

Introduction to QCD sum rule for heavy quark system near T_c

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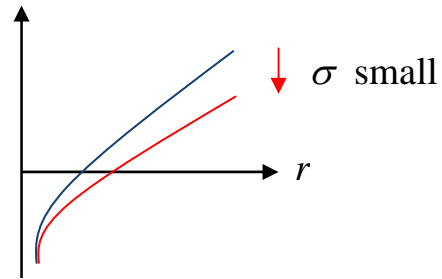
Early work on J/ψ (Hashimoto, Miyamura, Hirose, Kanki)

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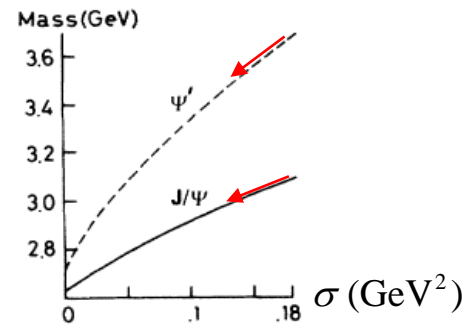
PHYSICAL REVIEW LETTERS

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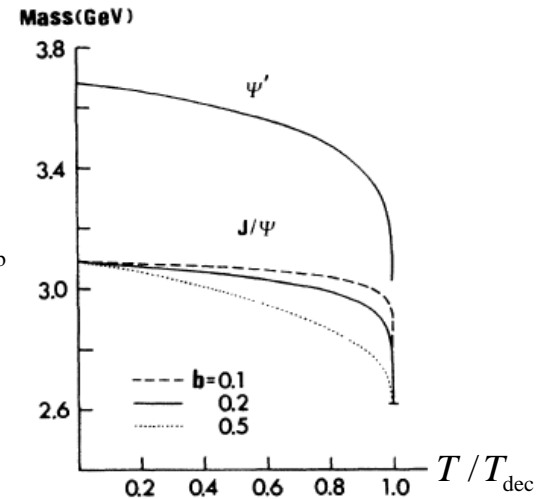
Mass Shift of Charmonium near Deconfining Temperature and Possible Detection in Lepton-Pair Production



$$V(r) = -\frac{4}{3} \frac{\alpha_s(r)}{r} + \sigma \times r$$



$$\sigma(T) = \sigma(0) \times \left[\frac{T_{\text{dec}} - T}{T_{\text{dec}}} \right]^b$$



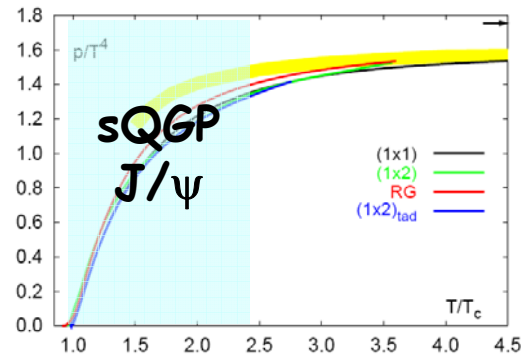
J/ ψ in Quark-gluon plasma

- Matsui and Satz: J/ ψ will dissolve at T_c due to color screening
- Lattice MEM : Asakawa, Hatsuda, Karsch, Petreczky
J/ ψ will survive up to $2 T_c$
- Potential models (Wong ...) :
Consistent with MEM Wong.
- Refined Potential models with lattice (Mocsy, Petreczky...)
: J/ ψ will dissolve slightly above T_c
- Lattice after zero mode subtraction (WHOT-QCD)
: J/ ψ wave function hardly changes at $2.3 T_c$
- AdS/QCD (Kim, Lee, Fukushima ..)
: J/ ψ mass change

Some perspectives on sQGP and relation to deconfinement

Some perspectives from Lattice data on (ε, p) near T_c

- Lattice data (Karsch et al) vs. Resummed perturbation (Blaizot et al.)

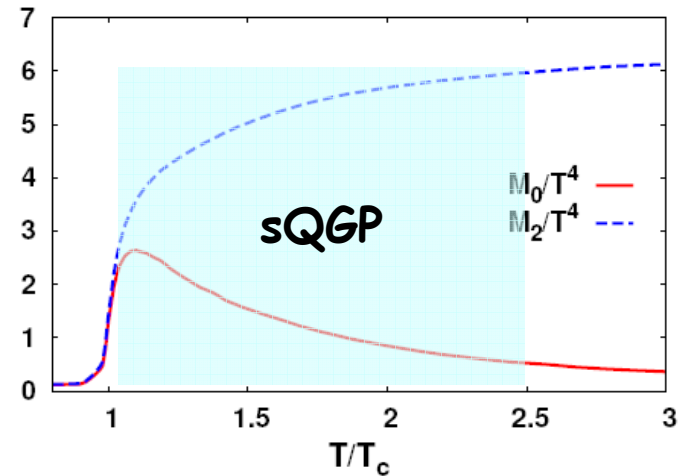


Karsch hep-lat/0106019

- Operator representation: Gluon condensates

$$M_2 = \left\langle -\text{ST}(G^{\alpha\mu} G^{\beta\mu}) \right\rangle_T = \varepsilon + p$$

$$M_0 = -\frac{11}{8} \left[\left\langle \frac{\alpha}{\pi} G^2 \right\rangle_T - \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0 \right] = \varepsilon - 3p$$



➤ M_0 and Bag pressure

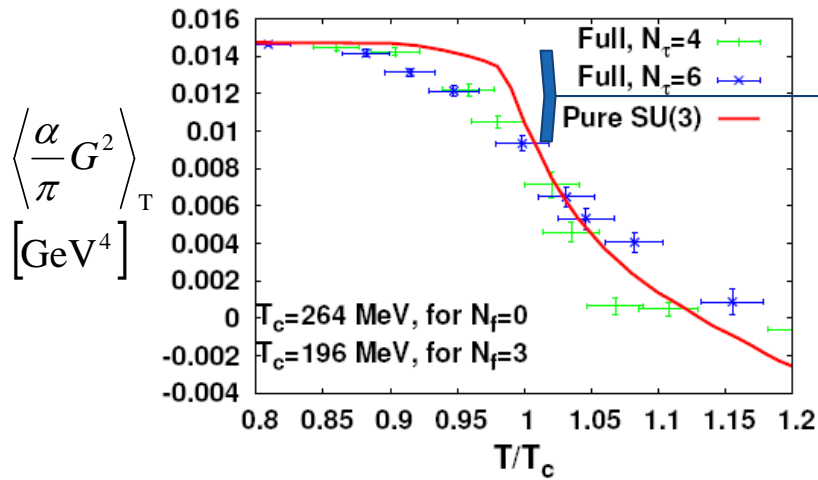
$$\varepsilon = \frac{3}{4}M_2 + \frac{1}{4}M_0$$

$$p = \frac{1}{4}M_2 - \frac{1}{4}M_0$$

$$B = \frac{1}{4}(M_0(T_c) - M_0(0))$$

$$= \frac{1}{4}\left(-\frac{9}{8}\Delta\left\langle\frac{\alpha}{\pi}G^2\right\rangle\right) \approx (189 \text{ MeV})^4$$

