Production of a transverse $\rho$-meson at the twist-3 level

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Introduction: phenomenology of exclusive processes within **collinear factorization**

- Experimental tests are possible in **fixed target** experiments
  - $e^\pm p$, $\mu^\pm p$: HERA (HERMES), JLab, COMPASS...
- as well as in **colliders**, mainly for medium $s$
  - $e^\pm p$ colliders: HERA (H1, ZEUS)
  - $e^+e^-$ colliders: LEP, Belle, BaBar, BEPC

**Collinear factorization** has been proven only for specific cases:
e.g.: $\rho_T$ production cannot directly be factorized (appearance of **end point singularities**)
⇒ improvement needed for a consistent approach of exclusive processes
At the same time, at large $s$, the interest for phenomenological tests of hard Pomeron and related resummed approaches has become pretty wide:

- inclusive tests (total cross-section) and semi-inclusive tests (diffraction, forward jets, ...)
- exclusive tests (meson production)

These tests concern all type of collider experiments:

- $e^\pm p$: HERA: (H1, ZEUS)
- $p\bar{p}$ and $pp$: TEVATRON (CDF, D0); LHC (CMS, ATLAS, ALICE)
- $e^+e^-$: (LEP, ILC)

These high energy exclusive processes in the perturbative Regge limit may provide new ideas when dealing with collinear factorization.
Polarization effects in $\gamma^* P \to \rho P$ at HERA

- one can experimentally measure all spin density matrix elements

- at $t = t_{\text{min}}$ one can experimentally distinguish

  $$\left\{ \begin{array}{l}
  \gamma_L^* \to \rho_L : \text{ dominates (twist 2 dominance)} \\
  \gamma_T^* \to \rho_T : \text{ sizable (twist 3)}
  \end{array} \right.$$  

- S-channel helicity conservation:

  $$\left\{ \begin{array}{c}
  \gamma_L^* \to \rho_L \quad (\equiv T_{00}) \\
  \gamma_T^* \to \rho_T,
  \end{array} \right.$$  

  Dominate with respect to all other transitions. Experimentally, $\gamma_T^* \to \rho_T$ is dominated by $\gamma_T^*(-) \to \rho_T(-)$ and $\gamma_T^*(+)^+ \to \rho_T(+) \quad (\equiv T_{11})$
The processes with vector particle such as rho-meson probe deeper into the fine features of QCD.
It deserves theoretical development to describe HERA data in its special kinematical range:

- large $s_{\gamma^* P}$ $\Rightarrow$ small-$x$ effects expected, within $k_t$-factorization
- large $Q^2$ $\Rightarrow$ hard scale $\Rightarrow$ perturbative approach and collinear factorization
  $\Rightarrow$ the $\rho$ can be described through its chiral even Distribution Amplitudes

$$\left\{ \begin{array}{l} \rho_L \text{ twist 2} \\ \rho_T \text{ twist 3} \end{array} \right.$$  

The main ingredient is the $\gamma^* \rightarrow \rho$ impact factor

SIMPLEST OBJECT: ONLY 1 SOFT PART

- For $\rho_T$, special care is needed: a pure 2-body description would violate gauge invariance.
- We show that:
  - Including in a consistent way all twist 3 contributions, i.e. 2-body and 3-body correlators, gives a gauge invariant impact factor
  - Our treatment is free of end-point singularities and does not violate the QCD factorization
QCD in perturbative Regge limit

- In this limit, the dynamics is dominated by gluons (dominance of spin 1 exchange in $t$ channel)
- BFKL (and extensions: NLL, saturations effects, ...) is expected to dominate with respect to Born order at large relative rapidity.
Impact factor for exclusive processes

\( \gamma^* \gamma^* \rightarrow \rho \rho \) as an example

- Use Sudakov decomposition \( k = \alpha p_1 + \beta p_2 + k_\perp \) (\( p_1^2 = p_2^2 = 0, \ 2p_1 \cdot p_2 = s \))
- write \( d^4k = \frac{s}{2} d\alpha d\beta d^2k_\perp \)
- \( t \)-channel gluons with non-sense polarizations \( (\epsilon^{up}_{NS} = \frac{2}{s} p_2, \ \epsilon^{down}_{NS} = \frac{2}{s} p_1) \)
  dominate at large \( s \)

