Properties of mesons in strongly coupled plasmas from AdS/CFT

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and Urs Wiedemann *arxiv:0712.0590*

TF, Hong Liu *arxiv:0807.0063*

TF, Hong Liu *arxiv:0812.4278*

Motivation: J/Ψ and the QGP

- Charmonium ($J/\Psi, \ldots$) and Bottomonium ($\Upsilon, \ldots$) *may* survive the deconfinement transition $T = T_c$
- Attributed to their small size compared to T_c^{-1}.
- Potentially good probes of the QGP *Matsui, Satz*
- Consider *medium* effects:
  1. Color screening - weakens interactions between quarks and anti-quarks
     $\rightarrow$ bound state eventually *dissociates* at some "$T \rightarrow T_{\mathrm{diss}}$"
     as size approaches screening length
  2. Collisions with deconfined thermal quarks and gluons can *break apart* bound state
     $\rightarrow$ medium induced width
     becomes more important close to T_{diss}

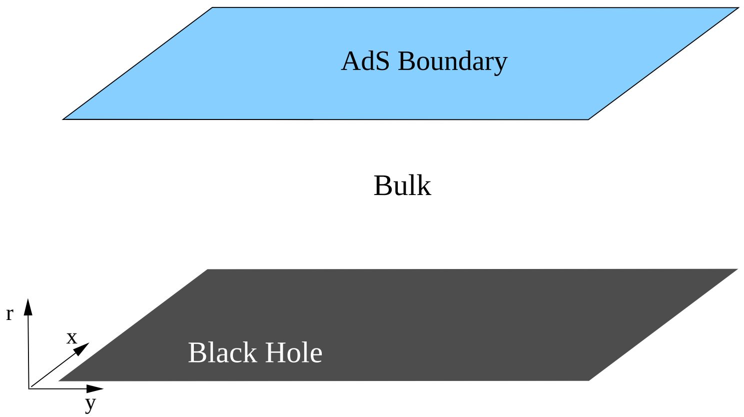
Theoretical challenges

- Difficulties with quantifying this picture from lattice spectral functions and potential models.
- Heavy quark mesons produced in heavy ion collisions can have large momentum relative to the plasma.
- Characterize screening length, dissociation temperature, meson width for a moving meson
- We study a toy model via AdS/CFT. Try to extract such medium effects focusing on their *momentum* dependence.

Outline

- Introduce **gauge gravity duality** (AdS/CFT)
- Meson screening length from **Semi-classical Strings**
- **Meson dispersion relations** - a subluminal limiting velocity
- **Meson widths** - a nonperturbative calculation
- Conclusions

Gauge gravity duality - basics *Maldacena et al.*



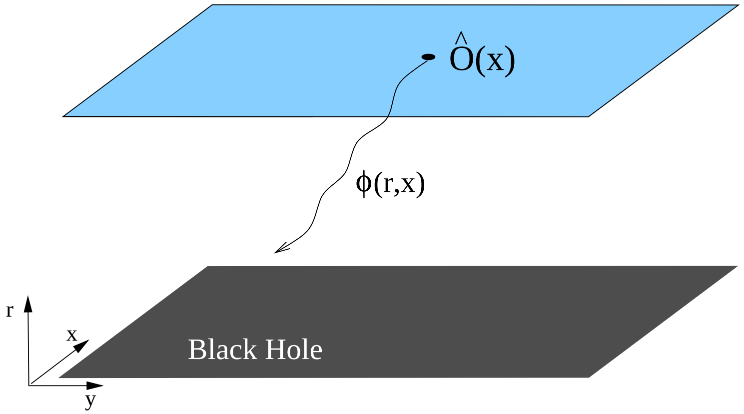
$\mathcal{N} = 4$ SYM	String theory on $AdS_5 \times S^5$
N_c	g_s^{-1}
$\sqrt{\lambda} = \sqrt{g_{\rm YM}^2 N_C}$	R^2/α'
T	$T_{\rm Hawking}$
energy scale	radial direction r

$$ds^2 = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2 dr^2}{r^2 f} + R^2 d\Omega_5^2$$

$$f = 1 - r_0^4/r^4$$

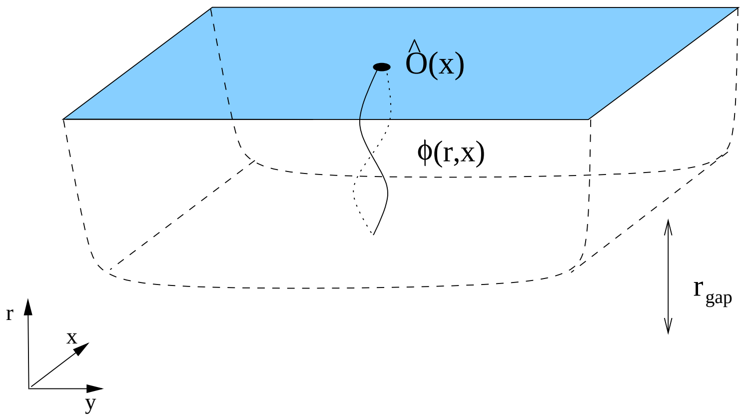
- Many different examples
- Classical gravity limit:$(g_s \to 0\,,\ \alpha' \to 0) \ \leftrightarrow \ (N_c \to \infty\,,\ \lambda \to \infty)$

Probing AdS/CFT with operators



$\mathcal{N} = 4$ SYM	String theory on $AdS_5 \times S^5$
$\hat{\mathcal{O}}(x)$	$\phi(x, r)$
thermalization	falling into BH

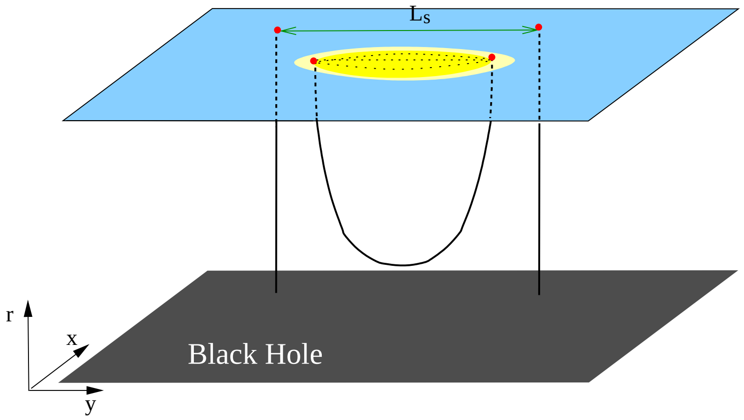
spectral function of $\hat{\mathcal{O}}$ continuous



bound states	resonances
(mesons, glueballs)	of AdS box

spectral functions of $\hat{\mathcal{O}}$ discrete.