➤ M_0 and Gluon condensate $\left\langle\frac{\alpha}{\pi}G^2\right\rangle_T = \left\langle\frac{\alpha}{\pi}G^2\right\rangle_0 - \frac{8}{11}M_0$



Dominated by non perturbative change at T_c

$$0.3 \times \left\langle\frac{\alpha}{\pi}G^2\right\rangle_0$$

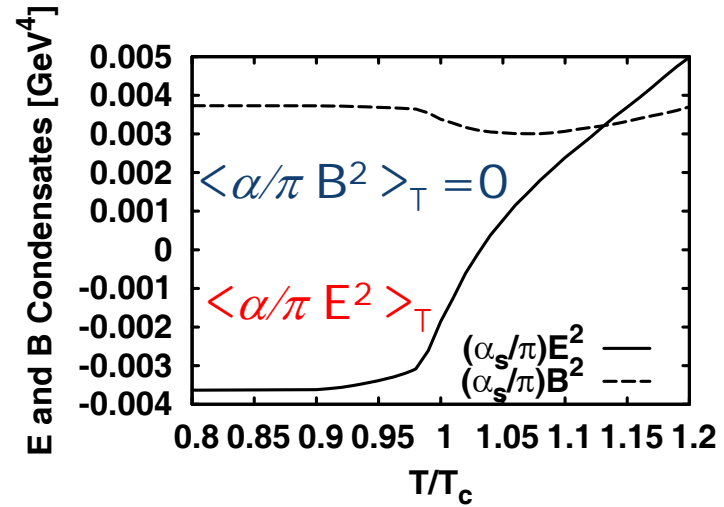
SHLee PRD40,2484(89)

➤ Relation to Electric and Magnetic condensate

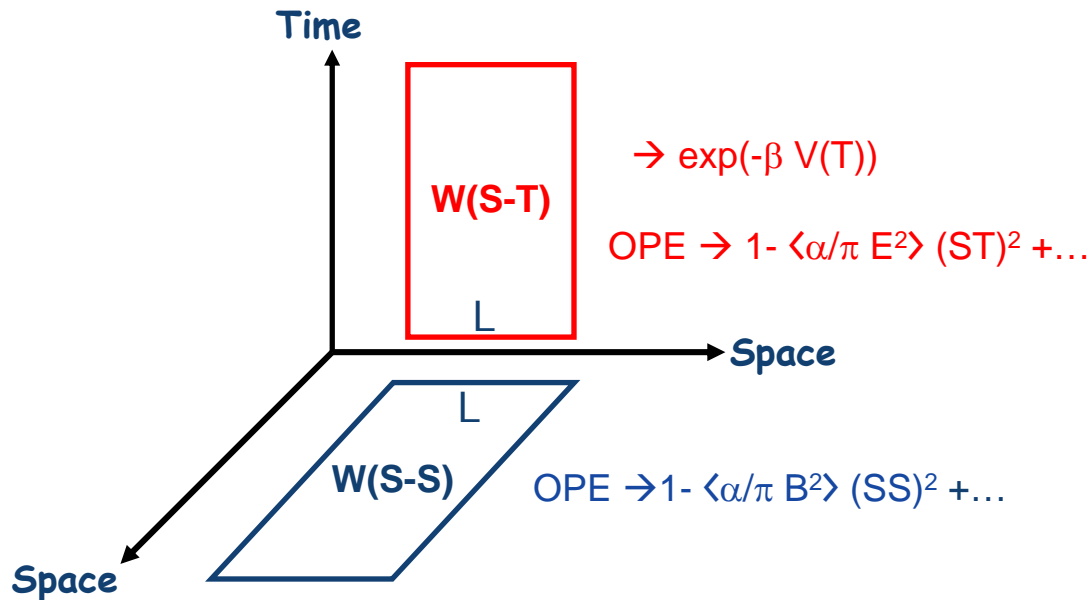
Kaczmarek et al (prd04)

$$\Delta \left\langle \frac{\alpha}{\pi} E^2 \right\rangle_T = +\frac{2}{9} M_0 + \frac{3}{4} \frac{\alpha_s(T)}{\pi} M_2$$

$$\Delta \left\langle \frac{\alpha}{\pi} B^2 \right\rangle_T = -\frac{2}{9} M_0 + \frac{3}{4} \frac{\alpha_s(T)}{\pi} M_2$$



➤ Relation to deconfinement



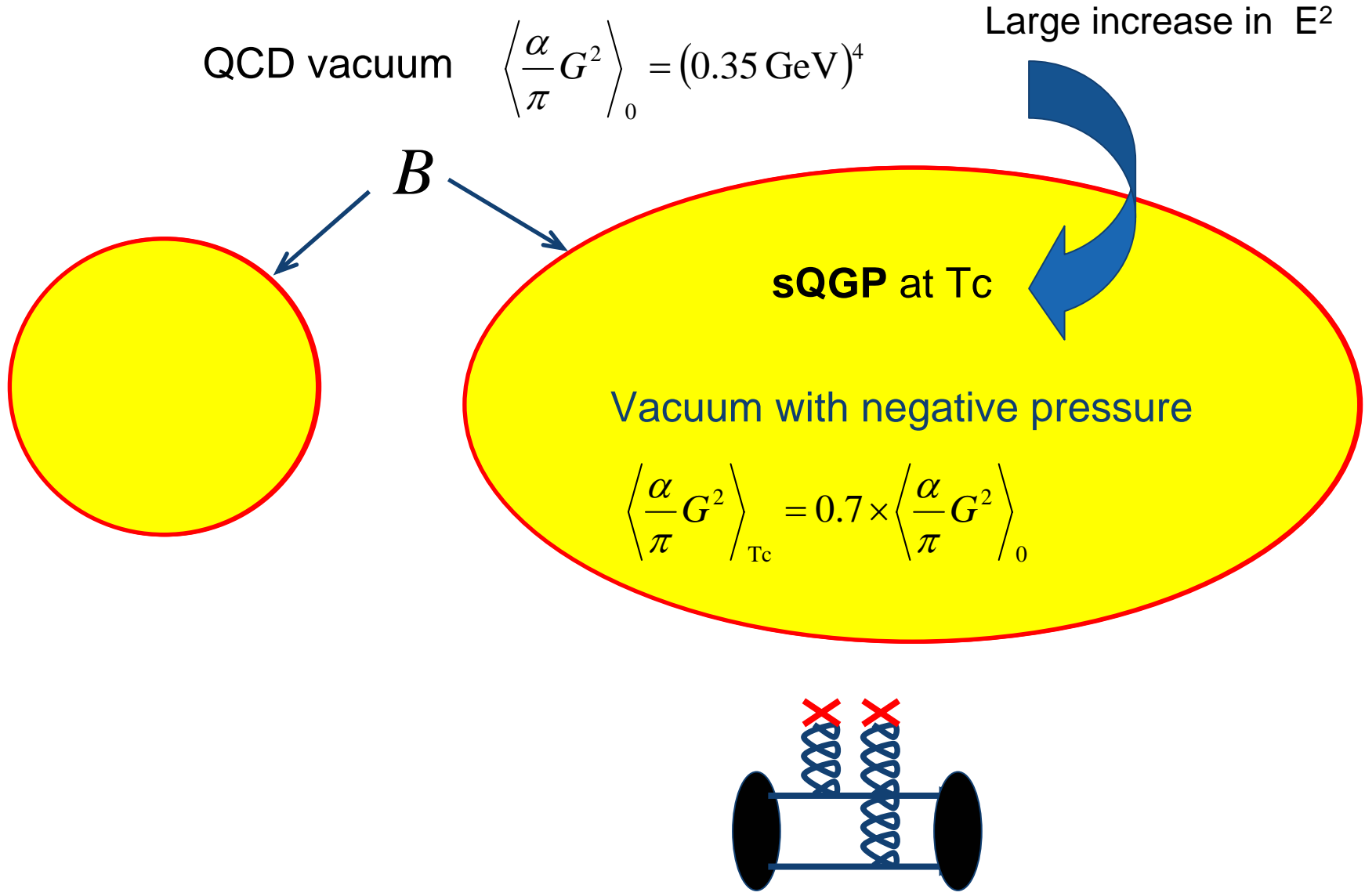
$$\Delta \langle E^2 \rangle = \int \frac{d^3k}{(2\pi)^3 E_k} \frac{(2k^2 + 3m^2)}{e^{E_k/T} - 1}$$