⇒ set \( \alpha_k = 0 \) and \( \int d\beta \)

⇒ set \( \beta_k = 0 \) and \( \int d\alpha \)
Impact factor for exclusive processes

$k_T$ factorization

impact representation \( \vec{k} = \text{Eucl.} \leftrightarrow \vec{k}_\perp = \text{Mink.} \)

\[
\mathcal{M} = is \int \frac{d^2 k}{(2\pi)^2 k^2 (r - k)^2} \Phi \gamma^*(q_1) \rightarrow \rho (p_1^\rho) (k, r - k) \ \Phi \gamma^*(q_2) \rightarrow \rho (p_2^\rho) (-k, -r + k)
\]

The \( \gamma_{L,T}^*(q) g(k_1) \rightarrow \rho_{L,T} g(k_2) \) impact factor is normalized as

\[
\Phi \gamma^* \rightarrow \rho (k_r^2) = e^{\gamma^* \mu} \frac{1}{2s} \int \frac{d\kappa}{2\pi} \text{Disc}_\kappa \mathcal{S}_{\mu}^{\gamma^* \rightarrow \rho} g(k_r^2),
\]

with \( \kappa = (q + k)^2 = \beta s - Q^2 - k^2 \)
Gauge invariance

- QCD gauge invariance (probes are colorless)
  \( \Rightarrow \) impact factor should vanish when \( k \to 0 \) or \( r - k \to 0 \)

- In the following we will restrict ourself to the case \( t = t_{min} \), i.e. to \( r = 0 \)

\[
\begin{align*}
k_1 &= \frac{\kappa + Q^2 + k^2}{s} p_2 + k_\perp \\
k_2 &= \frac{\kappa + k^2}{s} p_2 + k_\perp \\
k_1^2 &= k_2^2 = -k^2
\end{align*}
\]

This kinematics takes into account skewedness effects along \( p_2 \)

\( t = t_{min} \) \( \Rightarrow \) restriction to the transitions

\[
\begin{cases}
0 &\to 0 \quad \text{(twist 2)} \\
(\text{+ or -}) &\to (+ \text{ or -}) \quad \text{(twist 3)}
\end{cases}
\]

- At twist 3 level (for \( \gamma_T^* \to \rho_T \) transition), gauge invariance is a non trivial statement which requires 2 and 3 body correlators
The impact factor can be written as

\[ \Phi = \int d^4l \cdots \text{tr}[H(l \cdots) \ S(l \cdots)] \]

\text{hard part} \quad \text{soft part}

At the 2-body level:

\[ S_{q\bar{q}}(l) = \int d^4z \ e^{-il\cdot z} \langle \rho(p)|\psi(0)\bar{\psi}(z)|0\rangle, \]

\( H \) and \( S \) are related by \( \int d^4l \) and by the summation over spinor indices.
1 - Momentum factorization (1)

- Use Sudakov decomposition in the form ($p = p_1$, $n = 2p_2/s \Rightarrow p \cdot n = 1$)

\[ l_\mu = yp_\mu + l_\mu^\perp + (l \cdot p) n_\mu, \quad y = l \cdot n \]

scaling: \[1 \quad 1/Q \quad 1/Q^2\]

- decompose $H(k)$ around the $p$ direction:

\[ H(l) = H(yp) + \frac{\partial H(l)}{\partial l_\alpha} \bigg|_{l=yp} (l - yp)_\alpha + \ldots \quad \text{with} \quad (l - yp)_\alpha \approx l_\alpha^\perp \]

- In Fourier space, the twist 3 term $l_\alpha^\perp$ turns into a derivative of the soft term

