_i/a = 2n_i \pi / L \quad (n_i=0, \pm 1, \pm 2, \ldots) \] for Periodic boundary condition

- **infinite volume case**
  - discrete spectrum (bound states)
  - continuum spectrum (2-particle states)

- **finite volume case**
  - localized wave func.
  - broad wave func.

In a finite volume (e.g. Lattice simulations),
discrete spectra does not always indicate bound states!

Shape of wave functions may be good signature
to find out the charmonium melting.
Bound state or scattering state?

$\Phi(r)$: wave function

- $r$: c - c distance

Examples for:
- S-wave
- Periodic B.C

Localized wave function
- Small vol. dependence

Broad wave function
- Vol. dependence
Wave functions at finite temperature

Temp. dependence of (Bethe-Salpeter) “Wave function”

\[ BS(\vec{r}, t) = \sum_{\vec{x}} \langle \bar{q}(\vec{x} + \vec{r}, t) \Gamma q(\vec{x}, t) \bar{q}(\vec{0}, 0) \Gamma q(\vec{0}, 0) \rangle \]

\[ \Psi(|\vec{r}|, t) = BS(\vec{r}, t) / BS(\vec{r}_0, t) \]

\[ \Gamma = \begin{cases} \gamma_5 & (Ps) \\ \gamma_i & (Ve) \quad (i = 1, 2, 3) \\ \sum_j \left( \bar{\partial}_j \gamma_j - \bar{\partial}_j \gamma_j \right) & (Sc) \\ \sum_{j,k} \epsilon_{ijk} \left( \bar{\partial}_j \gamma_k - \bar{\partial}_j \gamma_k \right) & (Av) \quad (i = 1, 2, 3) \end{cases} \]

Remarks on wave function of quark-antiquark

- gauge variant \( \rightarrow \) Coulomb gauge fixing
- large or small components of quark/antiquark
  \( \rightarrow \) wave func. for large components
Technique to calculate wave function at T>0

It is difficult to extract higher states from lattice correlators (at T>0) even if we use MEM !!

It is important to investigate a few lowest states (at T>0)

Constant mode can be separated by the Midpoint subtraction

T. Umeda (2007)

In order to study a few lowest states, the variational analysis is one of the most reliable methods !

\[
N \times N \text{ correlation matrix} : C(t) \\
C(t) \psi = \lambda(t, t_0) C(t_0) \psi \\
\lambda_i(t, t_0) = e^{-E_i(t-t_0)}
\]
Lattice setup

- Quenched approximation (no dynamical quark effect)
- Anisotropic lattices
  - Lattice spacing: $a_s = 0.0970(5)$ fm
  - Anisotropy: $a_s/a_t = 4$
- $r_s=1$ to suppress doubler effects
- Variational analysis with 4 x 4 correlation matrix

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<th>$N_t$</th>
<th>32</th>
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<th>20</th>
<th>16</th>
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<td>1.40</td>
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<td>$V=32^3$</td>
<td>—</td>
<td>—</td>
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</tr>
</tbody>
</table>
Wave functions in free quark case

Test with free quarks ($L_s/a=20$, $m_a=0.17$) in case of S-wave channels

- Free quarks make trivial waves with an allowed momentum in a box

\[ \Psi_k(|r|, t) = \frac{\sum_{\vec{p}=\vec{k}} \cos (p_1 r_1) \cos (p_2 r_2) \cos (p_3 r_3)}{\sum_{\vec{p}=\vec{k}} 1} \]

- The wave function is constructed with eigen functions of 6 x 6 correlators

- 6 types of Gaussian smeared operators
  \[ \phi(x) = \exp(-A|x|^2), \]
  \[ A = 0.02, 0.05, 0.1, 0.15, 0.2, 0.25 \]

- Our method well reproduces the known result ( ! )
Charmonium wave functions at finite temperatures

- Small temperature dependence in each channels
- Clear signals of bound states even at \( T=2.3T_c \) ( ! )
- \((2\text{fm})^3\) may be small for P-wave states.
Volume dependence at $T=2.3T_c$

- Clear signals of bound states even at $T=2.3T_c$ (\^\_\^)
- Large volume is necessary for P-wave states.
Discussion

We found localized wave functions up to 2.3Tc for S- & P- wave channels.

(1) Does variational analysis work well?

- wave function for lowest/next-lowest state
  + contributions from higher states $\leftarrow$ contaminations

- our results suggest
  - there are no/small broad wave functions even in the higher states (!)

(2) Unbound state with localized wave function?

- anyway,
  - tight wave function is incompatible with the $J/\psi$ suppression (!)

(3) Small changes in wave functions contradict some potential model results with $T_{\text{dis}}(J/\psi) > T_c$ (!)
The idea has been originally applied for the charmonium study in H. Iida et al., Phys. Rev. D74 (2006) 074502.

Boundary condition dependence

The wave functions are localized, their energies are insensitive to B.C.

The momenta depends on BC, the scattering state energies are sensitive to B.C.
Variational analysis in free quark case

Test with free quarks ( $L_s/a=20$, $m_a=0.17$ )

- **S wave**
  - PBC: $b=(1,1,1)$
  - APBC: $b=(-1,-1,-1)$
  - MBC: $b=(-1,1,1)$

- **P wave**
  - PBC: $b=(1,1,1)$
  - APBC: $b=(-1,-1,-1)$
  - MBC: $b=(-1,1,1)$

Mass diff. between the lowest masses in each BC

$q(x_i + L_i) = b_i q(x_i)$

- $b_i = 1$: periodic
- $b_i = -1$: anti-periodic

An expected diff. in $V=(2fm)^3$
(free quark case)

$\sim 200\text{MeV}$
Temperature dependence of charmonium spectra

- No significant differences in the different B.C.
- Analysis is difficult at higher temperature (\(2T_C\))

\[ q(x_i + L_i) = b_i q(x_i) \]
\[ b_i = 1 \text{ : periodic} \]
\[ b_i = -1 \text{: anti-periodic} \]

PBC : \(b=(1, 1, 1)\)
APBC : \(b=(-1, -1, -1)\)
MBC : \(b=(-1, 1, 1)\)

an expected gap in \(V=(2fm)^3\)
(free quark case)
\(~200\text{MeV}\)
Summary and future plan

We investigated $T_{\text{dis}}$ of charmonia from Lattice QCD without Bayesian (MEM) analysis using...

- Bethe-Salpeter “wave function”
- Volume dependence of the “wave function”
- Boundary condition dependence

No evidence for unbound $c\bar{c}$ quarks up to $T = 2.3\ T_c$

→ The result may affect the scenario of $J/\psi$ suppression.

Future plan

- Possible scenarios for the experimental $J/\psi$ suppression
- Higher Temp. calculations ( $T/T_c=3\sim5$ )
- Full QCD calculations ( $N_f=2+1$ Wilson is now in progress )