$$\Delta \langle B^2 \rangle = \int \frac{d^3k}{(2\pi)^3 E_k} \frac{(2k^2)}{e^{E_k/T} - 1}$$

Heavy quark system in sQGP

OPE, QCD Stark Effect, and
QCD sum rules

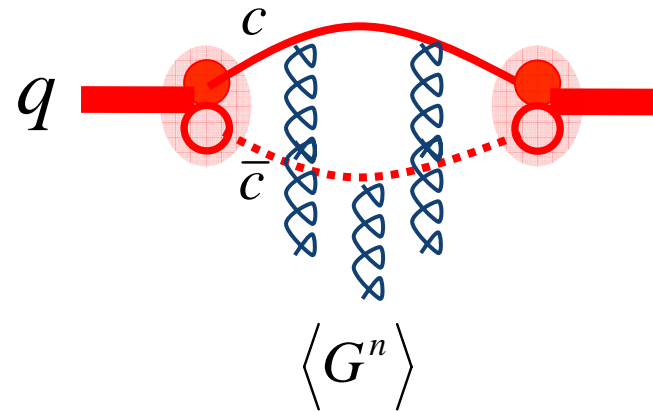
Heavy quark system near T_c



Heavy quark correlation function $\Pi(q^2)$

➤ Definition

$$\Pi(q) = \int dx e^{iqx} \langle \bar{c}c(x), \bar{c}c(0) \rangle$$



➤ Operator product expansion (OPE)

$$\Pi(q) = \dots + \int_0^1 dx \frac{F(q^2, x)}{(4m^2 - q^2 + (2x-1)^2 q^2)^n} \cdot \langle G^n \rangle + \dots$$

- OPE makes sense when $4m^2 - q^2 \gg \langle G \rangle_{\text{vacuum}} = \Lambda_{QCD}^2$

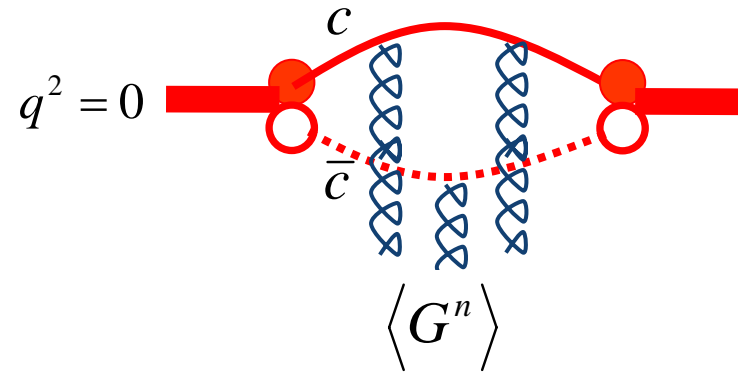
- Even at finite temperature or as long as

$$4m^2 - q^2 \gg \langle G \rangle_{\text{medium}} = (\Lambda_{QCD} + aT)^2$$

- $q^2=0$: photo production of open charm

$$4m^2 \gg \Lambda_{QCD}^2$$

$$\Pi(q^2 = 0) = \sum \frac{C_n}{(4m^2)^n} \langle G^n \rangle$$

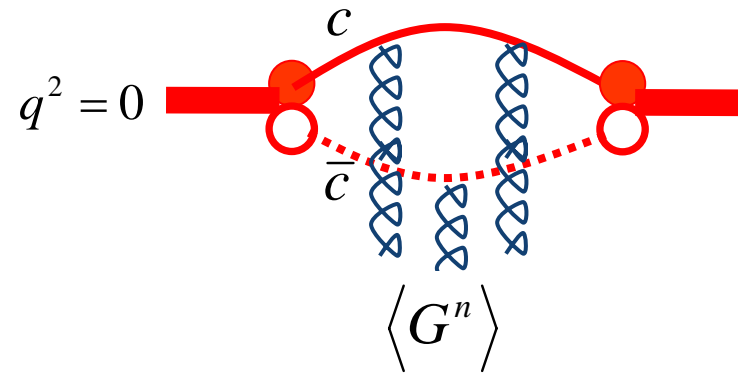


- $q^2=m_{J/\psi}^2$: OPE for bound state (Peskin 79)

$$4m^2 - q^2 \approx 2m \varepsilon \gg \Lambda_{QCD}^2$$

- $-q^2 > 0$: QCD sum rules for heavy quarks

$$4m^2 + Q^2 \gg \Lambda_{QCD}^2$$



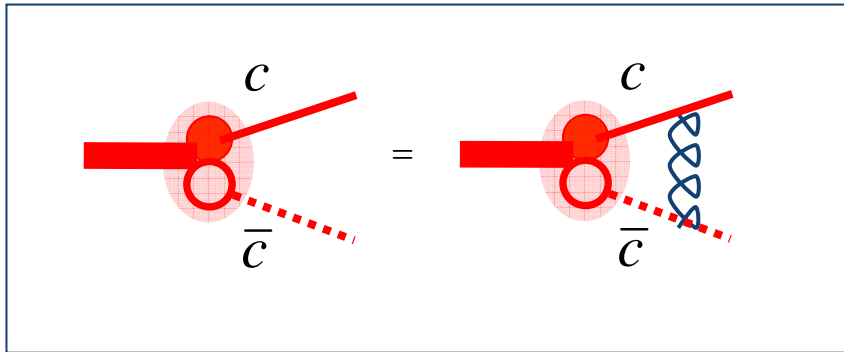
➤ $q^2 = m_{J/\psi}^2$: OPE for bound state (Peskin 79)