$\Rightarrow$ one will deal with \[ \int d^4z \ e^{-il \cdot z} \langle \rho(p) | \psi(0) i \partial_\alpha^\perp \bar{\psi}(z) | 0 \rangle \]
Collinear factorization
Light-Cone Collinear approach: 2 steps of factorization (2-body case)

1 - Momentum factorization (2)

- write

\[ d^4 l \longrightarrow d^4 l \, \delta(y - l \cdot n) \, dy \]

- \( \int d^4 l \, \delta(y - l \cdot n) \) is then absorbed in the soft term:

\[
(\tilde{S}_{q\bar{q}}, \partial_{\perp} \tilde{S}_{q\bar{q}}) \equiv \int d^4 l \, \delta(y - l \cdot n) \int d^4 z \, e^{-il \cdot z} \langle \rho(p)|\psi(0) (1, \hat{i} \partial_{\perp})\bar{\psi}(z)|0\rangle
\]

\[
(\delta(y - l \cdot n) = \int \frac{d\lambda}{2\pi} e^{-i\lambda(y-l \cdot n)} \Rightarrow) = \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \int d^4 z \, \delta^{(4)}(z - \lambda n) \langle \rho(p)|\psi(0) (1, \hat{i} \partial_{\perp})\bar{\psi}(z)|0\rangle
\]

\[
= \int \frac{d\lambda}{2\pi} e^{-i\lambda y} \langle \rho(p)|\psi(0) (1, \hat{i} \partial_{\perp})\bar{\psi}(\lambda n)|0\rangle
\]

- \( \int dy \) performs the longitudinal momentum factorization
2 - Spinorial (and color) factorization

- Use Fierz decomposition of the Dirac (and color) matrices $\psi(0)\bar{\psi}(z)$ and $\psi(0)i\partial_\perp\bar{\psi}(z)$:

- $\Phi$ has now the simple factorized form:

$$\Phi = \int dx \left\{ \text{tr} [H_{q\bar{q}}(x\, p) \, \Gamma] \, S_{q\bar{q}}^\Gamma(x) + \text{tr} [\partial_\perp H_{q\bar{q}}(x\, p) \, \Gamma] \, \partial_\perp S_{q\bar{q}}^\Gamma(x) \right\}$$

$$\Gamma = \gamma^\mu \text{ and } \gamma^\mu \gamma^5 \text{ matrices}$$

$$S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \, \Gamma \, \psi(0) | 0 \rangle$$

$$\partial_\perp S_{q\bar{q}}^\Gamma(x) = \int \frac{d\lambda}{2\pi} e^{-i\lambda x} \langle \rho(p) | \bar{\psi}(\lambda n) \, \Gamma \, i \quad \partial_\perp \psi(0) | 0 \rangle$$

- choose axial gauge condition for gluons, i.e. $n \cdot A = 0 \Rightarrow$ no Wilson line
Collinear factorization

Light-Cone Collinear approach: 2 steps of factorization (3-body case)

Factorization of 3-body contributions

- 3-body contributions start at genuine twist 3
  ⇒ no need for Taylor expansion

- Momentum factorization goes in the same way as for 2-body case

- Spinorial (and color) factorization is similar:
Collinear factorization
Parametrization of vacuum–to–rho-meson matrix elements (DAs): 2-body correlators

2-body non-local correlators

- vector correlator
  \[ \langle \rho(p) | \bar{\psi}(z) \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho \left[ \varphi_1(y) (e^* \cdot n)p_\mu + \varphi_3(y) e_\mu^* T \right] \]

- axial correlator
  \[ \langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi_A(y) \epsilon_{\mu \lambda \beta \delta} e_\lambda^* T p_\beta n_\delta \]

- vector correlator with transverse derivative
  \[ \langle \rho(p) | \bar{\psi}(z) \gamma_\mu i \partial_{\alpha}^\perp \psi(0) | 0 \rangle \equiv m_\rho f_\rho \varphi_1^T(y) p_\mu e_{\alpha}^* T \]