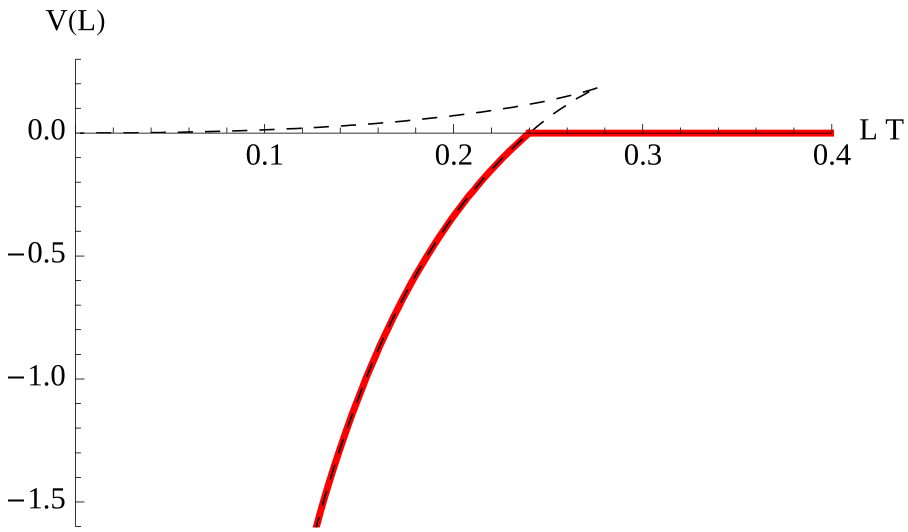
Semi-classical Strings



Heavy 'test' quark	String ending on bdry
Static quark potential potential $V(L)$	Area of hanging string

Maldacena

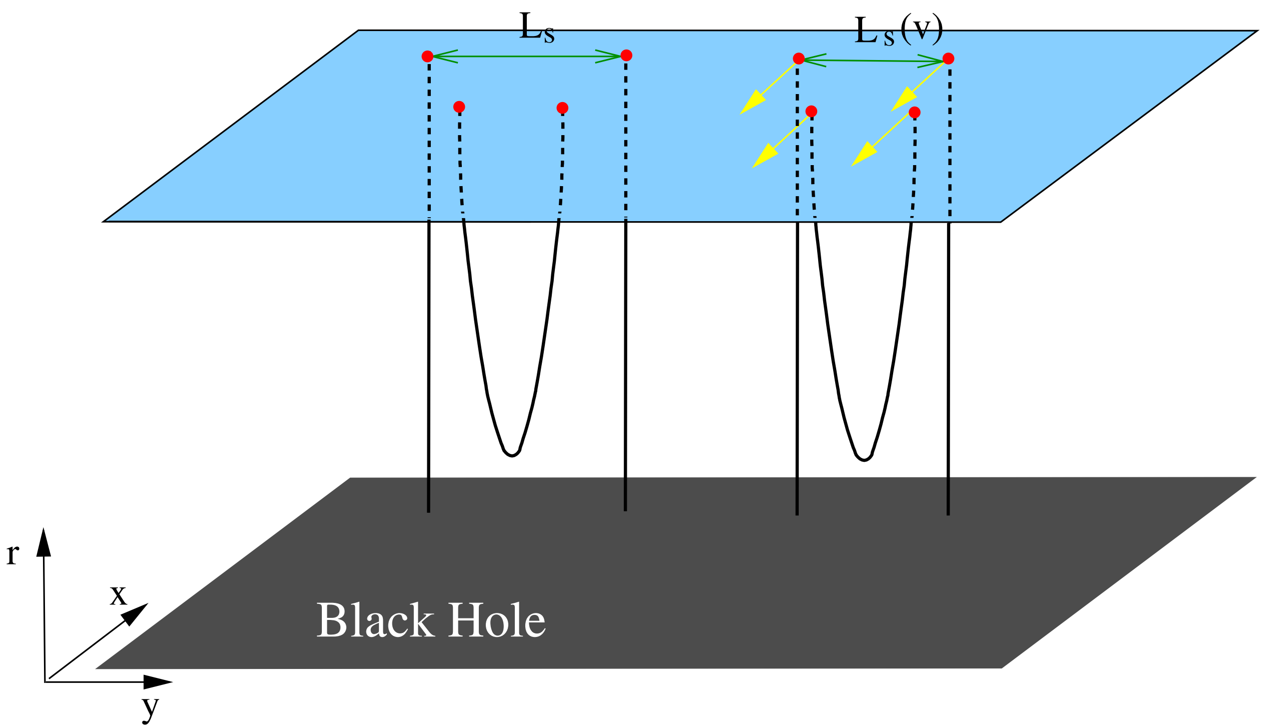
(define $V(L)$ from *real time* wilson loop with temporal extent $\to \infty$.)



strong coupling: $L_s \sim .24/T$

Rey, Theisen, Yee

Semi-classical Moving Strings



H. Liu, K. Rajagopal,
U. Wiedemann

Chernicoff, Garcia, Guijosa

Peeters, Sonnenshein, Zamaklar

Screening at zero velocity $L_S \sim 1/T$

At non-zero velocity $L_S(v) \sim (1/T)(1-v^2)^{1/4}$
Effective temperature $T(v) = T\sqrt{\gamma}$.

Semi-classical Moving Strings

Suggests mesons with higher velocity relative to plasma will dissociate at lower T:

Dissociation temperature such that $L_s(v) \sim$ *meson size* $\sim M^{-1} \rightarrow$

$T_{\text{diss}}(v) = T_{\text{diss}}(0)(1 - v^2)^{1/4}$.

- there exists a maximum *velocity* beyond which meson bound states do not exist.

$$v_{\text{max}} \sim 1 - \left(\frac{T}{M}\right)^4$$

- Mesons moving with large *momentum* $p > M(M/T)^2$ relative to medium should have weakened interaction - dissociate more easily.

With what is to come we will see evidence for *both*.

Meson dispersion relations

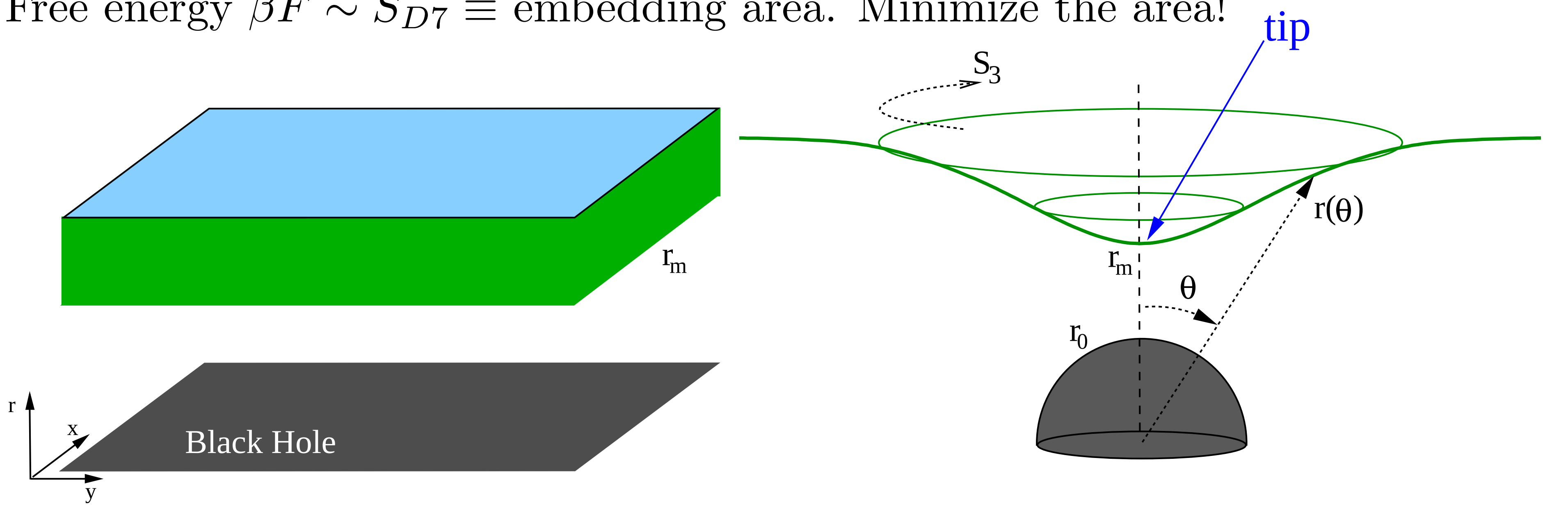
- instead of inferring results from the static quark potential study the spectrum directly
- need to add dynamical quarks

