Insight on quarkonium from the weak-coupling expansion

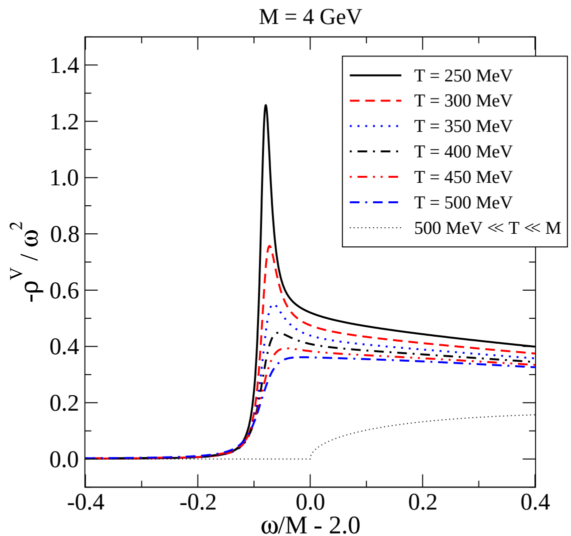
Mikko Laine
University of Bielefeld
Germany

(i) general *qualitative* patterns;

(ii) *quantitative* predictions for the bottomonium system;

(iii) appendix: what changes *out-of-equilibrium*?

Consider spectral function in the vector channel, in full thermal equilibrium, with $Q \equiv (\omega, \mathbf{0})$:

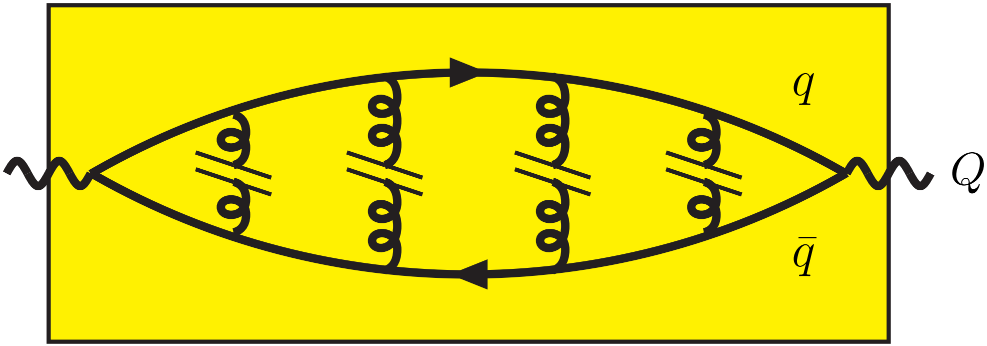


ML 0704.1720

What is the physics responsible for the broadening / disappearance of the quarkonium peak?

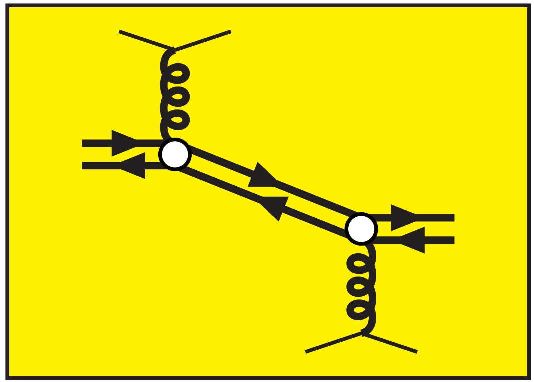
(a) Debye screening of the electric field binding together the quark and antiquark.