- axial correlator with transverse derivative
  \[ \langle \rho(p) | \bar{\psi}(z) \gamma_5 \gamma_\mu i \partial_{\alpha}^\perp \psi(0) | 0 \rangle \equiv m_\rho f_\rho i \varphi_A^T(y) p_\mu \epsilon_{\alpha \lambda \beta \delta} e_\lambda^* T p_\beta n_\delta, \]

where \( y (\bar{y} \equiv 1 - y) = \) momentum fraction along \( p \equiv p_1 \) of the quark (antiquark) and

\[ \bar{F} \equiv \int_0^1 dy \exp [i y p \cdot z], \text{ with } z = \lambda n \]

\Rightarrow 5 2-body DAs
Collinear factorization
Parametrization of vacuum–to–rho-meson matrix elements: 3-body correlators

3-body non-local correlators

- vector correlator

\[ \langle \rho(p) | \bar{\psi}(z_1) \gamma_\mu g A^T_\alpha (z_2) \psi(0) |0 \rangle \overset{F_2}{=} m_\rho \, f^V_3 \, B(y_1, y_2) \, p_\mu \, e^{*T}_\alpha, \]

- axial correlator

\[ \langle \rho(p) | \bar{\psi}(z_1) \gamma_5 \gamma_\mu g A^T_\alpha (z_2) \psi(0) |0 \rangle \overset{F_2}{=} m_\rho \, f^A_3 \, i \, D(y_1, y_2) \, p_\mu \, \varepsilon_\alpha \lambda \beta \delta \, e^{*T}_\lambda \, p_\beta \, n_\delta, \]

where \( y_1, \bar{y}_2, y_2 - y_1 = \) quark, antiquark, gluon momentum fraction

and \[ \overset{F_2}{=} \int_0^1 dy_1 \int_0^1 dy_2 \, \exp \left[ i \, y_1 \, p \cdot z_1 + i(y_2 - y_1) \, p \cdot z_2 \right], \] with \( z_{1,2} = \lambda n \)

\[ \Rightarrow \] 2 3-body DAs
Collinear factorization
Symmetry properties

From C-conjugation on the previous correlators, one gets:

- **2-body correlators:**

  \[
  \varphi_1(y) = \varphi_1(1 - y) \\
  \varphi_3(y) = \varphi_3(1 - y) \\
  \varphi_A(y) = -\varphi_A(1 - y) \\
  \varphi^T_1(y) = -\varphi^T_1(1 - y) \\
  \varphi^T_A(y) = \varphi^T_A(1 - y)
  \]

- **3-body correlators:**

  \[
  B(y_1, y_2) = -B(1 - y_2, 1 - y_1) \\
  D(y_1, y_2) = D(1 - y_2, 1 - y_1)
  \]
Equations of motion

- **Dirac equation** leads to

\[ \langle i(\not{D}(0)\psi(0))_\alpha \bar{\psi}_\beta(z) \rangle = 0 \quad (i \not{D}_\mu = i \not{\partial}_\mu + A_\mu) \]

- Apply the Fierz decomposition to the above 2 and 3-body correlators

\[- \langle \psi(x) \bar{\psi}(z) \rangle = \frac{1}{4} \langle \bar{\psi}(z) \gamma_\mu \psi(x) \rangle \gamma_\mu + \frac{1}{4} \langle \bar{\psi}(z) \gamma_5 \gamma_\mu \psi(x) \rangle \gamma_\mu \gamma_5.\]

- \[ \Rightarrow 2 \text{ Equations of motion:} \]

\[ \bar{y}_1 \varphi_3(y_1) + \bar{y}_1 \varphi_A(y_1) + \varphi_T(y_1) + \varphi_T^A(y_1) \]

\[ + \int dy_2 \left[ \zeta V(y_1, y_2) + \zeta A^D(y_1, y_2) \right] = 0 \quad \text{and} \quad (\bar{y}_1 \leftrightarrow y_1) \]