$$4m^2 - q^2 \approx 2m \varepsilon \gg \Lambda_{QCD}^2$$

QCD 2nd order Stark Effect : $\varepsilon > \Lambda_{\text{qcd}}$

- OPE for bound state: $m \rightarrow \text{infinity}$

$$\varepsilon_0 = m \left(N_c g^2 / 16\pi \right)^2 \rightarrow O(mg^4), \quad |\vec{k}| \rightarrow O(mg^2)$$



$$g^2 \frac{mg^4 (mg^2)^3}{(mg^4)(mg^4)(mg^2)^2} \rightarrow O(1)$$

- Attractive for ground state

$$\Delta M_i = \sum_{\bar{n}} \frac{|\langle i | z E | \bar{n} \rangle|^2}{E_i - E_{\bar{n}}}$$

$$\Delta m_{J/\psi} = -\frac{128}{9\pi^2} \frac{a_0^2}{\varepsilon_0} \int dx \frac{x^{3/2}}{(1+x)^6} \frac{1}{x + a_0^2 \varepsilon m} \times \left\langle \frac{\alpha}{\pi} E^2 \right\rangle_T$$

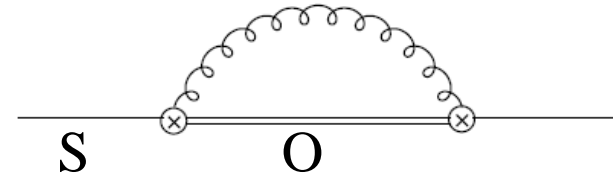
T/T _c	1.0	1.05	ε_0
$\Delta m_{J/\psi}$	-44 MeV	-105 MeV	311 MeV
Δm_Y	-4.3 MeV	-10 MeV	580 MeV

2nd order Stark effect from pNRQCD

- LO Singlet potential from pNRQCD : Brambilla et al. 1/r > Binding > Λ_{QCD} ,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^a F^{\alpha\mu\nu} + \sum_{i=1}^{n_f} \bar{q}_i i\not{D}q_i + \int d^3r \text{Tr} \left\{ S^\dagger \left[i\partial_0 + C_F \frac{\alpha_{V_s}}{r} \right] S + O^\dagger \left[iD_0 - \frac{1}{2N_c} \frac{\alpha_{V_o}}{r} \right] O \right\} \\ + V_A \text{Tr} \left\{ O^\dagger \vec{r} \cdot g\vec{E} S + S^\dagger \vec{r} \cdot g\vec{E} O \right\} + \frac{V_B}{2} \text{Tr} \left\{ O^\dagger \vec{r} \cdot g\vec{E} O + O^\dagger O \vec{r} \cdot g\vec{E} \right\} + \dots \quad (55)$$

$$[\delta V_S(r)]_{11} = -ig^2 \frac{T_F}{N_c} \frac{r^2}{d-1} \int_0^\infty dt e^{-it\Delta V} \left[\left\langle \vec{E}^a(t) \phi(t,0)_{ab} \vec{E}^b(0) \right\rangle_T \right]_{11},$$



➤ Derivation

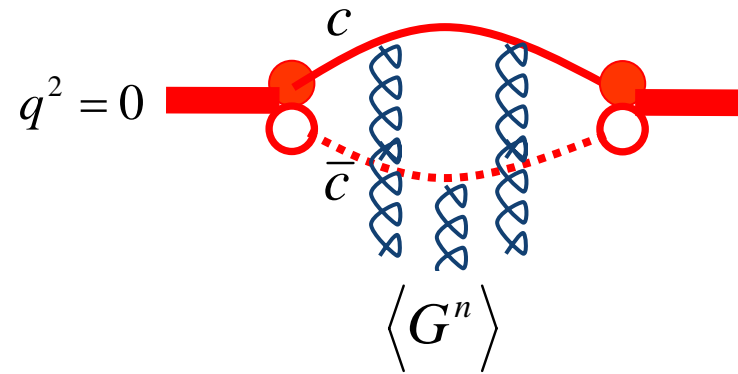
• Take expectation value $\delta E_{J/\psi} = -ig^2 \frac{T_F}{N_c} \frac{-i}{d-1} \int \frac{d^3p}{(2\pi)^3} \frac{r^2}{E_O - E_{J/\psi}} |\psi(p)|^2 \langle E^a(0) E^a(0) \rangle_T$

• Large N_c limit $\Delta V = \frac{1}{r} \left(\frac{\alpha_{V_o}}{2N_c} + C_F \alpha_{V_s} \right) \approx \frac{N_c \alpha_s}{2r}$

• Static condensate $\left[\left\langle \vec{E}^a(t) \phi(t,0)_{ab} \vec{E}^b(0) \right\rangle_T \right]_{11} \rightarrow \left[\left\langle \vec{E}^a(0) \vec{E}^a(0) \right\rangle_T \right]_{11}$

• Energy $E_{J/\psi} = 2m_c - \epsilon$
 $E_O = 2m_c + p^2/m_c,$

• $\rightarrow = -\frac{1}{18} \int_0^\infty dk^2 \left| \frac{\partial \psi(k)}{\partial k} \right|^2 \frac{k}{k^2/m_h + \epsilon_0} \left\langle \frac{\alpha_s}{\pi} \Delta E^2 \right\rangle_T$



- $-q^2 > 0$: QCD sum rules for heavy quarks

$$4m^2 + Q^2 \gg \Lambda_{QCD}^2$$

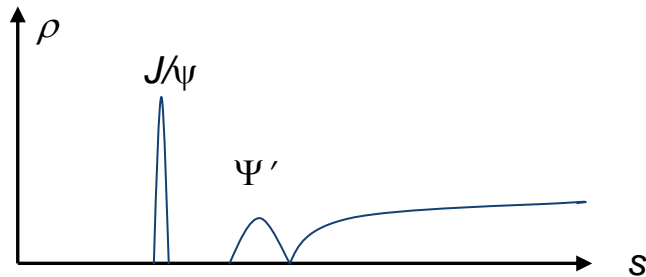
$Q^2 = -q^2 > 0$, QCD sum rules for Heavy quark system

- sum rule at $T=0$: can take any $Q^2 \geq 0$, $4m^2 + Q^2 \gg \langle G \rangle_{\text{vacuum}} = \Lambda_{\text{QCD}}^2$

$$M_n = \left(\frac{d}{dQ^2} \right)^n \langle J(Q), J(0) \rangle = \int ds \frac{\rho(s)}{(s+Q^2)^n}$$