Matsui Satz 1986



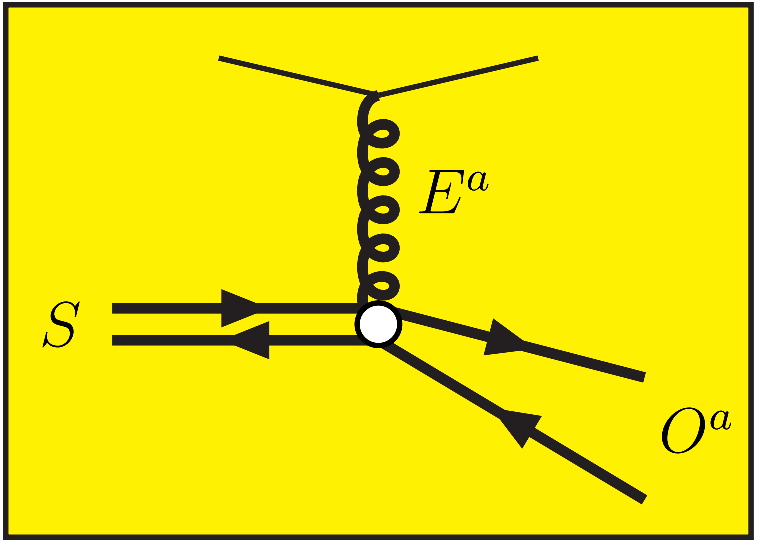
This is "classical" QED physics — certainly there, and cannot be "cancelled" by anything else.

(b) "Collisional broadening" (momentum transfer) due to hard particles in the plasma.



Again "classical" QED physics — bound state is charge-neutral but has a finite size and hence electric dipole moment; it gets kicked / experiences drag.

(c) Genuine QCD effects: $\mathbf{3} \otimes \mathbf{3}^* = \mathbf{1} \oplus \mathbf{8}$.



Color transfer in addition to **momentum** transfer!

How does Debye screening work? $(\alpha_s = \frac{g^2}{4\pi} \ll 1)$

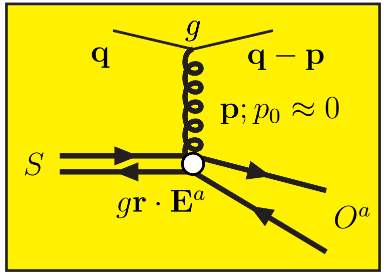
Bound state radius like Bohr for hydrogen: $r \sim \frac{1}{g^2 M}$.

Debye mass for a non-Abelian theory: $m_{\rm D} \sim gT$.

Dissociation takes place when Coloumb-potential is screened, i.e. $g^2 \exp(-m_{\rm D} r)/4\pi r \ll g^2/4\pi r \Rightarrow$

$$r m_{\rm D} \sim 1 \quad \Rightarrow \quad \frac{gT}{g^2 M} \sim 1 \quad \Rightarrow \quad \boxed{T \sim gM}\,.$$

How does momentum/color transfer work?



$$\Gamma \sim \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{g^4(\mathbf{p}\cdot\mathbf{r})^2}{(p^2+m_\mathrm{D}^2)^2} \underbrace{\int \frac{\mathrm{d}^3\mathbf{q}}{(2\pi)^3}\, \delta((\mathbf{q}-\mathbf{p})^2-\mathbf{q}^2)\, q\, n_\mathrm{B}(q)[1+n_\mathrm{B}(q)]}_{\sim T^3/p}$$

$$\sim g^4 T^3 r^2 \ln\frac{1}{g} \; .$$

from $\mathrm{Im}\, V_{\mathrm{QCD}}$: ML et al hep-ph/0611300

from $\mathrm{Im}\, V_{\mathrm{QED}}$: Beraudo et al 0712.4394

from PNRQED: Escobedo Soto 0804.0691

from PNRQCD: Brambilla et al 0804.0993

The width equals (unscreened!) binding energy for

$$g^4 T^3 r^2 \sim \frac{g^2}{r}$$

$$g^4 T^3 \frac{1}{g^4 M^2} \sim g^4 M$$

$$\boxed{T \sim g^{4/3} M < gM}$$

Burnier et al 0711.1743: $g^2 M < T < gM$

Escobedo Soto 0804.0691: $T \sim g^{4/3} M$

ML 0810.1112; Dominguez Wu 0811.1058: $T \sim g^{\frac{4}{3}} (\ln \frac{1}{g})^{-\frac{1}{3}} M$

