Canonical description of charm and bottom production in $e^+e^-$, $pA$, $\pi A$, $pp$ and $\bar{pp}$ collisions

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- Statistical Model and exact charge conservation laws: Its phenomenological consequences in HIC
- SM application to particle productions in $e^+e^-$ annihilation
  - light flavors production
  - heavy flavors production
- Relative production yields of charm and bottom hadrons in $pA$, $\pi A$, $pp$ and $\bar{pp}$ collisions and SM results

In collaboration with: A. Andronic, F. Beutler, P. Braun-Munzinger & J. Stachel


See also:

P. Braun-Munzinger et. al. in “Quark Gluon Plasma 3”
F. Beutler, diploma theses arXiv:0904.2885 [nucl-th]
Statistical operator and mass spectrum

- Resonance dominance

\[ \ln Z^{GC}(T, \mu) = \int \frac{2V \mu p^\mu}{(2\pi)^3} \tau(p^2, \mu_B, \mu_S, \mu_Q) e^{-\beta_{\mu}p^\mu} \]

- Approximate \( \tau(m^2, \mu) \) by experimentally known mass spectrum

\[ \ln Z(T, \mu) \approx \frac{VT}{2\pi^2} \sum_{i \in \text{hadrons}} \frac{Q_i \mu}{T} \int ds \frac{\sqrt{s}}{T} K_2(\sqrt{s}/T) F^{B-W}(m_i, s) \]

- Particle yield, thermal density, BR, thermal density of resonances

\[ < N_i > = V \left[ n_{i}^{\text{th}}(T, \mu_B) + \sum K \Gamma_{K \rightarrow i} n_{i}^{\text{th-Res.}}(T, \mu_B) \right] \]

- Only 2-parameters needed to fix all particle yield ratios
LGT phase boundary and chemical freezeout
\[ e^{i\phi S} He^{i\phi S} = H \leftrightarrow [S, H] = 0 \]

conservation on the average

\[ Z^{GC}(T, \mu_S, V) = \text{Tr} \left[ e^{-\beta(H-\mu_S S)} \right] \]

\[ Z^C(T, V) = \text{Tr}_S \left[ e^{-\beta H} \right] \]

\[ Z^{GC} = \sum_{S = -\infty}^{S = +\infty} e^{S \mu_S / T} Z^C_S \]

Consider thermal system with Total Strangenes \( S = 0 \)

\[ n^C_S \approx \gamma_s^S n^{GC} \]

\[ \frac{I_S(\gamma_s 2V_C \sqrt{n^{GC}_{s=-1} n^{GC}_{s=+1}})}{I_0(\gamma_s 2V_C \sqrt{n^{GC}_{s=-1} n^{GC}_{s=+1}})} \]

suppression factor \( \leq 1 \)

suppression increases with \( S \) and with decreasing collision energy
i) Strong, quadratic dependance of $|S|=1$ particles with $A_{part}$ at SIS

ii) strange anti-particle/particle ratios independent of $A_{part}$

$\pi + N \rightarrow K^+ + Y$

$\langle K^+ \rangle^C \equiv A_{part}^2 e^{-m_k/T} e^{-(m_Y - \mu_B)/T}$

$\langle \pi^+ \rangle \equiv A_{part} e^{-(m_\Delta - \mu_B)/T}$

J. Cleymans, H. Oeschler & K.R.
iii) Scaling properties of particle production yields

J. Cleymans, H. Oeschler & K.R.

Excellent description of kaon production from SIS to AGS

\[ \pi + N \square \quad K^+ + \Lambda \quad \Rightarrow \quad [K^+] = [\Lambda] \]

\[ \pi + \Lambda \square \quad K^- + N \quad \Rightarrow \quad \frac{[\pi]}{[N]} = \kappa \cdot \frac{[K^-]}{[\Lambda]} \quad \Rightarrow \text{Scaling:} \quad \frac{[K^-]}{[K^+]} = \kappa \cdot \frac{[\pi]}{[N]} \]

Similar scaling for \( [\Xi^-]/[K^+] = \kappa \cdot [K^+]/[N] \)

A. Andronic, P. Braun-Munzinger & K.R.
Strangeness enhancement from p-Be to central Pb-Pb collisions at $\sqrt{s_{NN}} = 17.3$ GeV

Canonical model with exact strangeness conservation at fixed $T \approx 168$ MeV and $\mu_B$ being centrality dependent provide good description of NA57 data if the correlation volume scales as:

$$V = \left(\frac{A_{\text{part}}}{2}\right)^\alpha \text{ with } \alpha \approx 1/3$$
Particles excitation functions in HG model

Braun-Munzinger, Cleymans, Oeschler & K.R.
Andronic, Braun-Munzinger & Stachel (09)