Phenomenological side

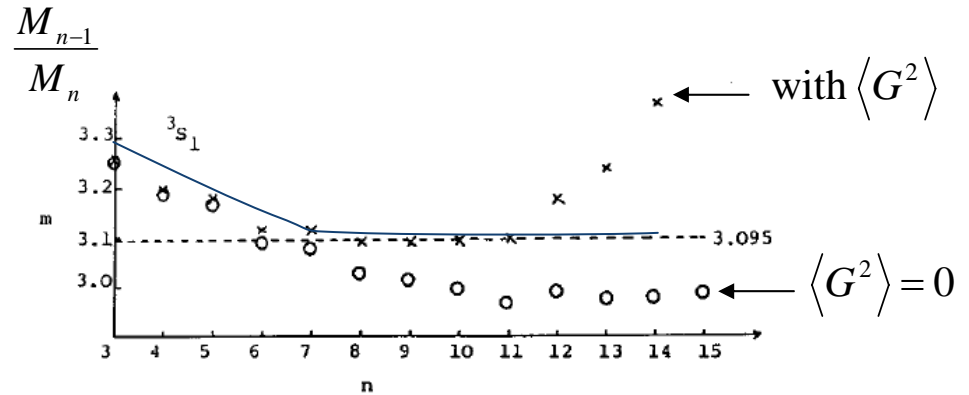
OPE



$$M_n = \frac{1}{(m_{J/\psi}^2)^n} \left(f_{J/\psi} + c \left(\frac{m_{J/\psi}^2}{m_{\psi'}^2} \right)^n \dots \right)$$

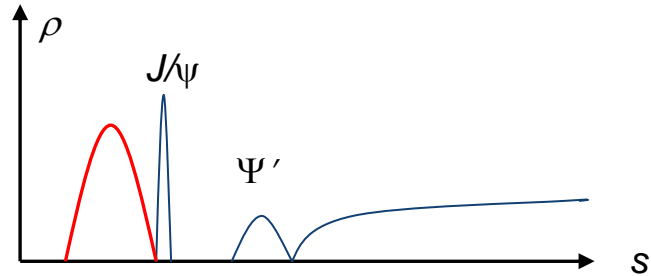
$$\frac{M_{n-1}}{M_n} = m_{J/\psi}^2 + \frac{(m_{\psi'}^2 - m_{J/\psi}^2)}{f_{J/\psi}} \left(\frac{m_{J/\psi}^2}{m_{\psi'}^2} \right)^n$$

$$M_n = a_n \left(1 + \alpha + \frac{(n+4)!}{n!} \frac{\langle G^2 \rangle}{(4m_c^2)^2} \dots \right)$$



➤ sum rule near T_c $4m^2 + Q^2 \gg \langle G \rangle_0 + \Delta \langle G \rangle_T$

Phenomenological side

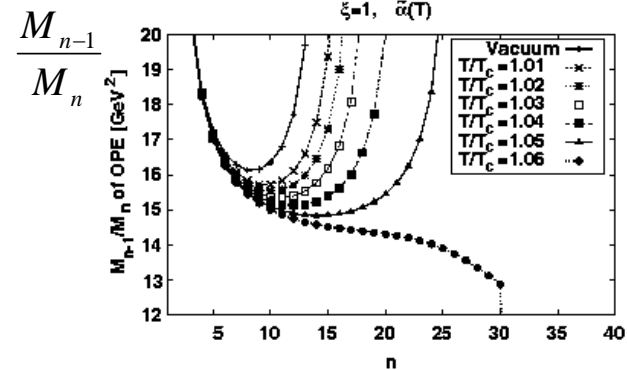
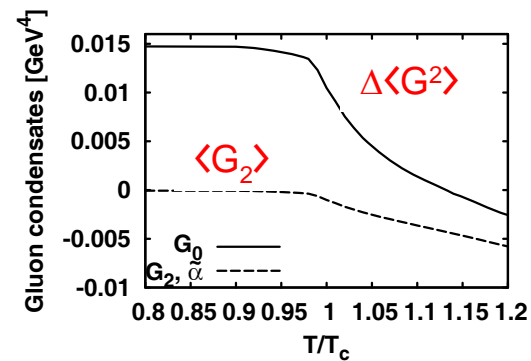


$$\rho(s) = \frac{f \sqrt{s} \Gamma}{(s - m_{J/\psi}^2)^2 + s \Gamma^2}$$

$$M_n = \int ds \frac{\rho(s)}{(s + Q^2)^n}$$

OPE

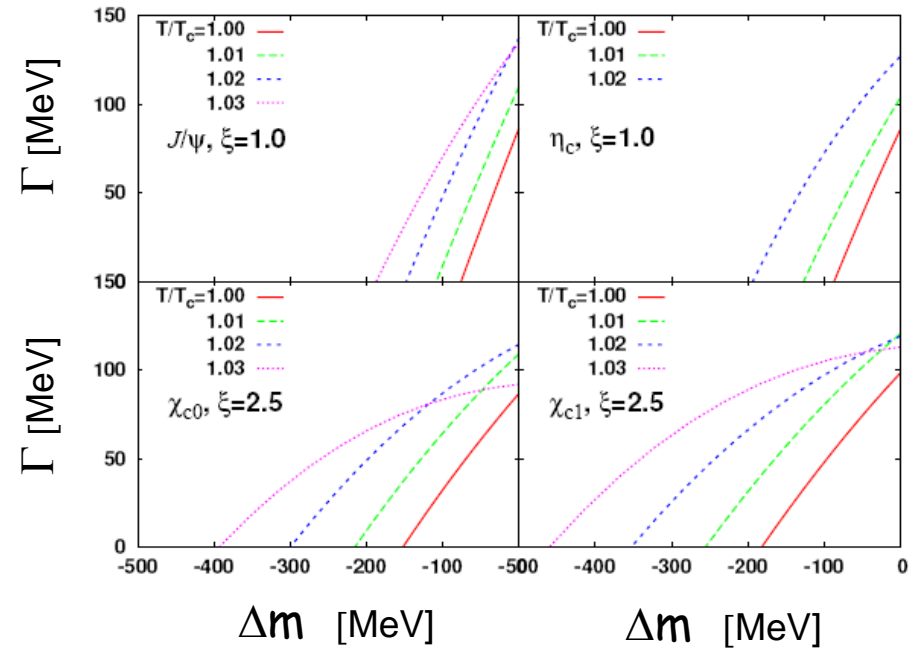
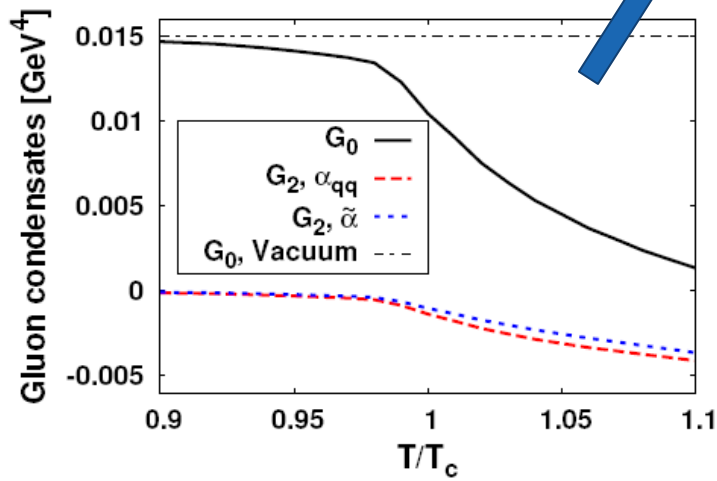
$$M_n = a_n \left(1 + \alpha + \frac{(n+4)!}{n!} \frac{\Delta \langle G^2 \rangle + c \langle G_2 \rangle}{(4m_c^2)^2} \dots \right)$$