- In **WW approximation**: genuine twist 3 = 0 i.e. \( B = D = 0 \)

\[
\begin{cases}
\varphi_T^A(y) = \frac{1}{2} [(y - \bar{y}) \varphi_A^{WW}(y) - \varphi_3^{WW}(y)] \\
\varphi_T^1(y) = \frac{1}{2} [(y - \bar{y}) \varphi_3^{WW}(y) - \varphi_A^{WW}(y)]
\end{cases}
\]
A minimal set of DAs

- The non-perturbative correlators cannot be obtained from perturbative QCD (!)
- one should reduce them to a minimal set before using any model
- this can be achieved by using an additional condition:
  
  independency of the full amplitude with respect to the light-cone direction \( n \)

\[ \Rightarrow \text{we prove that 3 independent Distribution Amplitudes are needed:} \]

\[ \phi_1(y) \quad \leftarrow \quad 2 \text{ body twist 2 correlator} \]

\[ B(y_1, y_2) \quad \leftarrow \quad 3 \text{ body genuine twist 3 vector correlator} \]

\[ D(y_1, y_2) \quad \leftarrow \quad 3 \text{ body genuine twist 3 axial correlator} \]
Collinear factorization

$n$-independence

$n$-independence in practice

- $n^\mu$, with $n^2 = 0$, $n \cdot p = 1$ is not fixed uniquely
  \[
n^\mu \rightarrow n'\mu = n^\mu + \frac{n^2}{2} p^\mu + n^T_T
  \]

- $\rho_T$ polarization: $e^{* T}_\mu = e^*_\mu - p_\mu e^* \cdot n$
  for the full factorized amplitude:
  \[
  A = H \otimes S \quad \frac{dA}{dn^\mu} = 0, \quad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial (e^* \cdot n)}
  \]

- rewrite hard terms in one single form, of 2-body type: use Ward identities
  Example: hard 3-body $\rightarrow$ hard 2-body

  \[
  \text{tr} \left[ H_{3\rho}(y_1, y_2) \right] \ p^\rho \ \vec{p} \ B(y_1, y_2) = \frac{1}{y_1 - y_2} \ (\text{tr} \left[ H_{2}(y_1) \right] \ p - \text{tr} \left[ H_{2}(y_2) \right] \ p) B(y_1, y_2),
  \]

  \[
  \frac{dS}{dn^\mu} = 0
  \]

  \[
  y_1 - y_2
  \]

  \[
  \frac{1}{y_1 - y_2}
  \]

  \[
  \frac{1}{y_1 - y_2}
  \]

  \[
  \frac{1}{y_1 - y_2}
  \]

  \[
  \frac{1}{y_1 - y_2}
  \]
Collinear factorization
*n*–independence

**Constraints from** **n**–**independence**

- **vector correlators**

  \[
  \frac{d}{dy_1} \varphi^T_1(y_1) = -\varphi_1(y_1) + \varphi_3(y_1)
  \]

  \[-\zeta_3 V \int_0^1 \frac{dy_2}{y_2 - y_1} \left( B(y_1, y_2) + B(y_2, y_1) \right) \]

- **axial correlators**

  \[
  \frac{d}{dy_1} \varphi^T_A(y_1) = \varphi_A(y_1) - \zeta_3 A \int_0^1 \frac{dy_2}{y_2 - y_1} \left( D(y_1, y_2) + D(y_2, y_1) \right) \]
\[- \int_0^1 dy_1 \int_0^1 dy_2 B(y_1, y_2) \times p_\mu \left[ \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} \right] + \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} + \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} + \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} + \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array}
\right] \]

\[
\times \left\{ \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} \right\} - (y_1 \leftrightarrow y_2)
\]

\[
= \int_0^1 dy_1 \int_0^1 dy_2 \frac{B(y_1, y_2)}{y_2 - y_1} \times \left[ \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} \right] - (y_1 \leftrightarrow y_2)
\]