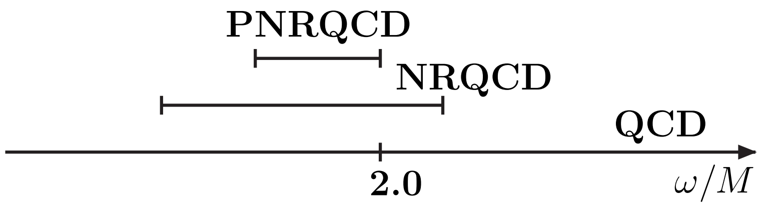
Intermediate summary

Debye screening is certainly a correct and **conservative** way to dissociate quarkonium ...

... but at least in the weak-coupling limit it does not appear to be the only relevant process.

Move now to the quantitative level

Want to use effective field theories in order to resum the perturbative series:



It is important to keep a controlled contact to QCD.

More specifically, the effective description involves an effective Lagrangian,

$$\mathcal{L}_{\text{QCD}} \longrightarrow \mathcal{L}_{\text{PNRQCD}} \ ,$$

At finite T: Escobedo Soto 0804.0691; Brambilla et al 0804.0993

but also a relation of the composite operators describing the physical currents,

$$\left[\bar{\psi}\gamma^k\psi\right]_{\text{QCD}} = \mathcal{Z}_V \times \left[\theta^\dagger\sigma_k\phi + \phi^\dagger\sigma_k\theta\right]_{\text{PNRQCD}} \ .$$

At zero T: e.g. Beneke et al 0706.2733

To determine the normalization factor $\mathcal{Z}_V$, let us compute ρ_V directly in QCD, and match to the resummed result at the edge of the resonance region!

Some fascinating history here...

The 2-loop $T = 0$ result is a classic: G. Källén and A. Sabry, "Fourth order vacuum polarization," Kong. Dan. Vid. Sel. Mat. Fys. Med. 29N17 (1955) 1.

Over 50 years later, still no full 3-loop result available!

(On the other hand Taylor expansions in ω^2/M^2; $(\omega^2 - 4M^2)/4M^2$; or M^2/ω^2 are known up to ≥ 4 loops.)

We updated 2-loop to finite temperature (for $T \ll M$.)