Dynamical quarks - the D3/D7 system

Adding quarks to $\mathcal{N} = 4$ SYM equivalent to adding N_f D7 branes (open strings) in the string theory, with $N_f \ll N_c$. *Karch, Katz*

Study embedding of $D7$ branes in $AdS_5 \times S^5$ - $r(\theta)$.

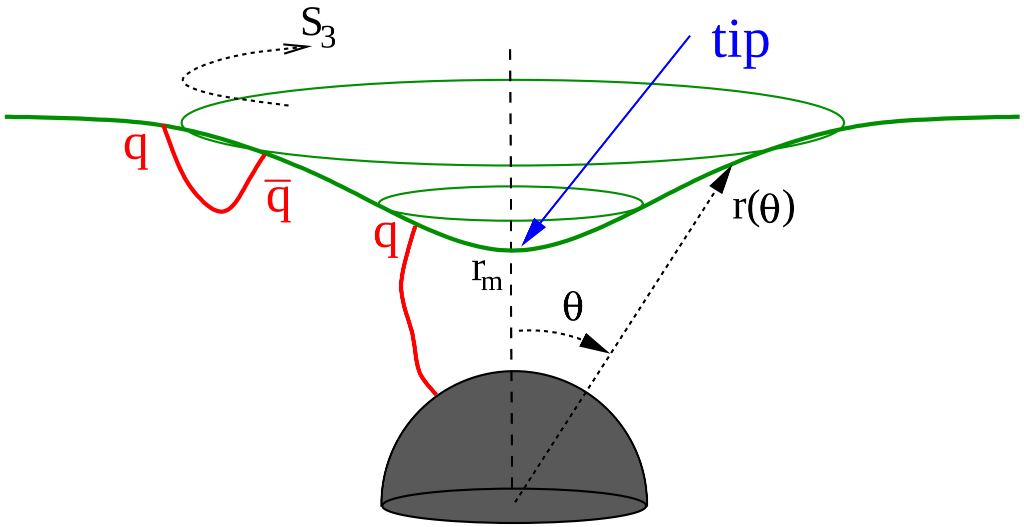
Free energy $\beta F \sim S_{D7} \equiv$ embedding area. Minimize the area!



$$\frac{dr^2}{r^2} + d\Omega_5^2 \to \frac{dr^2}{r^2} + (d\theta^2 + \cos^2(\theta) d\Omega_3^2 + \sin^2(\theta) d\phi^2)$$

Meson dictionary

Quark	strings ending on horizon.
$m_q^{(T)} =$	$(r_m - r_0)/\alpha'$
$\bar{q}q(x)$	$\chi(x,r)$ - fluctuation of D7 brane
Meson	resonant modes of D7 brane
$M =$	$(r_m - r_0)/R^2$



Rough scales:

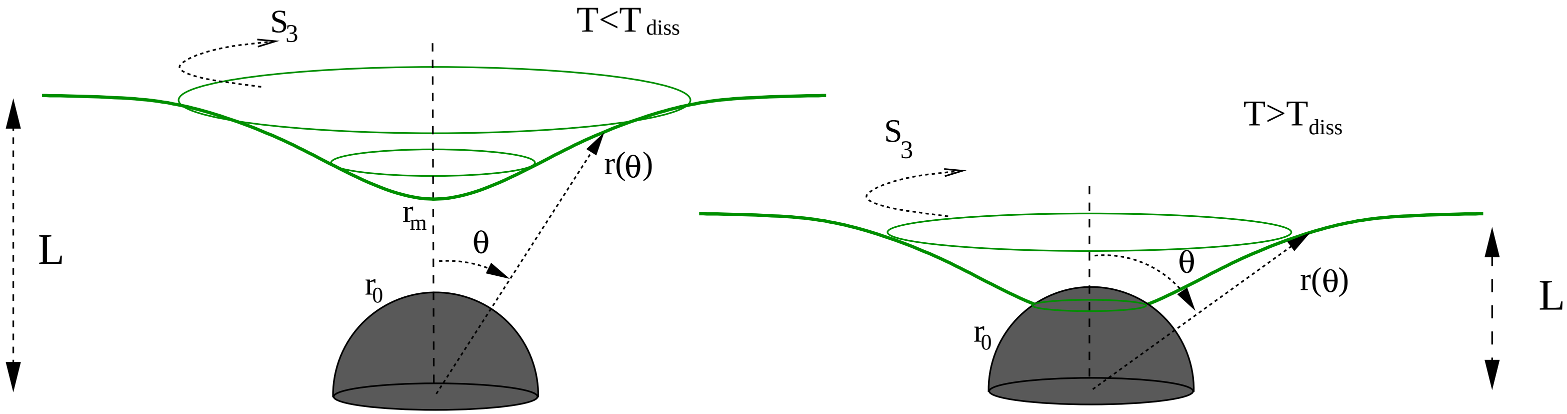
$M \sim m_q/\sqrt{\lambda} \rightarrow$

$m_q \gg M \rightarrow$

$E_{\rm BE} \sim 2m_q \gg M.$

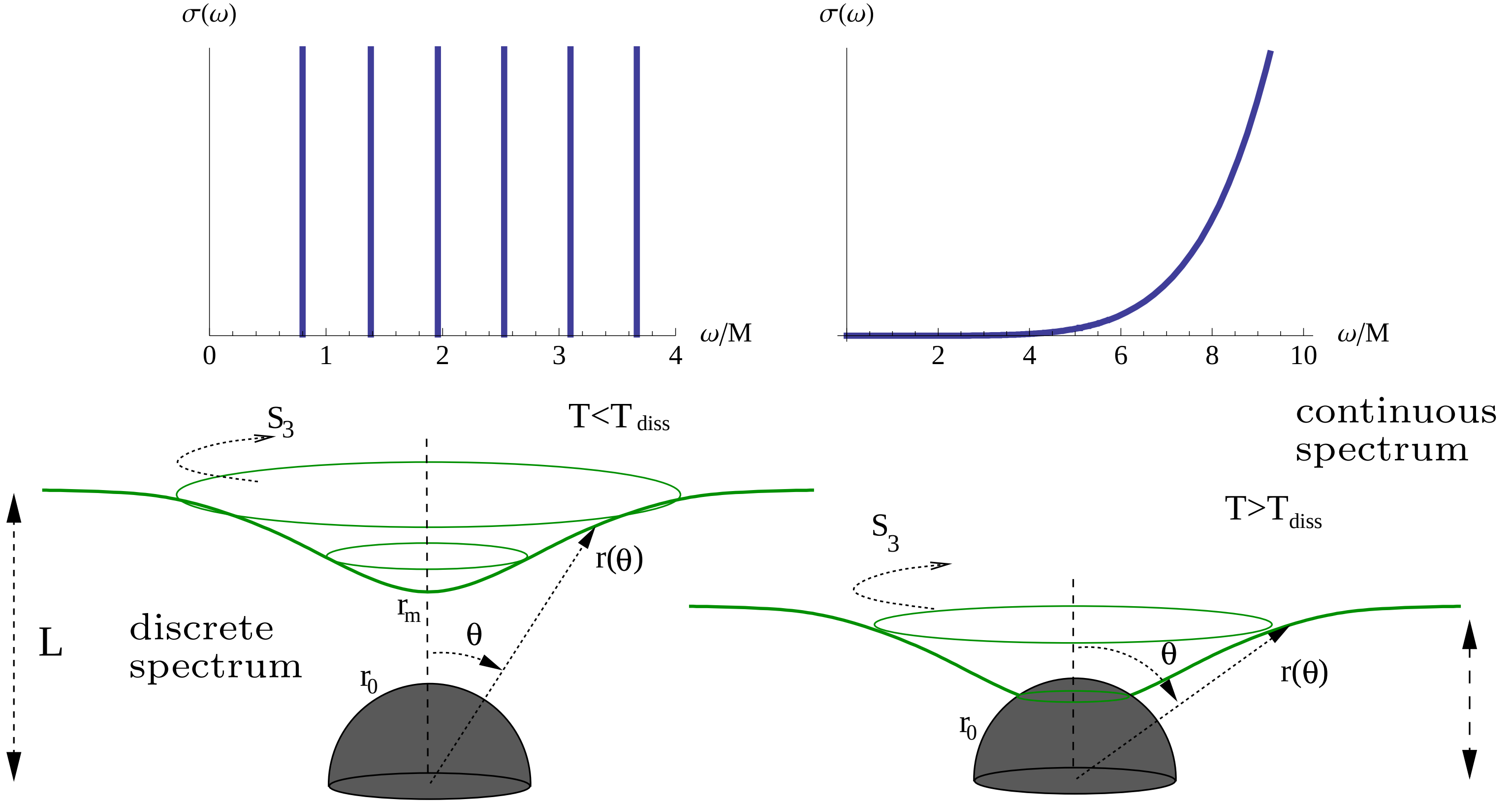
Dissociation

- Increase temperature (equivalent to decreasing the quark mass.)
- Eventually D7 brane will fall into horizon.
- First order phase transition, at $T_{diss} \approx 0.1 \times M$



Mateos, Myers, Thomson; Hoyos, Landsteiner, Montero

Dissociation



Mateos, Myers, Thomson; Hoyos, Landsteiner,Montero

Mesons dissociate above a critical temperature $T_{\text{diss}} \sim 1/M$.
Focus on when the embedding sits outside of horizon, pictured left.

Meson spectrum

2 transverse directions - radial mode χ_u and angular mode χ_ϕ.
$\rightarrow$ Couple to operators in the field theory $\sim \bar{q}q$.

Choose orthonormal basis of normal space:

$$\delta X^\mu = \chi_u n_u^\mu + \chi_\phi n_\phi^\mu$$

Embedding of D7: $K_u = 0$ and $K_\phi = 0$

Action for the fluctuations

$$S = -\frac{\mu_7}{2} \int d^8\xi \sqrt{h} \left((\partial\chi_u)^2 + (\partial\chi_\phi)^2 + m_u^2 \chi_u^2 + m_\phi^2 \chi_\phi^2 \right)$$

$$m_u^2 = R_{uu} + R_{u\phi\phi u} + 2R_{\phi\phi} + {}^{(8)}R - R$$

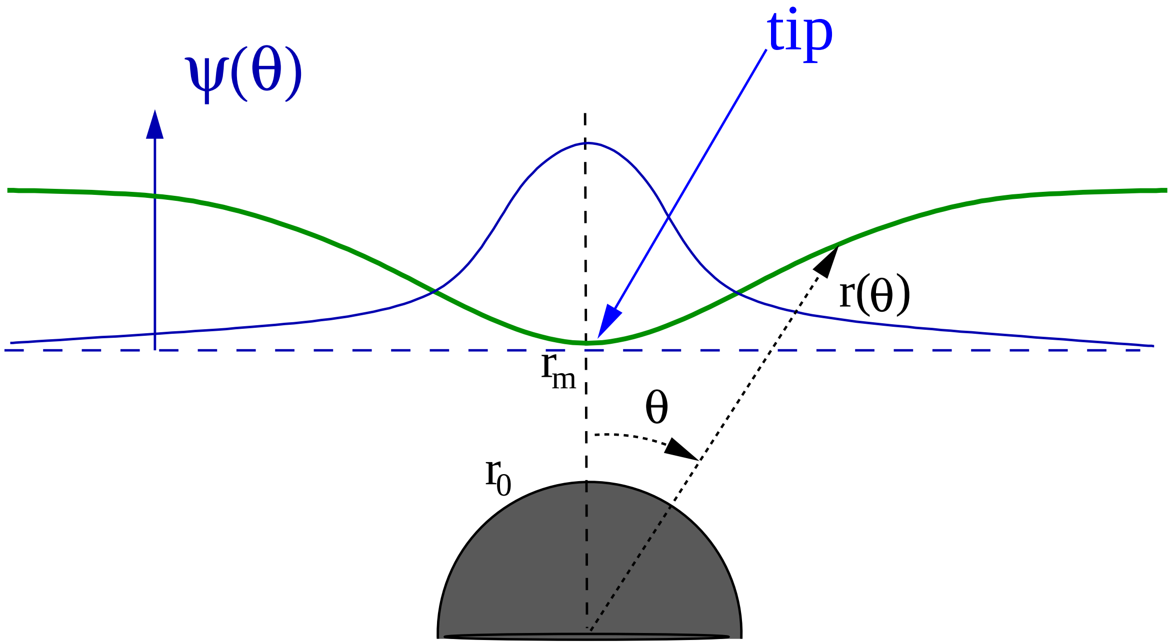
$$m_\phi^2 = -R_{\phi\phi} - R_{u\phi\phi u}$$

$m_i^2 \rightarrow -3$ which gives $\Delta_{\mathcal{O}} = 3$.