\[
= \int_0^1 dy_1 \int_0^1 dy_2 \frac{dy_2}{y_2 - y_1} \left[ B(y_1, y_2) + B(y_2, y_1) \right] \times \left[ \begin{array}{c}
\mu \\
y_2 - y_1 \\
y_2 - 1 \\
y_1 \end{array} \right]
\]

\[
(43)
\]
\[- \int_0^1 dy_1 \int_0^1 dy_2 \delta(y_1 - y_2) \varphi_T(y_1) \varphi_1(y_1) \times p_\mu \]

\[= \int_0^1 dy_1 \int_0^1 dy_2 \delta(y_1 - y_2) \varphi_T(y_1) \frac{\varphi_1(y_1)}{y_2 - y_1} \times \left\{ \begin{array}{c}
\end{array} \right\} - (y_1 \leftrightarrow y_2) \]
Collinear factorization
A set of independent non-perturbative correlators

Solution

- the set of 4 equations (2 EOM + 2 \( n \)-independence relations) can be solved analytically
- \( 7 \rightarrow 3 \) independent DAs

**twist 2**
- kinematical twist 3 (WW)
- genuine twist 3
- genuine + kinematical twist 3
Wandzura-Wilczek

\[ \varphi(y) = \varphi^{WW}(y) + \varphi^{gen}(y), \quad \varphi(y) = \varphi_3(y), \varphi_A(y), \varphi_1^T(y), \varphi_A^T(y) \]

where \( \varphi^{WW}(y) \) and \( \varphi^{gen}(y) \) are contributions in the so called Wandzura-Wilczek approximation and the genuine twist-3 contributions.

\( WW = \) vanishing 3-parton distributions \( B(y_1, y_2) \) and \( D(y_1, y_2) \), i.e. which satisfy the equations

\[ \bar{y}_1 \varphi^3_{WW}(y_1) + \bar{y}_1 \varphi_A^WW(y_1) + \varphi_1^T_{WW}(y_1) + \varphi_A^T_{WW}(y_1) = 0 \]
\[ y_1 \varphi^3_{WW}(y_1) - y_1 \varphi_A^WW(y_1) - \varphi_1^T_{WW}(y_1) + \varphi_A^T_{WW}(y_1) = 0. \]

\[
\frac{d}{dy_1} \varphi_1^T_{WW}(y_1) = -\varphi_1(y_1) + \varphi^3_{WW}(y_1), \quad \frac{d}{dy_1} \varphi_A^T_{WW}(y_1) = \varphi_A^WW(y_1).
\]

Solutions:

\[ \varphi_A^{WW}(y_1) = \frac{1}{2} \left[ \int_0^{y_1} \frac{dv}{v} \varphi_1(v) - \int_{y_1}^{1} \frac{dv}{v} \varphi_1(v) \right] \]
\[ \varphi_3^{WW}(y_1) = \frac{1}{2} \left[ \int_0^{y_1} \frac{dv}{v} \varphi_1(v) + \int_{y_1}^{1} \frac{dv}{v} \varphi_1(v) \right] \]

From these expr. the remaining \( \varphi_A^{WW T} \) and \( \varphi_1^{WW T} \) are

\[ \varphi_A^{WW T}(y_1) = \frac{1}{2} \left[ -\bar{y}_1 \int_0^{y_1} \frac{dv}{v} \varphi_1(v) - y_1 \int_{y_1}^{1} \frac{dv}{v} \varphi_1(v) \right] \]
\[ \varphi_1^{WW T}(y_1) = \frac{1}{2} \left[ -\bar{y}_1 \int_0^{y_1} \frac{dv}{v} \varphi_1(v) + y_1 \int_{y_1}^{1} \frac{dv}{v} \varphi_1(v) \right]. \]
Genuine twist-3

\[ \bar{y}_1 \varphi_{3}^{\text{gen}}(y_1) + \bar{y}_1 \varphi_{A}^{\text{gen}}(y_1) + \varphi_{1}^{T \text{gen}}(y_1) + \varphi_{A}^{T \text{gen}}(y_1) \]