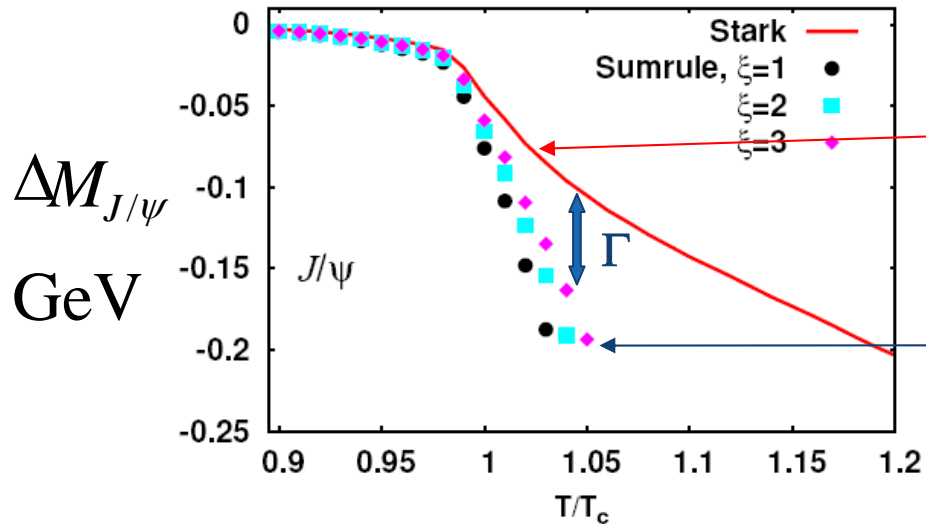
Matching M_{n-1}/M_n from Phen to OPE
 → Obtain constraint for $\Delta m_{J/\psi}$ and Γ

QCD sum rule constraint (Morita, Lee 08)

$$\frac{1}{n!} \left(-\frac{d}{dQ^2} \right)^n \Pi(Q^2) = \int ds \frac{\rho(s)}{(s+Q^2)^{n+1}}$$

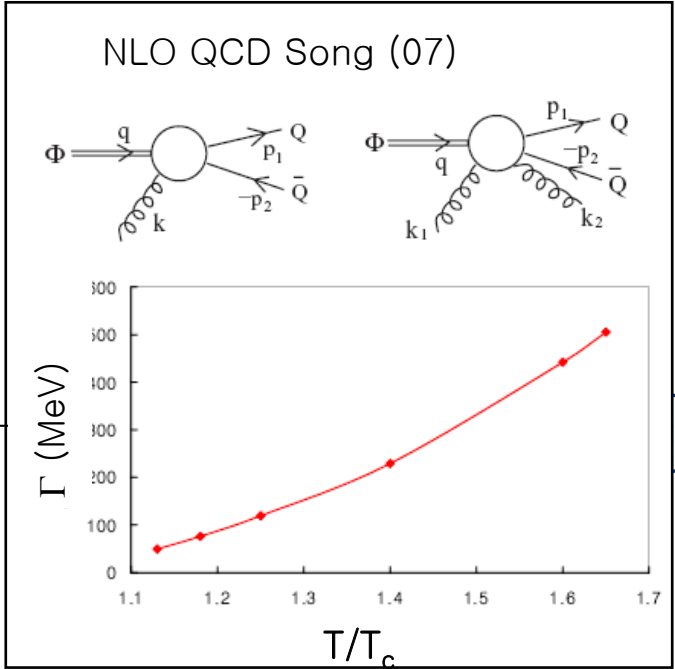
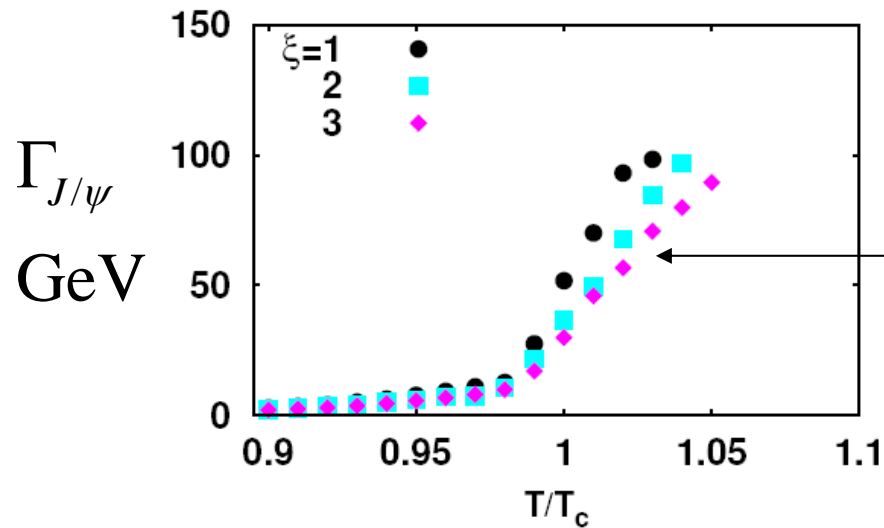


Mass and width of J/ψ near T_c (Morita, Lee 08)