Nu Xu & K.R.
H.Oeschler et al.
Hadron production in $e^+e^-$ annihilation

Jet structure of hadrons production

2 jets : 3 jets : 4 jets $= \mathcal{O}(\alpha_s^0) : \mathcal{O}(\alpha_s^1) : \mathcal{O}(\alpha_s^2)$

\[ \alpha_s \text{ at } \sqrt{s}=91 \text{ GeV is } 0.12 \pm 0.0031 \]

\[ \Gamma_{q\bar{q}} = \frac{G_F m_{Z}^2 \beta_q}{2\pi\sqrt{2}} [(1 + 2\eta^2) (g_V^q)^2 + (g_A^q)^2) - 6\eta^2 (g_A^q)^2] \]

Flavor content of the jets:

**up type quarks** $= (u, c)$

**down type quarks** $= (d, s, b)$

\[ \Gamma_{u\bar{u}}/\Gamma_{hadron} \approx 17\% \]
\[ \Gamma_{d\bar{d}}/\Gamma_{hadron} \approx 22\% \]

Can we quantify light and heavy flavor particles within Statistical Model??

Canonical model for particle production yields in $e^+ e^-$


$$Z^{GC}_{N,S,Q,C,B}(T,V) = \frac{1}{(2\pi)^5} \int_0^{2\pi} d^5 \phi \, e^{i\phi \cdot \bar{X}} \exp \left( \sum_i \frac{d_i \cdot V}{(2\pi)^3} \int d^3 p \ln(1 \pm e^{-\beta E_i - i x_i \cdot \phi_i})^{\pm 1} \right)$$

- The integral is not convenient for numerical analysis as the integrand is a strongly oscillating function
- The partition function and particle multiplicity can be expressed as multi-sum and products of different Bessel functions
- Because of the high precision of the LEP measurements, a high mass cut $M \approx 3 GeV$ and quantum statistic are required
Quantum statistic and Mass cut effects on particle yields

Strong, 8-15% deviation of baryon yields if cutting the mass spectrum at $M \approx 1.7\text{GeV}$

Quantum statistic needed for Pions and Kaons

\[ \ln(1-x)^{-1} = \sum_{k=1}^{\infty} \frac{x^k}{k!} \]
Most hadronic events in high energy $e^+e^-$ collisions are two-jet events

Each jet represents an independent fireball

Problem: Open Charm an Bottom shows dramatic deviations from data

\[ < N_B >^{model} / data \approx 10^{-20} \]
\[ < N_{D^\pm} >^{model} / data \approx 10^{-2} \]

Subtract the contributions from charm an bottom to lighter particles

e.g. C,B contributions to

\[ \pi^+ - 11.7\%, \quad K^+ - 30.0\% \]
\[ K^{*0}(892) - 16.5\%, \quad \phi(1020) - 68.0\% \]
Canonical effects and charm/bottom mesons

\[ < N_i >^Q_{Q_i=\pm 1} = \gamma V \cdot Z_i^{Q_i} \frac{Z_{\pm 1}}{x} \frac{I_Q^{\mp 1} (2\gamma V x)}{I_Q (2\gamma V x)} \]

\[ z_i^{Q_i} = \frac{d_i}{(2\pi)^3} \int d^3 p \exp(-\beta (E_i + \vec{b}_i \cdot \vec{\mu}_i)) \]

\[ Z_{\pm 1} = \sum_i z_i^{Q_i=\pm 1} \]

\[ x = \sqrt{Z_{-1} Z_1} \]

\[ Q = 0 \]

Total charge of the system

And small \( 2V_c x << 1 \)

\[ Q = +1 \]

\[ < N_i >^Q_{Q_i=+1} \approx \frac{V \gamma \cdot z_i}{V \gamma \cdot Z_{+1}} \]

Strong Suppression of thermal particle phase-space

Strong Enhancement of thermal particle phase-space

Charge \( Q=+1 \) redistributed between different particle species
Charm and Bottom particles at LEP at $\sqrt{s} = 91 \, GeV$

Open charm and bottom well described by thermalization of the fireball with overall Charm = ±1 and Bottom = ±1

$J/\psi$, $\psi$ and $\chi_c$ are entirely coming from Bottom’s decays and agree with model

Hidden charm, $Y$ is of non-thermal origin, thus it does not fit to model systematics!

Very good agreement with $\chi^2 / \text{dof} = 34 / 18$ for all data and $\chi^2 / \text{dof} = 22 / 16$ when excluding $Y$ and $J/\psi$
Statistical Model and relative charm and bottom production in $hA$ and $hh$ collisions at different $\sqrt{s}$

- Fixed thermal parameters in $hA$ and $hh$ collisions within the Statistical model
- Consider particle ratios of open charm only
- Consider ratios of charmonia or bottomonia

Remark:

In hadronic collisions the relative production cross sections between bottom and charm are much smaller than in $e^+ e^-$ collisions. Even at the LHC energies $\sigma_b/\sigma_c \simeq 1/10$, while in 91 GeV $e^+ e^- B(Z^0 \rightarrow b)/B(Z^0 \rightarrow c) = 0.22/0.17$. 
Fixing thermal parameters in h-A and hh:  

- Temperature independent of system size and centrality
- Strong variation of baryon chemical potential with centrality and system size for mid-rapidity data
Centrality dependence of baryon chemical potential

For NA57 the temperature is fixed to $T \approx 168 \ MeV$ from central Pb-Pb collisions: (the value consistent with recent analysis of A. Andronic et al.)
Open charm production ratios in p(π)A collisions

Andronic, Beutler, Braun-Munzinger, Stachel & K.R.

