Quarkonium Production Mechanism and Reliability of pQCD Calculation

Jianwei Qiu
Iowa State University

Based on work done with Kang, Nayak and G. Sterman
Good probes for a dense medium

- **Basic requirements:**
  - Can be cleanly measured experimentally
  - Can be reliably calculated theoretically

- **Necessary conditions:**
  - Sensitive to the scales and properties of the strongly interacting matter – low momentum scale (a few hundred MeV)
  - Large momentum transfer to ensure pQCD calculation
    - a hard probe sensitive to low momentum physics

- **Potentially good probes:**
  - Have two observed scales (one hard and one soft)
  - Have one observed hard scale and a steeply falling distribution
Quarkonium could be a good probe

- It has two intrinsic scales:
  - **Heavy quark mass:**
    - Heavy quark pairs are produced at a distance scale much less than fm
    
    \[ \Delta r \sim \frac{1}{2m_Q} \leq 0.1 \text{ fm (for a charm-quark pair)} \]
    
    \[ \leq 0.025 \text{ fm (for a b-quark pair)} \]

    - PQCD is expected to work for the production of heavy quarks
  - **Quarkonium binding energy:**
    
    \[ \frac{|M^2 - 4m_Q^2|}{4m_Q^2} \ll 1 \]
    
    for both charm and bottom quarkonia

- The transition from a heavy quark pair to a quarkonium should be sensitive to the soft physics at the medium temperature
J/ψ Suppression in QGP

- Heavy quarkonium provides a non-relativistic system
  – controlled approximation: potential model, EFT, ...

\[ \frac{v^2}{c^2} \sim \frac{k_Q^2}{m_Q^2} \sim \frac{|M^2 - 4m_c^2|}{4m_c^2} \sim 0.3 \quad \text{Bottom:} \quad \frac{v^2}{c^2} \sim 0.1 \]

- Color screening in QGP suppresses the formation of J/ψ
  - Potential: \( V_{Q\bar{Q}}(r) \Rightarrow V_{Q\bar{Q}}(r, T) \)
  - Wave function: \( \Phi_{Q\bar{Q}}(r) \Rightarrow \Phi_{Q\bar{Q}}(r, T) \)
  - J/ψ formation rate \( \propto \left| \Phi_{Q\bar{Q}}(r, T) \right|^2 \)

J/ψ suppression \( \Leftrightarrow \) medium properties

Matsui & Satz (1986)

- Calibration:
  - Do we understand the production mechanism of J/ψ well enough to calibrate the production rate and extract the information on QGP?
The basic production mechanism

- **Production of an off-shell heavy quark pair:**

  ![Diagram showing the production process](image)

  - Coherent soft interaction
  - Perturbative
  - Non-perturbative

  \[ \Delta r \leq \frac{1}{2m_q} \]

- **Approximation:** **on-shell heavy quark pair + hadronization**

  \[ \sigma_{AB \rightarrow h} = \sum_{states} \int d\Gamma_{Q\bar{Q}} \frac{d\sigma_{AB \rightarrow states(Q\bar{Q})}}{d\Gamma_{Q\bar{Q}}} F_{states(Q\bar{Q}) \rightarrow h}(p_Q, p_{\bar{Q}}, p_h) \]

Different models ⇔ Different assumptions/treatments on how the heavy quark pair becomes a quarkonium?
Popular production models

- **Color singlet model:**
  - Only pairs with right quantum number can become quarkonia
  - Non-perturbative part $\sim$ decay wave function squared
  
- **Color evaporation model:**
  - All colored or color singlet pairs with invariant mass less than open charm threshold could become bound quarkonia
  - Non-perturbative part $= \text{one constant per quarkonium state}$

- **NRQCD model:**
  - All colored or color singlet pairs could become quarkonia
  - Power expansion in relative velocity of heavy quark pairs
  - Non-perturbative part $= \text{one matrix element per } Q\bar{Q} \text{ state}$

\[
\sigma_{AB \rightarrow J/\psi} (M_{J/\psi}) \approx \sum_{[O]} \sigma_{AB \rightarrow [O]} \left( m_{c\bar{c}}^2 = M_{J/\psi} \right) \langle O_{J/\psi} (0) \rangle
\]
CSM: Huge high order corrections