Meson spectrum - dispersion relations

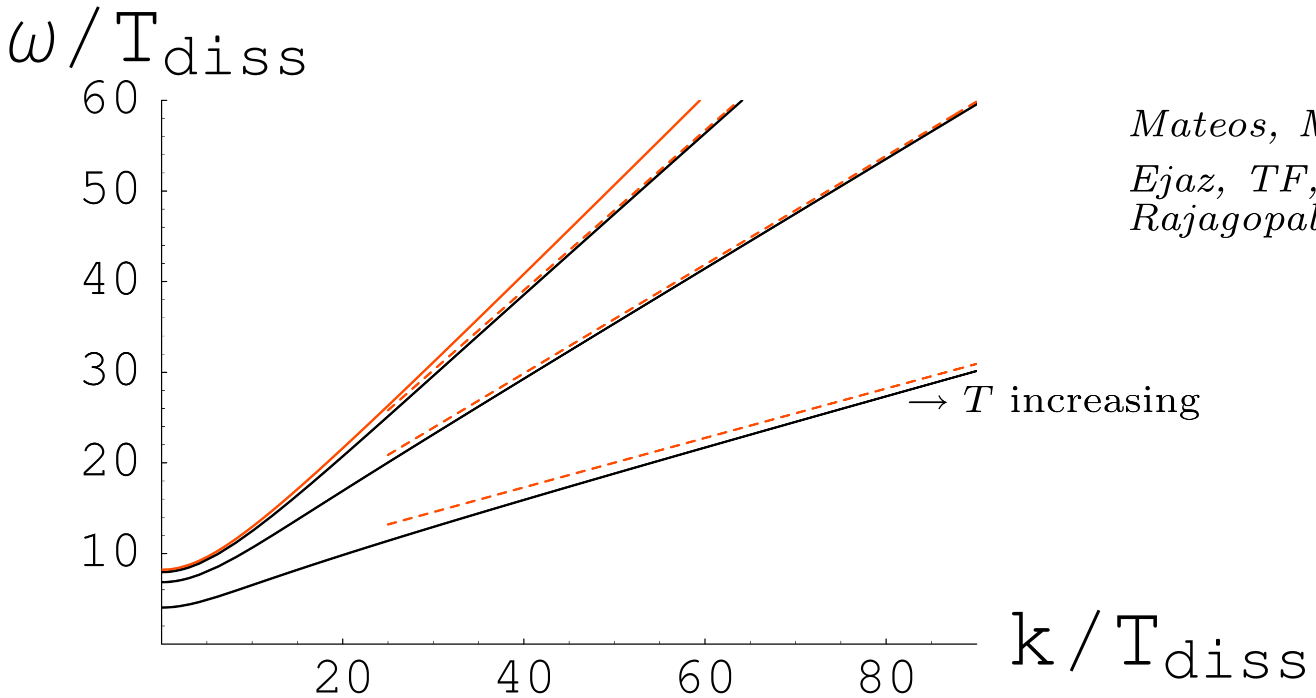
Concentrate on transverse scalar χ_ϕ : $\partial_\alpha(\sqrt{g}\partial^\alpha \chi_\phi) + \sqrt{g} m_\phi^2 \chi_\phi = 0$

$$\chi_\phi = e^{-i\omega t + i\vec{k}\cdot\vec{x}} Y_l(S_3)\psi(\theta) \quad \rightarrow \quad \hat{H}(\vec{k}, l)\psi(\theta) = \omega^2 \psi(\theta)$$



$T < T_{\text{diss}}$ - discrete spectrum, $\omega = \omega_n(\vec{k}, l)$.

Meson spectrum - dispersion relations



Mateos, Myers, Thomson
Ejaz, TF, Liu
Rajagopal, Wiedemann

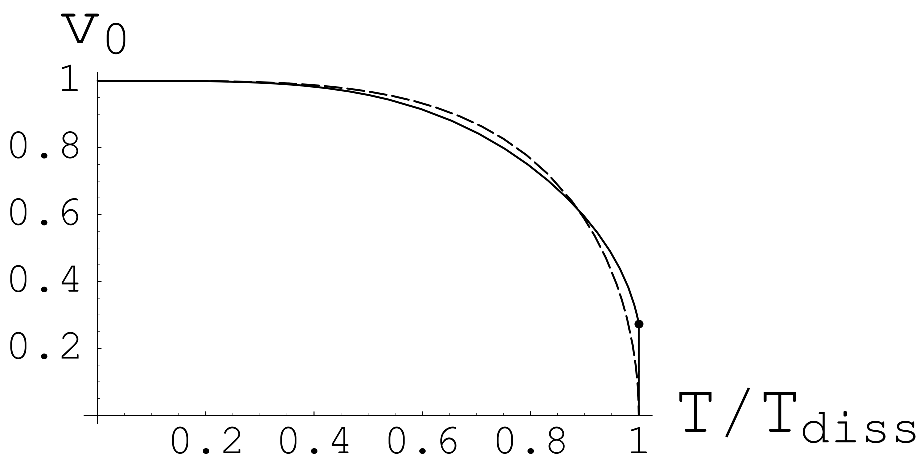
General argument: Large k - wave function localized at "tip" $\rightarrow$
$\omega = kv_0$ with $v_0 = \sqrt{\frac{-g_{tt}}{g_{xx}}}\Big|_{\text{tip}}$

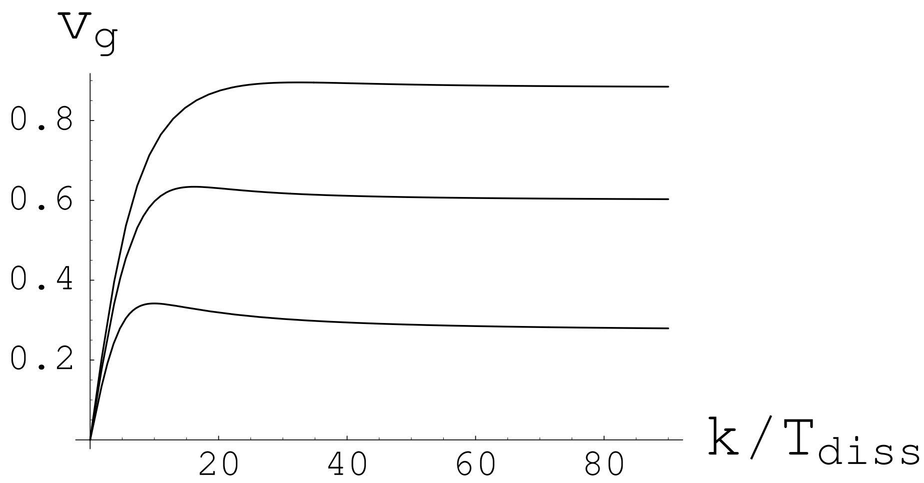
The limiting velocity

Can analytically solve for the large k dispersion relation:

$$\omega^2 = k^2 v_0^2 + k\Omega(n+2) + \mathcal{O}(1/k)\,, \qquad v_0 \sim \frac{1-(T/M)^4}{1+(T/M)^4}$$

Consistent with semiclassical strings interpretation of a maximum velocity





Maximum velocity slightly higher than limiting velocity.

Meson widths

Puzzle I - zero width?