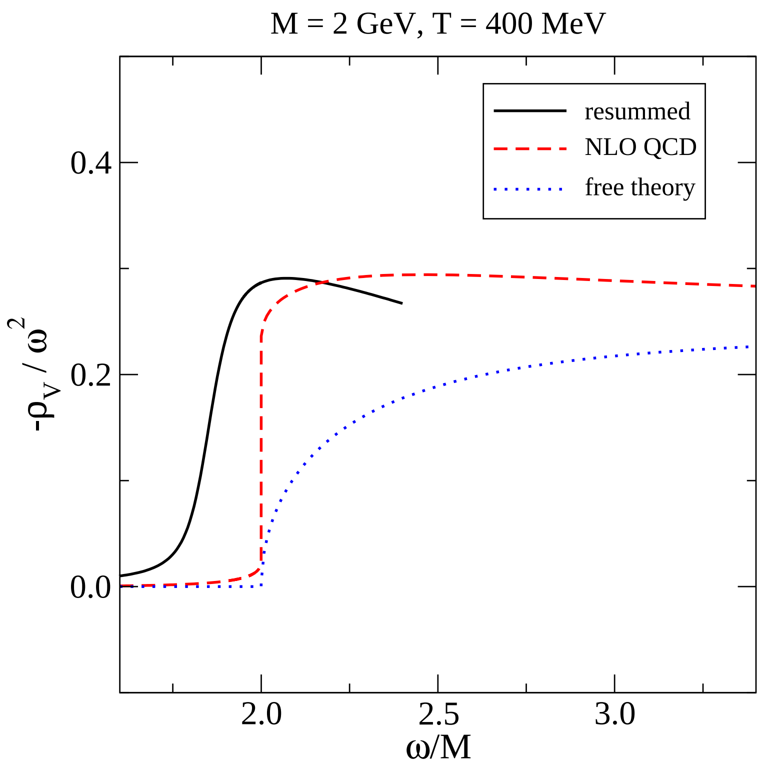
Burnier et al 0812.2105

After 1y of work, result for the thermal correction:

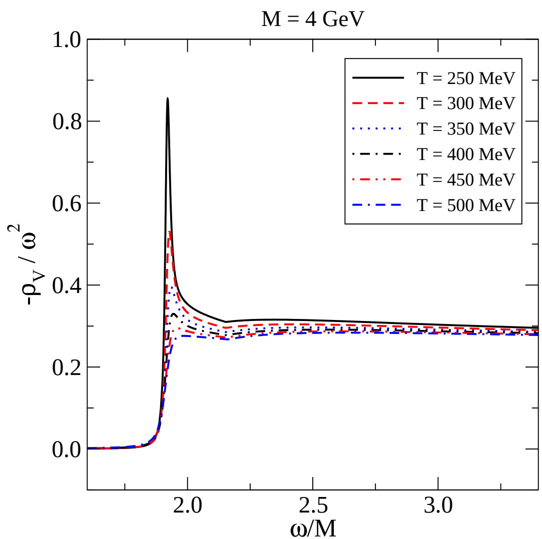
$$
\begin{aligned}
\rho_V(\omega)|^T \;=\; & \frac{8g^2 C_A C_F}{(4\pi)^3\omega^2}\int_0^\infty \mathrm{d}k\,\frac{n_{\mathrm{B}}(k)}{k}\Bigg\{ \\
& \theta(\omega)\,\theta\Big(k-\frac{4M^2-\omega^2}{2\omega}\Big)\Bigg[2\omega^2k^2\sqrt{1-\frac{4M^2}{\omega(\omega+2k)}} \\
& \quad +(\omega^2+2M^2)\sqrt{\omega(\omega+2k)}\sqrt{\omega(\omega+2k)-4M^2} \\
& \quad -2\Big(\omega^4-4M^4+2\omega k(\omega^2+2M^2)+2\omega^2k^2\Big)\,\mathrm{acosh}\sqrt{\frac{\omega(\omega+2k)}{4M^2}}\Bigg] \\
+\; & \theta(\omega-2M)\,\theta\Big(\frac{\omega^2-4M^2}{2\omega}-k\Big)\Bigg[2\omega^2k^2\sqrt{1-\frac{4M^2}{\omega(\omega-2k)}} \\
& \quad +(\omega^2+2M^2)\sqrt{\omega(\omega-2k)}\sqrt{\omega(\omega-2k)-4M^2} \\
& \quad -2\Big(\omega^4-4M^4-2\omega k(\omega^2+2M^2)+2\omega^2k^2\Big)\,\mathrm{acosh}\sqrt{\frac{\omega(\omega-2k)}{4M^2}}\Bigg] \\
+\; & \theta(\omega-2M)\Bigg[-2(\omega^2+2M^2)\,\omega\sqrt{\omega^2-4M^2} \\
& \quad +4\Big(\omega^4-4M^4+2\omega^2k^2\Big)\,\mathrm{acosh}\Big(\frac{\omega}{2M}\Big)\Bigg]\Bigg\}+\mathcal{O}(e^{-\beta M},g^4)\;.
\end{aligned}
$$