Color-singlet contribution for J/ψ and Upsilon production at Tevatron

P. Artoisenet, F. Maltoni, et.al. 2007

Large uncertainty band
⇒ strong scale dependence

Large NLO, NNLO contribution
⇒ how perturbative series converge?
CEM: OK for inclusive production

- Good for total cross section, ok for $P_T$ distribution:

Amundson et al, PLB 1997
CEM: Resummation of pQCD logs

CEM with all order resummation of soft gluon shower

Berger, Qiu, Wang, 2005
NRQCD Model: Best fit to Tevatron data

Unpolarized $J/\psi$ at the Tevatron:

NRQCD model gave the best description of $P_T$ distribution of various inclusive heavy quarkonium production at Tevatron, with matrix elements fixed by data.
Polarization of quarkonium at Tevatron

- Measure angular distribution of $\mu^+\mu^-$ in $J/\psi$ decay

![Diagram of $J/\psi$ decay with $\mu^+$, $\mu^-$, and directions indicated.]

- Normalized distribution:

$$I(\cos \theta^*) = \frac{3}{2(\alpha + 3)} \left( 1 + \alpha \cos \theta^* \right)$$

$$\alpha = \begin{cases} 
+1 & \text{fully transverse} \\
0 & \text{unpolarized} \\
-1 & \text{fully longitudinal} 
\end{cases}$$
Surprises from polarization measurements

- Transverse polarization at high $p_T$?

NRQCD: Cho & Wise, Beneke & Rothstein, 1995, ...

CDF Collaboration, PRL 2007
Exclusive production in $e^+e^-$

- **Double charm production:**
  
<table>
<thead>
<tr>
<th>$J/\psi$ c$\bar{c}$</th>
<th>$\eta_c(1S)$</th>
<th>$\chi_{c0}$</th>
<th>$\eta_c(2S)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BABAR</td>
<td>17.6 ± 2.8$^{+1.5}_{-2.1}$</td>
<td>10.3 ± 2.5$^{+1.4}_{-1.8}$</td>
<td>16.4 ± 3.7$^{+2.4}_{-3.0}$</td>
</tr>
<tr>
<td>Belle [14]</td>
<td>25.6 ± 2.8 ± 3.4</td>
<td>6.4 ± 1.7 ± 1.0</td>
<td>16.5 ± 3.0 ± 2.4</td>
</tr>
<tr>
<td>NRQCD [6]</td>
<td>2.31 ± 1.09</td>
<td>2.28 ± 1.03</td>
<td>0.96 ± 0.45</td>
</tr>
<tr>
<td>NRQCD [4]</td>
<td>5.5</td>
<td>6.9</td>
<td>3.7</td>
</tr>
</tbody>
</table>

- **Possible resolution for $J/\psi + \eta_c$:**
  
  - NLO correction: $K_{\text{factor}} = 1.96$
  - Relativistic Correction:
    - X-section: $K_{\text{factor}} = 1.34$
    - Wave func: $K_{\text{factor}} = 1.32$
  - Combined: $K_{\text{factor}} = 4.15$

\[
\sigma[e^+e^- \rightarrow J/\psi + \eta_c] = 17.5 \pm 5.7 \text{ fb}
\]

Zhang, Gao, Chao, PRL

Bodwin et al. hep-ph/0611002
Inclusive production in $e^+e^-$

- **Charm associated production:**
  \[ \sigma(e^+e^- \rightarrow J/\psi c\bar{c}) \]
  - **Belle:** $(0.87^{+0.21}_{-0.19} \pm 0.17)$ pb
  - **NRQCD:** $\sim 0.07$ pb

- **Ratio to light flavors:**
  \[ \frac{\sigma(e^+e^- \rightarrow J/\psi c\bar{c})}{\sigma(e^+e^- \rightarrow J/\psi X)} \]
  - **Belle:** $0.59^{+0.15}_{-0.13} \pm 0.12$

- **Message:**
  Production rate of $e^+e^- \rightarrow J/\psi c\bar{c}$ is larger than all these channels: $e^+e^- \rightarrow J/\psi gg$, $e^+e^- \rightarrow J/\psi q\bar{q}$, ... combined?
Questions

- Is the key approximation:
  production of an on-shell heavy quark pair
  + hadronization for the pair to a quarkonium valid?