- For $T < T_{\rm diss}$, mesons have zero width. (Recall gluons are deconfined.)
- Due to topology of D7 brane: fluctuations on brane see an induced geometry which is asymptotically AdS and smoothly capped off at some finite radius r_m (in AdS/CFT this is usually associated with confinement.)
- Relax $\alpha' \to 0$ limit : derivative expansion on D brane. Mesons still stable to any order in a perturbative α' expansion.
- But, bound states should always have a non-zero width in a finite temperature medium.
- Implies;
  $\to$ g_s correction
  $\to$ *Nonperturbative* α' correction.

Puzzle II - chemical potential

- Gauge field $A_\mu(r)$ on D7 brane $\leftrightarrow$ conserved baryon/quark current J_μ in field theory. More specifically the behavior of the gauge field at the boundary is,

$$A_t(r) \rightarrow \mu + n_q/r^2 + \dots$$

- Turn on constant gauge field $A_t(r) = \mu$: still a solution of $S_{D7} \sim \beta F$! Free energy does not depend on μ so $n_q = 0$. This is a puzzle at non-zero T.

- Again true in a perturbative α' expansion.

(For $\mu > m_q^{(T)}$ a new phase where D7 falls into horizon is thermodynamically preferred - see comment at end of talk.)

Expectations from field theory

- Thermal contribution:
  1. Recall $\beta m_q^{(T)} \sim \sqrt{\lambda} M/T$, so quark densities are suppressed,

     $$n_\pm \sim e^{-\beta m_q^{(T)} \pm \beta\mu} \to \mathcal{O}(e^{-\sqrt{\lambda}})$$

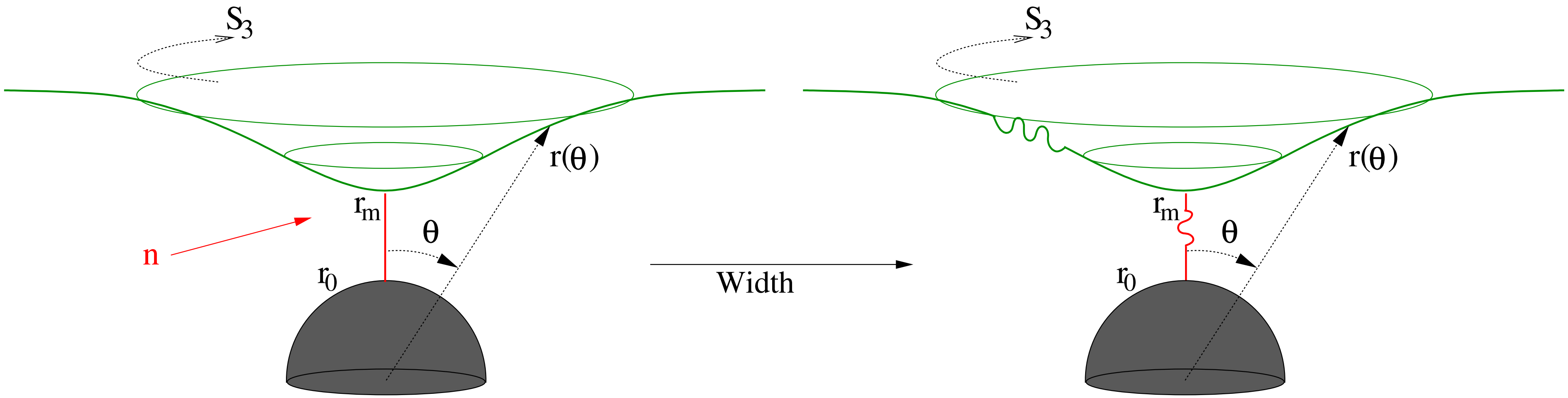
  2. Similarly $E_{\mathrm{BE}} \sim 2m_q$. At zero T mesons stable as tightly bound. At finite temperature thermal fluctuations can tear quarks apart, but fluctuations also exponentially suppressed in $\sqrt{\lambda}$.

  Not visible in perturbative $1/\sqrt{\lambda}$ expansion. Can only have non-perturbative origins on the worldsheet.

- $1/N_c$: the annihilation process, meson $\to$ meson + meson etc.
- $1/N_c^2$: there are thermal contributions to the width from processes that do not break apart the meson; see *Dusling,Erdmenger, Kaminski, Rust, Teaney, Young*

Add in the quarks by hand

- We are clearly missing the thermal quarks, since they are heavy.
- Add a finite density of strings to the D7 brane.

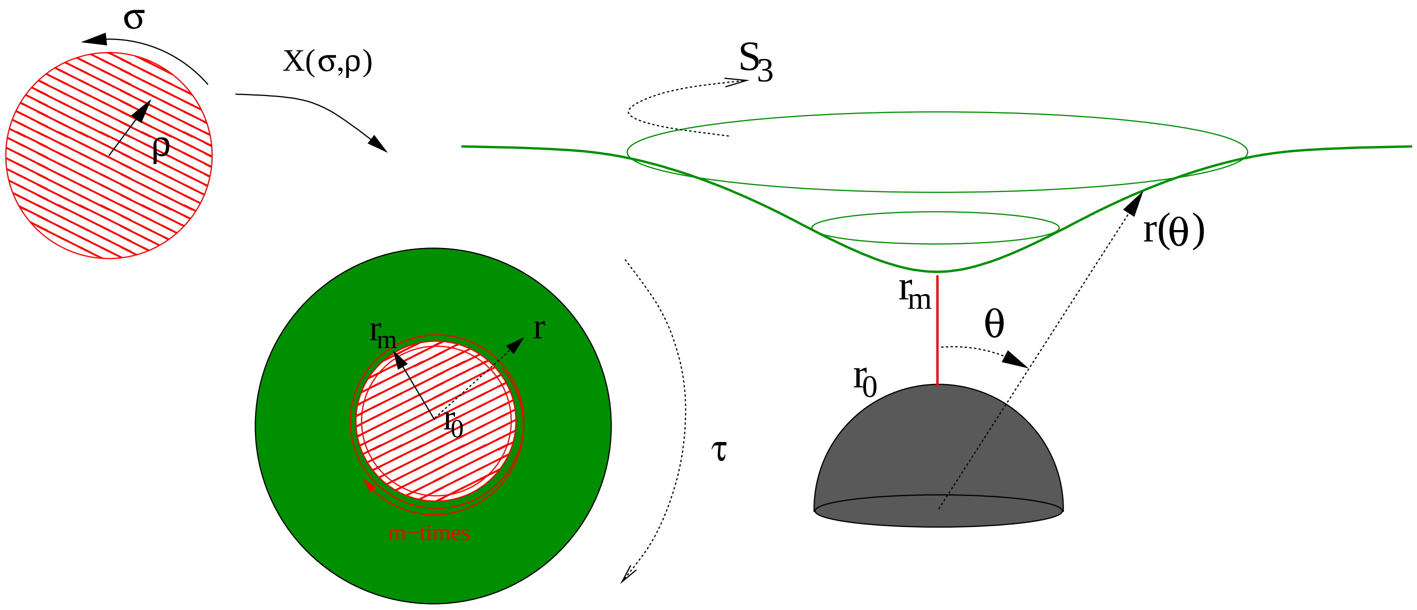


Effectively changes the topology of the D7 brane.