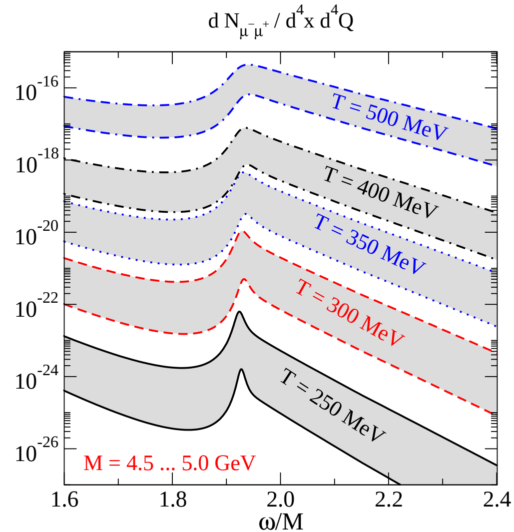
2h after appearance, limit for $\omega \gg M$ crosschecked by S. Caron-Huot 0903.3958.

Matching of QCD and resummed ($\sim$ PNRQCD) results, after tuning of $\mathcal{Z}_V$:



Final result for the spectral function and dilepton rate (in full equilibrium):





Burnier et al 0812.2105

Note that the absolute magnitude of the dilepton rate appears to be a more sensitive thermometer than the line shape around threshold!

Conclusions

Weak-coupling computations can yield both qualitative insights **and** quantitative results on hot QCD.

Appendix A: What changes out of equilibrium?

$$\frac{1}{\exp(\frac{p}{T}) - 1} \longrightarrow \frac{1}{\exp(\frac{\sqrt{p^2+\xi p_z^2}}{T'}) - 1}$$

where $\xi > 0$ characterizes a Bjorken expansion induced anisotropy ($p_z =$ momentum component along beam axis).

Romatschke Strickland hep-ph/0304092

Early times (?): T' is just a parameter with no connection to a physical "temperature"; all modes anisotropic.

Late times (?): T' close to the physical temperature T; strongly interacting soft modes thermalized but weakly interacting hard modes remain anisotropic.

For the late-time situation we find:

$$T \sim g^{\frac{4}{3}}(\ln \tfrac{1}{g})^{-\frac{1}{3}} M \times \left(\frac{s_{\text{eq}}}{s_{\text{non-eq}}} \right)^{2/9}$$

Burnier ML Vepsäläinen 0903.3467

For the early-time case no “temperature” exists; yet anisotropy decreases Γ so binding is stronger.

Dumitru Guo Strickland 0903.4703

So, in an ideal world, a quantitative discrepancy between lattice prediction and experimental observation could point towards non-equilibrium phenomena.

Appendix B: Main definitions

$$\rho_V(Q) \equiv \int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^3\mathbf{x}\, e^{iQ\cdot x} \left\langle \frac{1}{2}[\hat{\mathcal{J}}^{\mu}(x), \hat{\mathcal{J}}_{\mu}(0)] \right\rangle \ ,$$

$$\hat{\mathcal{J}}^{\mu} \equiv \hat{\bar{\psi}}\, \gamma^{\mu}\, \hat{\psi} \ , \quad \langle \ldots \rangle \equiv \frac{1}{\mathcal{Z}} \mathrm{Tr}\left[(\ldots) e^{-\beta \hat{H}} \right] \ , \quad \beta \equiv \frac{1}{T} \ ,$$

$$\frac{\mathrm{d}N_{\mu^+\mu^-}}{\mathrm{d}^4x\,\mathrm{d}^4Q} = \frac{-2e^4 Z^2}{3(2\pi)^5 Q^2} \Big(1 + \frac{2m_\mu^2}{Q^2}\Big) \Big(1 - \frac{4m_\mu^2}{Q^2}\Big)^{\frac{1}{2}} n_{\mathrm{B}}(q^0) \rho_V(Q) \ ;$$

$Q \equiv (\omega, \mathbf{0}) \equiv$ four-momentum of dilepton pair,

$Z \equiv$ heavy quark electric charge in units of e,

$M \equiv$ heavy quark pole mass.