  The approximation of producing a long-lived
  “on-shell” pair is necessary for the factorization

- Is there a better description of the nonperturbative
  hadronization of the QQ pair?
None of the factorized production models, including NRQCD model, were proved theoretically.

Factorization of NRQCD model fails for low $p_T$.

Factorization of NRQCD model might work for large $p_T$. Spectator interactions are suppressed by $(1/p_T)^n$.

Factorization is necessary for the predictive power.
Long-lived parton states

- **Perturbative pinch singularity:**
  \[
  \propto \int d^4k \, \mathcal{H}_{gg \to g}(Q, k) \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - i\epsilon} \mathcal{D}_{g \to J/\psi}(k, P)
  \]

- **Dominated by** \(k^2 \sim 0\) **region**

- **Parton model collinear factorization:**
  \[
  \approx \int \frac{dz}{z} d^2k_\perp \mathcal{H}_{gg \to g}(Q, k^2 = 0) \int dk^2 \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - i\epsilon} \mathcal{D}_{g \to J/\psi}(k, P)
  \]

- **Long-lived parton state**

- **Fragmentation function**

- **Short-distance part**
Production of heavy quark pairs

- Perturbative pinch singularity: Kang, Qiu and Sterman, 2009

\[ P^\mu = (P^+, 4m^2/2P^+, 0_\perp) \]
\[ q'^\mu = (q^+, q^-, q_\perp) \]
\[ q \neq q' \]
\[ D_{ij}(P, q) \propto \langle J/\psi | \psi_i^\dagger(0) \chi_j(y) | 0 \rangle \]

- Scattering amplitude:

\[ \mathcal{M} \propto \int \frac{d^4q}{(2\pi)^4} \text{Tr} \left[ \hat{H}(P, q, Q) \frac{\gamma \cdot (P/2 - q) + m}{(P/2 - q)^2 - m^2 + i\varepsilon} \hat{D}(P, q) \frac{\gamma \cdot (P/2 + q) + m}{(P/2 + q)^2 - m^2 + i\varepsilon} \right] \]

- Potential poles:

\[ q^- = \frac{q^2_\perp - 2m^2(q^+/P^+)}{(P^+ + 2q^+)} - i\varepsilon \theta(P^+ + 2q^+) \rightarrow q^2_\perp/P^+ - i\varepsilon \]
\[ q^- = -\frac{q^2_\perp + 2m^2(q^+/P^+)}{(P^+ - 2q^+)} + i\varepsilon \theta(P^+ - 2q^+) \rightarrow -q^2_\perp/P^+ + i\varepsilon \]

- Condition for pinched poles:

\[ P^+ \gg q^+(2m^2/q^2_\perp) \geq 2m \quad \text{High } P_T \]
Sources of contributions

- **Fragmentation contribution:**

- **Direct contribution:**

- **S-channel contribution:**
**Factorization: fragmentation contribution**

- **Fragmentation contribution at large $P_T$**

  Fragmentation function – gluon to a hadron $H$ (e.g., $J/\psi$):

  \[
  d\sigma_{A+B\to H+X}(p_T) = \sum_i d\bar{\sigma}_{A+B\to i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + O(m_H^2/p_T^2)
  \]

  Cannot get fragmentation func. from PDFs or decay matrix elements
The proof works in two steps