We can justify this *addition* by studying the theory in the Euclidean.

World sheet instanton

In *Euclidean time* there exists a non trivial "cycle with boundary" albeit wrapping Euclidean time,



Extremal solution to the world sheet action: a state that will contribute to the free energy.

From string theory perspective, this is a *world sheet instanton.* From field theory perspective, this is a thermal quark.

Spacetime Action

Calculate string partition function to find the spacetime action (free energy)[a]

$$S_E[\chi] = \int_{\text{disk}} DX\, e^{-I[X]-I_{\text{bdry}}[\chi,X]}$$

- X denotes the worldsheet fields and χ spacetime fields
- $I[X]$ is the world sheet action (Nambu-Goto),

$$I[X] = \frac{1}{2\pi\alpha'}\mathcal{A}(C) + \oint_{\Sigma} dx^{\mu} A_{\mu} + \dots$$

- $I_B[\chi, X]$ action on boundary of worldsheet, allowing for presence of some background (mesonic) fields χ.

[a]Note: string theory is a first quantized formalism, where the string partition function gives βF and not $\exp(-\beta F)$

Free Energy ($\chi = 0$)

Compute in saddle point approximation,

$$\beta F = S_E = S_{m=0} + S_{m=1} + S_{m=-1} + \ldots \qquad \text{and} \quad S_{m=0} \to S_{D7}$$

where $S_m \sim \exp(-I_m)$ for I_m the action of the world sheet *solution*.
In $m = \pm 1$ sectors, $I_{m=\pm 1} = \beta m_q^{(T)} \pm \mu\beta \to$ thermally suppressed.

Extra contribution to free energy,

$$\beta F_{\pm}(\beta, \mu) = S_{m=\pm 1} = e^{\pm\mu\beta} e^{-\beta m_q^{(T)}} \frac{1}{g_s} D V_3 \quad \to \quad n_{\pm} = e^{\pm\mu\beta} e^{-\beta m_q^{(T)}} \frac{1}{g_s} D$$

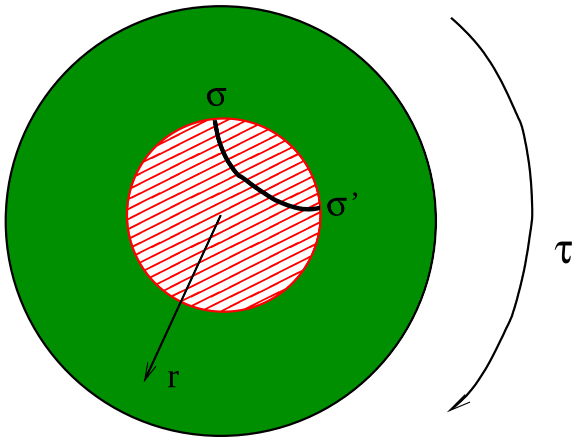
- $\pm$ - quarks, antiquarks
- g_s^{-1} - disk worldsheet! $n_{\pm} \propto N_c$
- D - functional determinant, includes fermions, hard!
- V_3 - three bosonic zero modes

Width Calculation

Integrate out fluctuations on the world sheet.

Quadratic action for χ^ϕ schematically, $S_{D7}[\chi^\phi]+$

$$S_{m=\pm1}[\chi_\phi] \sim \int d^3x_0 d\tau d\tau' \left(\chi^\phi(\tau,\vec{x}_0)\tilde{G}_D(\tau,\tau')\chi^\phi(\tau',\vec{x}_0)\right)_{\theta=0}$$



$\tilde{G}_D(\tau,\tau')$ ~ "boundary" to "boundary" propagator on the worldsheet disk.

Compute meson correlators in Euclidean. After analytic continuation find poles shifted by small imaginary part:

$$\left\langle \bar{q}q(\omega,\vec{k})\bar{q}q(-\omega,-\vec{k})\right\rangle \sim \frac{Z_n^2}{\omega^2-\omega_n^2(\vec{k})+i\omega\Gamma_n}$$

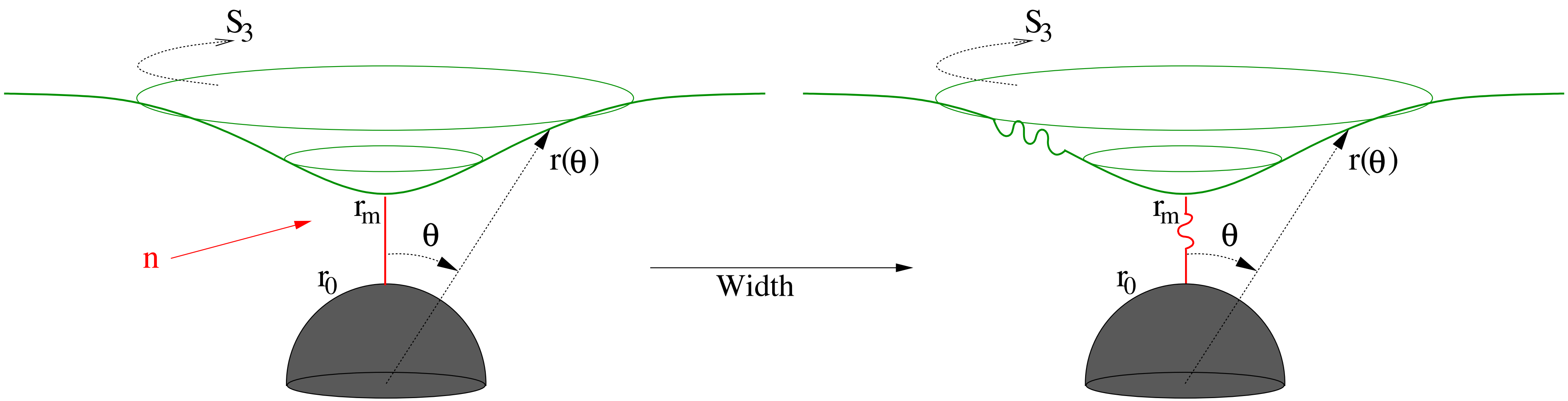
Width

Total width:

$$\Gamma_n = \Gamma_n^+ + \Gamma_n^- + \ldots \qquad\qquad \Gamma_n^{\pm 1} = \frac{32\pi^3\sqrt{\lambda}}{N_c m_q^2}|\psi_n(\theta = 0)|^2 n_\pm$$

Recall $n_\pm \propto N_c$ so $\Gamma \sim \mathcal{O}(1)$ in N_c.

Physical picture:

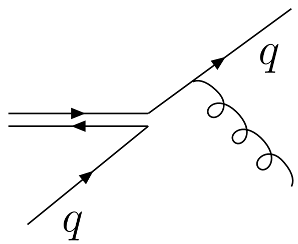


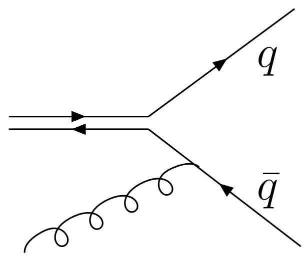
Wavefunction at tip controls *tunneling probability*.