The energy dependence of thermal parameters taken from systematic analysis of heavy ion data on different particle yields by: A. Andronic, P. Braun-Munzinger & J. Stachel


Good agreement of Statistical Model and data for the relative production cross section of charged to neutral D-mesons: Particularly with data average by PDG

Data compilation by:
The $\chi_c$ contributions to $J/\psi$ meson

A. Andronic, F. Beutler, P. Braun-Munzinger, J. Stachel & K.R.

The fraction of $J/\psi$ from radiative decays of $\chi_{c1}$ and $\chi_{c2}$ states:

$$R_{\chi_c} = \frac{\sum_{J=1}^{2} \sigma(\chi_{cJ}) Br(\chi_{cJ} \rightarrow J/\psi \gamma)}{\sigma(J/\psi)}$$

is far above the statistical model predictions

**Data compilation by HERA-B Coll.:**


**Data average** $0.361 \pm 0.015$, with a $\chi^2$/dof$=1.21$

**Data average at lower energies:** $0.25 \pm 0.05$

**From:** P. Faccioli, C. Lourenço, J. Seixas, H.K. Wöhri, JHEP **0810** (2008) 004

Clear, non-thermal origin of charm in $hA$ collisions in $\bar{p}p$ and $pp$ collisions.
Production cross section of $\psi'$ relative to $J/\psi$

The ratio for the Tevatron energy was derived from the CDF data on $J/\psi$ and $\psi'$, and is for $p_t > 1.25$ GeV:

We have extrapolated the $\psi'$ measurements from $p_t \geq 2$ GeV down to 1.25 GeV.

- Strong suppression of $\psi'/(J/\psi)$ ratio in PbPb relative to pA, pp and $\bar{pp}$
- Different production mechanism in elementary and heavy ion collisions
- The nuclear modification canceled out in the $\psi'/(J/\psi)$ ratio as the pp value is the same as in the pA
- Good agreement of Statistical Model and data in PbPb collisions

Data pA compilation by:
The relative production of $\chi_{c1}$ to $\chi_{c2}$ charmonia

The relative production of $\chi_{c1}$ to $\chi_{c2}$ is seen to be consistent with Statistical Thermal Model. However, due to similar masses

$$m_{\chi_{c1}} \, \square \, 3510 \, \text{MeV} \quad m_{\chi_{c1}} \, \square \, 3556 \, \text{MeV}$$

is almost consistent with the spin statistic ratio, thus this agreement is trivial

$$\frac{\chi_{c1}}{\chi_{c2}} = \frac{d_{J}^{\chi_{c1}}}{d_{J}^{\chi_{c2}}} \approx \frac{m_{\chi_{c1}}^{2} K_{2}(m_{\chi_{c1}} / T)}{m_{\chi_{c1}}^{2} K_{2}(m_{\chi_{c1}} / T)} = \frac{d_{J}^{\chi_{c1}}}{d_{J}^{\chi_{c2}}} = 0.6$$

Data compilation by HERA-B Coll.:


Data average $0.674 \pm 0.084$, with a $\chi^{2}/dof = 1.39$
The relative production of bottomonia is strongly inconsistent with Statistical Thermal Model results. Their ratios described by

$$\frac{Y^i}{Y} = \frac{d_j^{Y^i}}{d_j^Y} \ast \frac{m_{Y^i}^2 K_2(m_{Y^i} / T)}{m_Y^2 K_2(m_Y / T)}$$

with $T \approx 170$ $MeV$ exceed the data by one and even two orders of magnitude indicating:

A non-thermal origin of charmonia and bottomonia production in elementary and $hA$ collisions.

Data are from E866 and CDF Coll.
Conclusions

- An excellent description of open charm and bottom meson yields in $e^+e^-$ annihilations by the thermal model.
- The above yields are almost independent of the fireball volume and/or any factor quantifying deviations from chemical equilibrium.
- Hidden bottom in $e^+e^-$ annihilations is not described by the thermal model. Hidden charm is consistent with the model results as it is almost entirely coming from the open bottom decays.
- The relative production of D-mesons in hh and hA collisions is well described by the model.
- The charmonia and bottomonia ratios in hh and hA collisions are not consistent with the model predictions.
  This is in contradiction with AA data at the SPS where the ratio $\psi'/\psi(J/\psi)$ is well quantified by the model.
- The above indicates a different production mechanism of hidden charm and bottom in hh, hA, and AA collisions.