- **Step 1:** Fragmentation factorizes from the rest

  - Reduced diagram
  - Still has long-distance physics due to incoming hadrons

  - Fragmentation function
Step 2: Cancellation of remaining IR final state:

Note: Uncut loops are short distance

Remaining soft-interaction absorbed into the Wilson lines of PDFs

H is IR safe!
The Wilson line in $x^-$ direction ($n^\mu = \delta_{\mu -}$)

$$\Phi^{(g)}(x^-) = P \exp \left[ -ig \int_0^\infty n \cdot A^{(adj)} \left( (x^- + \lambda)n \right) \right]$$

Which depends on the “direction” vector: $n^\mu$

For the fragmentation function, or the jet, all that is left is gluon source:

A necessary condition for the factorization, or the universality of the fragmentation function is:

The fragmentation function is independent of the $n^\mu$
Connection to NRQCD Factorization

- Proposed NRQCD factorization:
  \[ d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_n d\tilde{\sigma}_{A+B\rightarrow c\bar{c}[n]+X}(p_T) \langle O^H_n \rangle \]

- Proved pQCD factorization for single hadron production:
  \[ d\sigma_{A+B\rightarrow H+X}(p_T) = \sum_i d\tilde{\sigma}_{A+B\rightarrow i+X}(p_T/z, \mu) \otimes D_{H/i}(z, m_c, \mu) + O(m^2_H/p^2_T) \]

- Prove NRQCD Factorization

  To prove:
  \[ D_{H/i}(z, m_c, \mu) = \sum_n d_{i\rightarrow c\bar{c}[n]}(z, \mu, m_c) \langle O^H_n \rangle \]

  with
  - IR safe
  - gauge invariant and universal
  - independent of the direction of the Wilson lines
Gauge Invariance and Wilson lines

- Conventional operator definition (in Q̄Q rest frame)
  \[ \mathcal{O}_n^H(0) = \chi^\dagger \mathcal{K}_n \psi(0) \left( a_H^\dagger a_H \right) \psi^\dagger \mathcal{K}_n' \chi(0) \]

- \( \psi \), \( \chi \) are heavy quark, antiquark fields

- \( \mathcal{K}_n, \mathcal{K}_n' \): Products of color and spin matrices, covariant derivatives

- Fields at \( x = 0 \) but \( \mathcal{O}_n^H \) is not truly local

- Operator-valued gauge transformations (as to \( A^+ = 0 \) gauge) do not commute with \( a_H^\dagger a_H \)

- Only color-singlet \( \mathcal{K}' \)s give gauge invariant \( \mathcal{O}' \)s
  or, the color-octet operators are not gauge invariant
Resolution: supplement fields by Wilson lines:

$$\Phi_l[x, A] = \exp \left[ -ig \int_0^\infty d\lambda \cdot A(x + \lambda l) \right]$$

Our new, gauge invariant operators:

$$\mathcal{O}^H_n(0) \to \chi^\dagger \mathcal{K}_{n,c} \psi(0) \Phi_l^\dagger[0, A]_{cb} \left(a_H^\dagger a_H\right) \Phi_l[0, A]_{ba} \chi^\dagger \mathcal{K}'_{n,a} \psi(0)$$

Two remaining questions for NRQCD factorization:

- Are the “coefficient” functions $d_{g\to c\bar{c}[n]}(z, \mu, m_c)$ IR safe?
  - Our NNLO answer is no The lines are necessary
- Do the lines absorb all IR divergences?
  - Can’t tell yet for sure. OK at NNLO in $\alpha_s$ and all powers in $v$

Key difficulty:

Cancelation of $l \cdot q$ dependence – line direction
Factorization at NNLO and all orders in $v^2$

- Calculation with a finite $v$

\[ \mathcal{I}^{(8\rightarrow 1)} = \frac{\alpha_s^2}{4\varepsilon} \left[ 1 - \frac{1}{2 f(|\vec{v}|)} \ln \left[ \frac{1 + f(|\vec{v}|)}{1 - f(|\vec{v}|)} \right] \right] \]

with

\[ f(v) = \frac{2v}{1 + v^2} \quad \vec{v} = \vec{q}/E^* \]

$2E^*$ is the total energy of the heavy quark pair

(QQ rest frame)

- Reproduce the $v^2$ result when expanded
Factorization at a finite $v$?