Interpretation

- String theory perspective: open string modes living on a D brane which lies outside a black hole can *tunnel* into the black hole.
- Field theory perspective:

$$\Gamma \sim e^{-\beta m_q^{(T)}}$$





$$\Gamma \sim e^{-\beta E_{\rm BE}} \sim e^{-2\beta m_q^{(T)}}$$

1. Our width from *left* process. Dominant thermal process which breaks apart meson.
2. However *right* process more interesting from QCD perspective. Subdominant in *this* strongly coupled gauge theory.

Momentum dependence of width

Note $n_\pm(T,\mu)$ not calculated; depends on functional determinant. Cannot extract width directly; However $n_\pm$ is k independent, so we take the ratio:

$$\frac{\Gamma_n(k)}{\Gamma_n(0)} = \frac{|\psi_n(\theta=0;\vec{k})|^2}{|\psi_n(\theta=0;\vec{k}=0)|^2}$$

Use large k analytic results for wave function, grows quadratically;

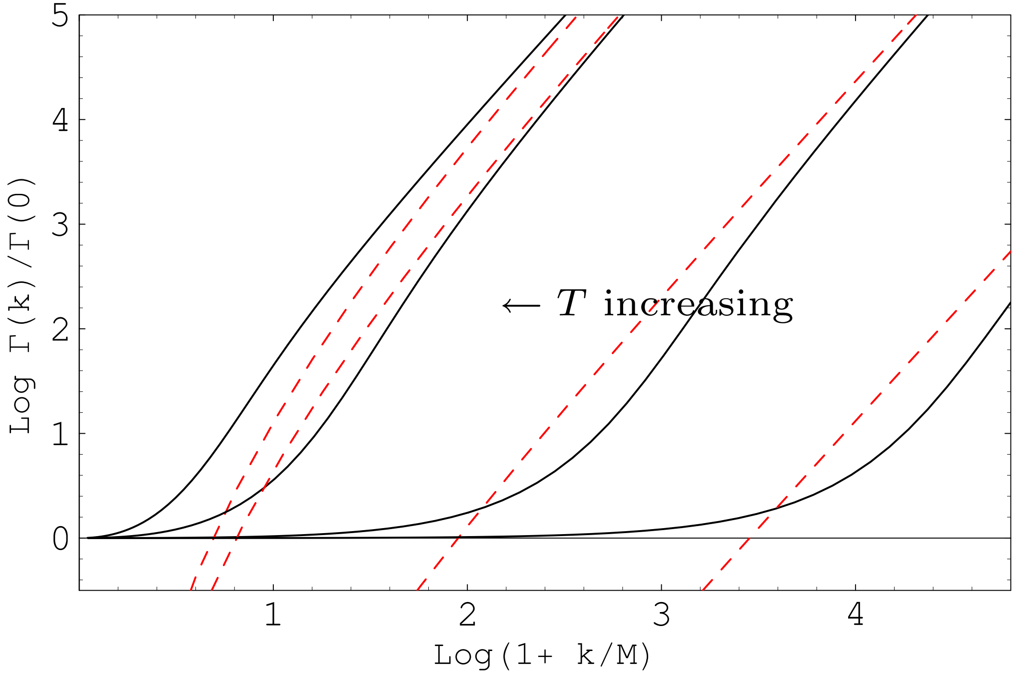
$$\Gamma_n(k)/\Gamma_n(0) = R_n[T/M](k/M)^2 + \mathcal{O}(k)$$

Temperatures $T \ll M$ and for $k \gg \frac{M^3}{T^2}$;

$$\Gamma_n(k)/\Gamma_n(0) \approx \frac{2(4\pi)^4}{(n+2)(n+3/2)}\frac{T^4k^2}{M^6}$$

General argument: wavefunction more peaked at the “tip” for higher momentum.

Momentum dependence of width

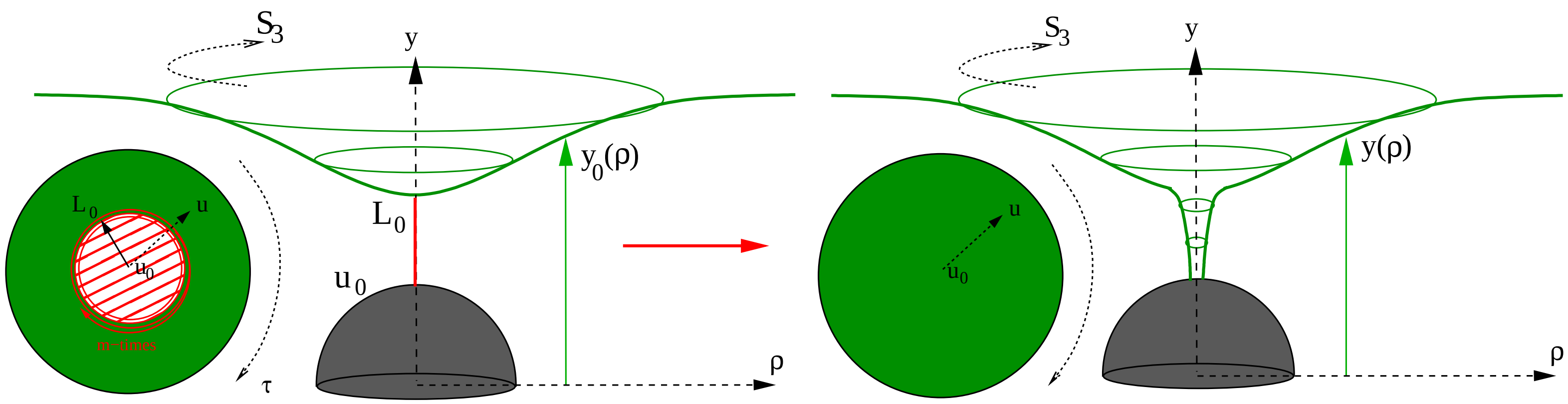


- Width turns up at same momentum that the meson group velocity reaches its maximum.
- $\Gamma \sim T^4 k^2 / M^6$ consistent with semiclassical string analysis of screening length: interactions weaker for meson with higher *momentum*

Phase transition driven by thermal quarks *TF, Hong Liu*

Recall contribution to free energy:$n_\pm \propto \exp\left(\pm\mu\beta - \beta m_q^{(T)}\right)$

- exponentially small/large for $\mu < m_q^{(T)}$ / $\mu > m_q^{(T)}$



In fact for $\mu > m_q^{(T)}$ a small neck made of "D-brane" is generated (it is no longer made of strings.)

Can compute width here too. Find same result as before $\Gamma_n = \Gamma_n^+$ (Note $\Gamma_n^- \ll \Gamma_n^+$ here.)

Conclusions/Open questions

- studied dynamical properties of mesons via AdS/CFT
- characterized the limiting velocity at high momentum
- found a width via a nonperturbative correction, width grows like k^2 for large k.
- do these features apply to QCD? If so, might be seen in the transverse momentum dependence of J/Ψ suppression.
- gluon contribution to breakup of the meson. Compare to weak coupling calculation. (Should also grow with k.)