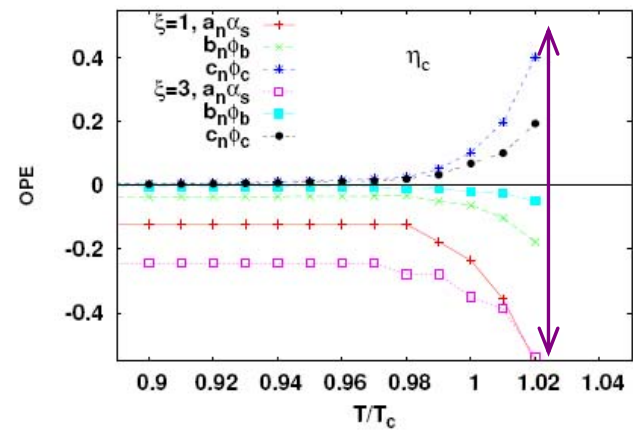
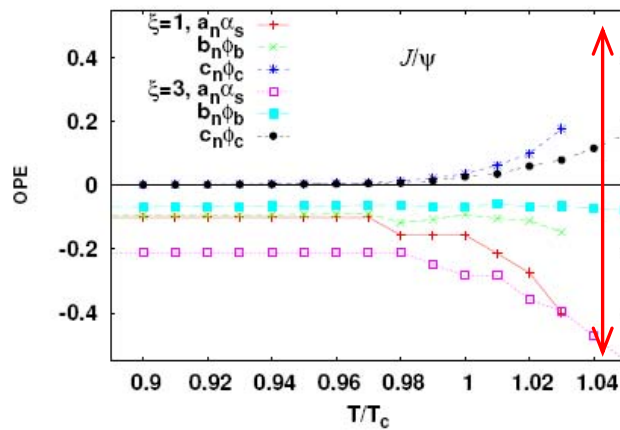
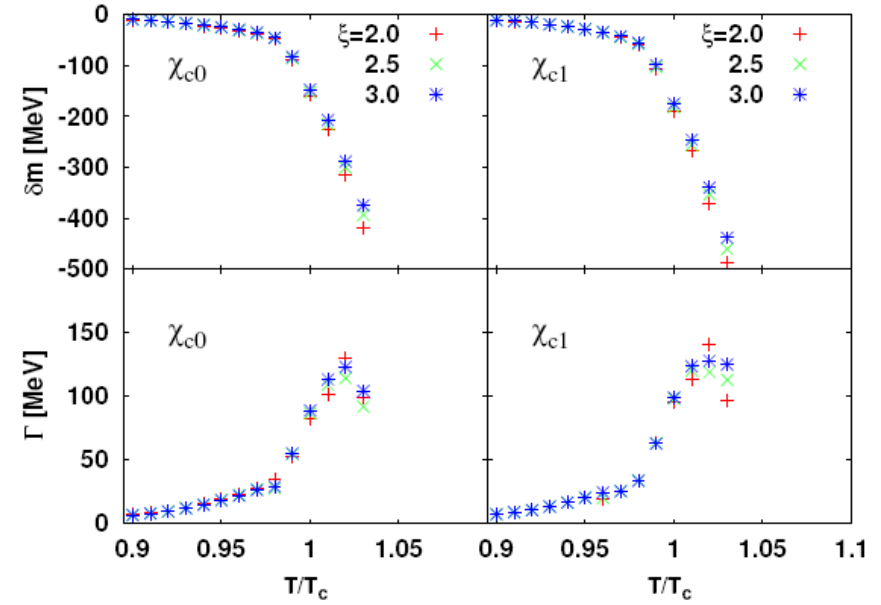
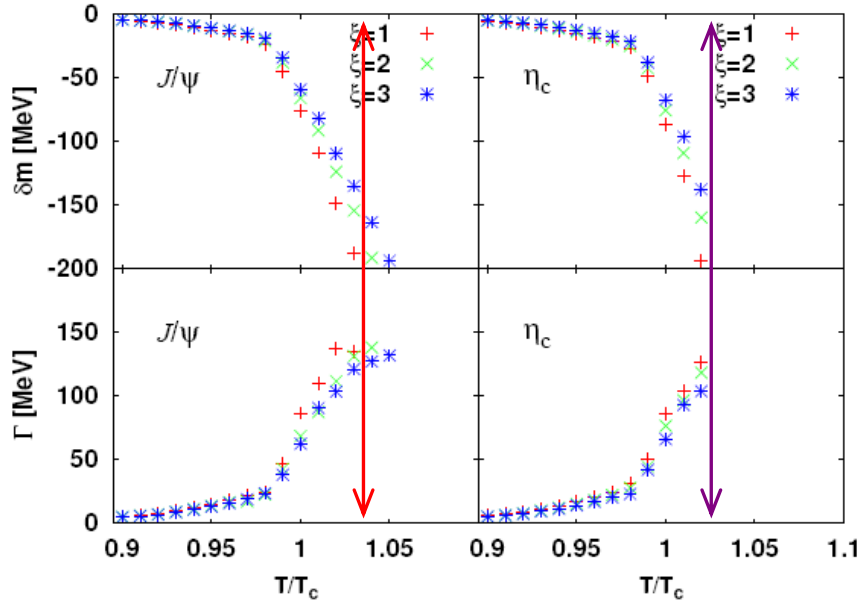
Δm from QCD Stark Effect

QCD sum rule limit with $\Delta\Gamma = 0$

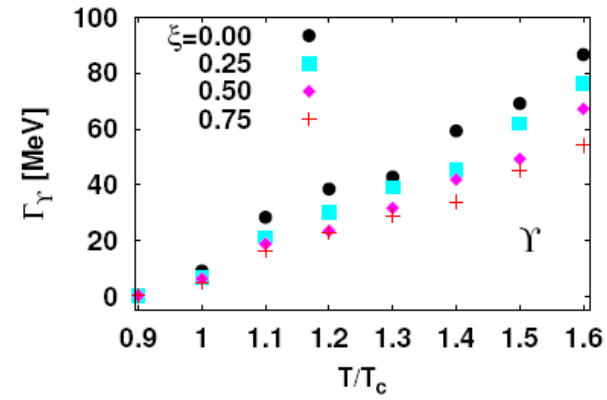
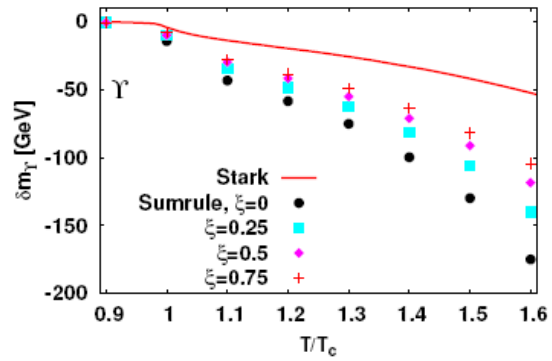


S-wave vs. P-waves

OPE breaks down



bb system



$$-\delta m + \frac{4}{3}\Gamma \simeq 10 + 200 \left(\frac{T}{T_c} - 1 \right) \text{ MeV, for S-waves}$$

$$-\delta m + 4\Gamma \simeq 40 + 800 \left(\frac{T}{T_c} - 1 \right) \text{ MeV, for P-waves}$$

Experimental observation from RHIC is difficult at present

1. **Expected mass shift for J/ψ is order 50 MeV at T_c**
2. **Larger effect for excited states**
3. **Small effect for Y but larger χ_b**