- Velocity expansion is not efficient for charmonium prod.
  - Large phase space available for gluon radiation:
    \[ Q^2 - 4m_c^2 \Rightarrow 4M_D^2 - 4m_c^2 \approx 6\text{GeV}^2 \]
  - Large possible velocity in production:
    \[ v_{\text{prod}} \sim \frac{|k_c|}{m_c} \sim \sqrt{\frac{4M_D^2 - 4m_c^2}{4m_c^2}} \sim 0.88 \]
  - Very different from decay:
    \[ v_{\text{decay}} \sim \sqrt{\frac{M_{J/\psi}^2 - 4m_c^2}{4m_c^2}} \sim 0.48 \]

- Polarization at high $P_T$:
  - Understand $D_{f \rightarrow J/\psi}(z, \mu_0, m_c)$ with $\mu_0 > 2m_c$
  - DGLAP resums $\ln^n(\mu/\mu_0)$ and does not generate the “longitudinal” polarization seen at Tevatron

Qiu, Rodriguez, Zhang, 2001

June 22, 2009

Jianwei Qiu, ISU
Factorization for total cross section

- Total cross section of heavy quark pairs:
  - Examined structure of low order diagrams
  - Conjecture:
    Cross section can be reliably computed in QCD by using the same factorization formula
  - But, all order proof in perturbation theory is lacking
  - Corrections: \((1/m_c)^2\)

- Usefulness:
  - Total quark cross section \(\rightarrow\) CEM

Collins, Soper and Sterman:
Nucl. Phys. B263, 37, 1986
Quarkonium production in cold medium

- Medium size and $X_F$ dependence of suppression:

$$\sigma_A \equiv \sigma_N A^{\alpha}$$

$\sigma_{\text{abs}} = 4.18 \pm 0.35 \text{ mb}$

NA38/NA50

Leitch, ECT*
Quarkonium production in sQGP

Suppression in A-A collisions:

![Graph showing suppression in A-A collisions with different datasets and error bars.](image)

Leitch, ECT*

Jianwei Qiu, ISU
Transverse momentum distribution

- **Multiple scattering in medium:**
  - Each scattering is too soft to calculate perturbatively
  - Resummation + multiple scattering (not yet achieved)

- **Moment of $P_T$-distribution:**
  - More inclusive – calculable
  - Based on observed particles only
  - Less sensitive to hadronization

- **Broadening:**
  - Sensitive to the medium properties
  - Perturbatively calculable

\[
\langle (q_T^2)^n \rangle = \frac{\int dq_T^2 (q_T^2)^n \frac{d\sigma}{dq_T^2}}{\int dq_T^2 \frac{d\sigma}{dq_T^2}}
\]

\[
\Delta \langle q_T^2 \rangle = \langle q_T^2 \rangle_{AB} - \langle q_T^2 \rangle_{NN}
\]
Pure initial-state multiple scattering

- **Drell-Yan in d+A collision:**
  
  \[ \Delta \langle q_T^2 \rangle_{\text{DY}} \approx C_F \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right) \]

  \[ \Delta \langle q_T^2 \rangle_{E772} \approx 0.027 A^{1/3} \]

  Theory and Experiment are consistent, clear \( A^{1/3} \) dependence

- **Drell-Yan (W/Z at the LHC) in d+A and A+A collision:**

  - If the medium was formed after the hard collision,
    - multiple scattering in cold nuclear matter
    - broadening in AA = superposition of pA

    \[ \Delta \langle q_T^2 \rangle_{dA} \approx C_F \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2(Q) A^{1/3} \right) \]

    \[ \Delta \langle q_T^2 \rangle_{AB} \approx C_F \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2(Q)[A^{1/3} + B^{1/3}] \right) \]

  - If the medium (its coherence) was formed before hard collision,
    fast parton in hot dense medium – thermal energy

    \[ \Delta \langle q_T^2 \rangle_{AA} \quad \text{Saturates as a function of the centrality} – \text{model needed} \]
Broadening of heavy quarkonia

- **Initial-state only:**
  \[
  \Delta \langle q_T^2 \rangle_{J/\psi}^{(I)} = C_A \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right)
  \]
  \[
  \Delta \langle q_T^2 \rangle_{DY} \approx C_F \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right)
  \]

- **Experimental data from d+A:**
  - Clear $A^{1/3}$ dependence
  - But, wrong normalization!
    \[
    \frac{\Delta \langle q_T^2 \rangle_{J/\psi}^{(I)}}{\Delta \langle q_T^2 \rangle_{DY}} \bigg|_{thy} = \frac{C_A}{C_F} = 2.25
    \]
    \[
    \frac{\Delta \langle q_T^2 \rangle_{J/\psi}^{(I)}}{\Delta \langle q_T^2 \rangle_{DY}} \bigg|_{exp} = \frac{0.133}{0.027} \approx 4.9
    \]
  - Final-state effect?
  - Only depend on observed quarkonia

Kang, Qiu, PRD77(2008)

J.C.Peng, hep-ph/9912371

June 22, 2009

Jianwei Qiu, ISU
Final-state multiple scattering

- Heavy quarkonium is unlikely to be formed when the heavy quark pair was produced

$$r_H \leq \frac{1}{2m_c} \sim \frac{1}{15} \text{fm}$$

- If the formation length: $$r_F \leq R_N \sim 1 \text{fm}$$
  no A-enhancement from final-state interaction
- If the formation length: $$r_F \geq R_A$$
  additional A$^{1/3}$ enhancement from the final-state interaction

- Final-state effect depends on how quarkonium is formed
  NRQCD model, color evaporation model, ...
Double scattering – $A^{1/3}$ dependence:

$$\Delta \langle q_T^2 \rangle_{\text{CEM}} \approx \int dq_T^2 dq_T^2 \int_{4m_Q^2}^{4M_Q^2} dQ^2 \frac{d\sigma_{hA \rightarrow Q\bar{Q}}^D}{dQ^2 dq_T^2} \Big/ \int_{4m_Q^2}^{4M_Q^2} dQ^2 \frac{d\sigma_{hA \rightarrow Q\bar{Q}}}{dQ^2}$$

Multiparton correlation:

$$T^{(F)}_{g/A}(x) = T^{(l)}_{g/A}(x) = \int \frac{dy^-}{2\pi} e^{ixp^+y^-} \int \frac{dy_1^- dy_2^-}{2\pi} \theta(y^- - y_1^-)\theta(-y_2^-) \times \frac{1}{xp^+} \langle p_A | F^+(y_2^-)F^+(0)F^+ \sigma(y^-)F^{+\alpha}(y_1^-) | p_A \rangle = \lambda^2 A^{4/3} \phi_{g/A}(x)$$

Broadening – twice of initial-state effect:

$$\Delta \langle q_T^2 \rangle_{\text{CEM}} = \left( \frac{8\pi^2\alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right) \left( C_F + C_A \right) \sigma_{q\bar{q}} + 2C_A \sigma_{gg}$$

\[ \approx 2C_A \left( \frac{8\pi^2\alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right) \quad \text{if gluon-gluon dominates, and if } r_F > R_A \]
NRQCD model

- **Cross section:**
\[
\sigma^{NRQCD}_{hA \rightarrow H} = A \sum_{a,b} \int dx' \phi_{a/h}(x') \int dx \phi_{b/A}(x) \left[ \sum_n H_{ab \rightarrow Q\bar{q}[n]} \langle \mathcal{O}^H(n) \rangle \right]
\]