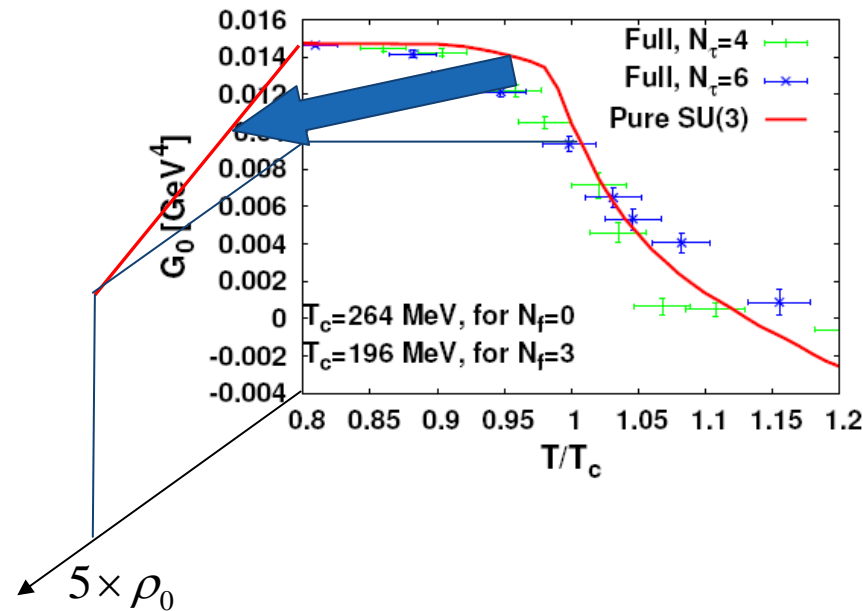
Gluon condensate in nuclear matter

- Linear density approximation

$$\langle \text{Op} \rangle_{\text{n.m.}} = \langle \text{Op} \rangle_0 + \frac{\rho_N}{2m_N} \langle \text{N} | \text{Op} | \text{N} \rangle, \quad M_0 \propto \langle \text{N} | \text{T}_\mu^\mu(\text{Chiral}) | \text{N} \rangle = m_N^0 \rightarrow 750 \text{ MeV}$$

$$M_2 \propto m_N \int dx x G(x, \mu^2) \rightarrow 0.45 m_N$$

- Gluon condensate at finite density $\left\langle \frac{\alpha}{\pi} G^2 \right\rangle = \left\langle \frac{\alpha}{\pi} G^2 \right\rangle_0 \left(1 - 0.061 \frac{\rho}{\rho_{\text{n.m.}}} \right)$

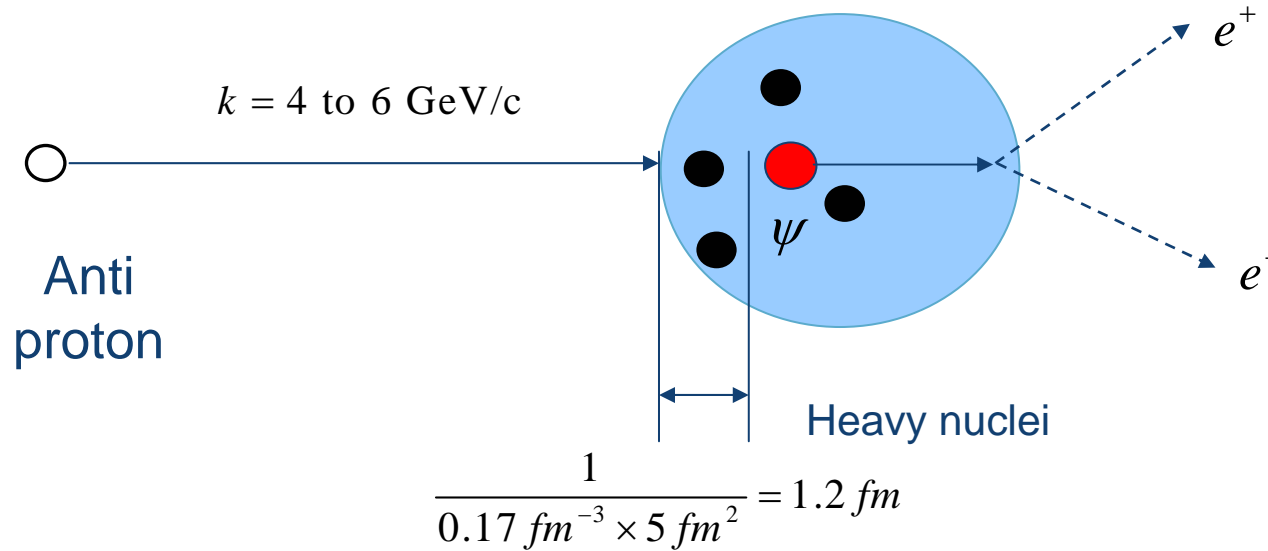


RHIC energy scan
FAIR

Other approaches for mass shift in nuclear matter

	Quantum numbers	QCD 2 nd Stark eff.	Potential model	QCD sum rules	Effects of DD loop
η_c	0^{-+}	-8 MeV		-5 MeV (Klingl, SHL, Weise, Morita)	No effect
J/ψ	1^{--}	-8 MeV (Peskin, Luke)	-10 MeV (Brodsky et al).	-7 MeV (Klingl, SHL, Weise, Morita)	< 2 MeV (SHL, Ko)
χ_c	$0, 1, 2^{++}$	-40 MeV		-15 MeV (Morita, Lee)	No effect on χ_{c1}
$\psi(3686)$	1^{--}	-100 MeV			< 30 MeV
$\psi(3770)$	1^{--}	-140 MeV			< 30 MeV

Observation of Δm through \bar{p} -A reaction



Can be done at J-PARC

Table 2: Summary of parameters and resultant cross sections.

	J/ψ	η_c	χ_{c0}	χ_{c1}	χ_{c2}
m [MeV]	3097	2980	3415	3511	3556
δm [MeV]	-7	-4	-15	-15	-15
Γ_{tot} [MeV]	0.0934	25.5	10.4	0.89	2.05
Final State	e^+e^-	$\gamma\gamma$	$J/\psi\gamma$	$J/\psi\gamma$	$J/\psi\gamma$
$\langle\sigma_{\text{BW}}\rangle_{\text{peak}}$ [pb]	0.435	10.7	17.0	4.25	18.8
Events/day	7.5	184	294	74	326

Expected luminosity at GSI $2 \times 10^{32} \text{ cm}^{-2} \text{ s}^{-1}$ \longrightarrow

Summary

1. Properties of $Q\bar{Q}$ system in sQGP is still controversial
2. The mass and width will suddenly change at $T_c \rightarrow$ different for s p wave and bottonium \rightarrow can probe confinement physics
3. Partial observation at nuclear matter through $\bar{p} A$ reaction might be possible.
FAIR, J-PARC
4. A new constraint for heavy quark system near T_c
5. \rightarrow to be continued by K. Morita