- **Broadening:**
\[
\Delta\langle q_T^2 \rangle_{HQ}^{NRQCD} = \left( \frac{8\pi^2 \alpha_s}{N_c^2 - 1} \lambda^2 A^{1/3} \right) \frac{(C_F + C_A)\sigma_{q\bar{q}}^{(0)} + 2C_A\sigma_{gg}^{(0)} + \sigma_{q\bar{q}}^{(1)}}{\sigma_{q\bar{q}}^{(0)} + \sigma_{gg}^{(0)}}
\]

**Hard parts:**
\[
\hat{\sigma}_{q\bar{q}}^{(0)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{16}{27} \delta(\hat{s} - M^2) \langle \mathcal{O}^H(3S_1^{(8)}) \rangle
\]
\[
\hat{\sigma}_{q\bar{q}}^{(1)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{80}{27} \delta(\hat{s} - M^2) \langle \mathcal{O}^H(3P_0^{(8)}) \rangle
\]
\[
\hat{\sigma}_{gg}^{(0)} = \frac{\pi^3 \alpha_s^2}{M^3} \frac{5}{12} \delta(\hat{s} - M^2) \left[ \langle \mathcal{O}^H(1S_0^{(8)}) \rangle + \frac{7}{m_Q^2} \langle \mathcal{O}^H(3P_0^{(8)}) \rangle \right]
\]

- **Leading features:**
\[
\Delta\langle q_T^2 \rangle_{HQ}^{NRQCD} \approx \Delta\langle q_T^2 \rangle_{HQ}^{CEM} \approx (2C_A/C_F)\Delta\langle q_T^2 \rangle_{DY}
\]
Broadening of heavy quarkonia in d+A

- Final-state effect is important:
  \[ \frac{\Delta \langle q_T^2 \rangle_{J/\psi}^{(I+F)}}{\Delta \langle q_T^2 \rangle_{DY}} \bigg|_{thy} \approx 2C_A/C_F = 4.5 \]

- Mass – independence, not very sensitive to the feeddown
Broadening of quarkonia in A+A

- If no hot medium was formed:
  - broadening in AA = superposition of pA
  - $\Delta \langle p_T^2 \rangle_{AA} \propto L_{eff}$

- If hot medium is formed:
  - $\Delta \langle p_T^2 \rangle_{final} \sim 0$
  - $\Delta \langle p_T^2 \rangle_{initial} \lesssim$ superposition of $\Delta \langle p_T^2 \rangle_{pA}$

Some kind of slow moving medium was produced at RHIC! $\Delta \langle q_T^2 \rangle_{AA}$ could be as small as 0!

final-state energy loss, initial-state thermal medium?
Summary and outlook

- Heavy quarkonium provides a “non-relativistic” system, and could offer some important perspectives to the formation of QCD bound states.
- Heavy quarkonium has two intrinsic scales, and could be a good probe of QGP or other dense medium.
- But, after 30 years, since the discovery of J/ψ, we still have not been able to fully understand the production mechanism of heavy quarkonia.
- None of the factorized production models, including NRQCD model, were proved theoretically.
- RHIC is offering an excellent opportunity to learn and examine the formation of QCD bound states – nuclear matter could be an effective filter to distinguish the production models.
Backup slices
Works for other states too:

\[ \text{E. Braaten et al. Annu. Rev. Nucl. Part. Sci. 46, 197 (1996)} \]
Same problem for other states:

CDF Collaboration, PRL 2007

Braaton & Lee, PRD63, 071501 (2001)
LEP data on $J/\psi$ photo-production: $\gamma\gamma \rightarrow J/\psi + X$
Kinematically preferred configuration:

Production rate of a singlet charm quark pair is dominated by the phase space where $s_3=(P_1+P_2+P_3)^2$ or $s_4=(P_1+P_2+P_4)^2$ near its minimum.

NRQCD formalism does not apply when there are more than one heavy quark velocity involved.

Color transfer enhances associated heavy quarkonium production.

A heavy quark as a color source to enhance the transition rate for an octet pair to become a singlet pair.

Nayak, Qiu, Sterman, PRL 2007