\[ = - \int_{0}^{1} dy_2 \left[ \zeta_{3}^{V} B(y_1, y_2) + \zeta_{3}^{A} D(y_1, y_2) \right] \]

\[ y_1 \varphi_{3}^{\text{gen}}(y_1) - y_1 \varphi_{A}^{\text{gen}}(y_1) - \varphi_{1}^{T \text{gen}}(y_1) + \varphi_{A}^{T \text{gen}}(y_1) \]

\[ = - \int_{0}^{1} dy_2 \left[ -\zeta_{3}^{V} B(y_2, y_1) + \zeta_{3}^{A} D(y_2, y_1) \right] . \]

\[ \frac{d}{dy_1} \varphi_{1}^{T \text{gen}}(y_1) = \varphi_{3}^{\text{gen}}(y_1) - \zeta_{3}^{V} \int_{0}^{1} \frac{dy_2}{y_2 - y_1} (B(y_1, y_2) + B(y_2, y_1)) , \]

\[ \frac{d}{dy_1} \varphi_{A}^{T \text{gen}}(y_1) = \varphi_{A}^{\text{gen}}(y_1) - \zeta_{3}^{A} \int_{0}^{1} \frac{dy_2}{y_2 - y_1} (D(y_1, y_2) + D(y_2, y_1)) . \]
Solution for genuine twist-3

\[ \varphi_{3 \, gen}^Y(y) = \]

\[ -\frac{1}{2} \int_{y}^{1} \frac{du}{u} \left[ \int_{0}^{u} dy_2 \frac{d}{du} (\zeta_3^V B - \zeta_3^A D)(y_2, u) \right. \]

\[ - \left. \int_{0}^{u} \frac{dy_2}{y_2 - u} (\zeta_3^V B - \zeta_3^A D)(y_2, u) \right] \]

\[ - \frac{1}{2} \int_{0}^{y_1} \frac{du}{u} \left[ \int_{u}^{1} dy_2 \frac{d}{du} (\zeta_3^V B + \zeta_3^A D)(u, y_2) \right. \]

\[ - \left. \int_{u}^{1} \frac{dy_2}{y_2 - u} (\zeta_3^V B + \zeta_3^A D)(u, y_2) \right] \]

Finally, the solution for \( \varphi_{1 \, gen}^Y \)

\[ \varphi_{1 \, gen}^Y(y) = \int_{0}^{y} du \varphi_{3 \, gen}^Y(u) - \zeta_3^V \int_{0}^{y} dy_1 \int_{0}^{1} dy_2 \frac{B(y_1, y_2)}{y_2 - y_1} . \]
Computation and results

Computation of the hard part

2-body diagrams

- without derivative

- practical trick for computing $\partial_{\perp} H$ : use the Ward identity

\[
\frac{\partial}{p_\mu} \rightarrow = \rightarrow \bullet \rightarrow 
\]

where

\[
\rightarrow = \frac{1}{m - \not{p} - i\epsilon}
\]
3-body diagrams

- "abelian" type

- "non-abelian" type
**Computation and results**

Recall: $\gamma^*_L \rightarrow \rho_L$ impact factor

$$\Phi^{\gamma^*_L \rightarrow \rho_L}(k^2) = \frac{2e}{Q} \frac{g^2}{f_{\rho}} \frac{\delta^{ab}}{2N_c} \int dy \varphi_1(y) \frac{k^2}{y \bar{y} Q^2 + k^2}$$

pure twist 2 scaling (from $\rho$-factorization point of view)
Computation and results
Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

$\gamma_T^* \rightarrow \rho_T$ impact factor:

Spin Non-Flip/Flip separation appears

$$\Phi_{\gamma_T^* \rightarrow \rho_T} (k^2) = \Phi_{n.f.} (k^2) T_{n.f.} + \Phi_{f.} (k^2) T_f.$$ 

where

$$T_{n.f.} = -(e_\gamma \cdot e^*)$$ and

$$T_f = \frac{(e_\gamma \cdot k)(e^*k)}{k^2} + \frac{(e_\gamma \cdot e^*)}{2}$$

non-flip transitions $\{ + \rightarrow + , - \rightarrow - \}$

flip transitions $\{ + \rightarrow - , - \rightarrow + \}$
Computation and results

Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

pure twist 3 scaling (from $\rho$-factorization point of view)

$$\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = \frac{e g^2 m_\rho f_\rho}{2 \sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \left\{ -2 \int dy_1 \frac{(k^2 + 2 Q^2 y_1 (1 - y_1)) k^2}{y_1 (1 - y_1) (k^2 + Q^2 y_1 (1 - y_1))^2} \left[ (2y_1 - 1) \varphi^T_1(y_1) + \varphi^T_A(y_1) \right] 
+ 2 \int dy_1 \, dy_2 \left[ \zeta^V_3 B(y_1, y_2) - \zeta^A_3 D(y_1, y_2) \right] \frac{y_1 (1 - y_1) k^2}{k^2 + Q^2 y_1 (1 - y_1)} \left[ \frac{2 - N_c/C_F) Q^2}{k^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} \right]
- \frac{N_c}{C_F} \frac{Q^2}{y_2 k^2 + Q^2 y_1 (y_2 - y_1)} \left[ \frac{y_1 Q^2}{k^2 + Q^2 y_1 (1 - y_1)} \left( \frac{(2 - N_c/C_F) y_1 k^2}{k^2 (y_1 - y_2 + 1) + Q^2 y_1 (1 - y_2)} - 2 \right) \right]
+ \frac{N_c (y_1 - y_2) (1 - y_2)}{C_F (1 - y_1)} \frac{Q^2}{k^2 (1 - y_1) + Q^2 (y_2 - y_1) (1 - y_2)} \right\} \right.$$
Computation and results

Results: $\gamma_T^* \rightarrow \rho_T$ impact factor

**WW limit**

- **WW limit:** keep only twist 2 + kinematical twist 3 terms (i.e. $B = D = 0$)

- The only remaining contributions come from the two-body correlators

- **non-flip** transition

\[
\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = \frac{-e m_\rho f_\rho}{2 \sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \left\{ (y - \bar{y}) \varphi_1^{TWW} (y) + 2 y \bar{y} \varphi_3^{WW} (y) + \varphi_A^{TW} (y) \right\}
\]

- which simplifies, using equation of motion:

\[
\int dy [(y - \bar{y}) \varphi_1^{TWW} (y) + 2 y \bar{y} \varphi_3^{WW} (y) + \varphi_A^{TWW} (y)] = 0
\]

\[
\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = \frac{e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2 k^2 (k^2 + 2 Q^2 y \bar{y})}{y \bar{y} (k^2 + Q^2 y \bar{y})^2} [(2 y - 1) \varphi_1^{TWW} (y) + \varphi_A^{TWW} (y)]
\]

- **flip** transition:

\[
\Phi_{n.f.}^{\gamma_T^* \rightarrow \rho_T} (k^2) = \frac{-e m_\rho f_\rho}{\sqrt{2} Q^2} \frac{\delta^{ab}}{2 N_c} \int_0^1 dy \frac{2 k^2 Q^2}{(k^2 + Q^2 y \bar{y})^2} [(1 - 2 y) \varphi_1^{TWW} (y) + \varphi_A^{TWW} (y)]
\]
The obtained results are gauge invariant:

\[ \Phi \gamma_T^* \rightarrow \rho_T \rightarrow 0 \quad \text{when} \quad k \rightarrow 0 \]

- this is straightforward in the \( WW \) limit
- at the full twist 3 order:
  - the \( C_F \) part of the abelian 3-body contribution cancels the 2-body contribution after using the equation of motion
  - the \( N_c \) part of the abelian 3-body contribution cancels the 3-body non-abelian contribution
  - thus \( \gamma_T^* \rightarrow \rho_T \) impact factor is gauge-invariant only provided the 2 and 3-body contributions have been taken into account in a consistent way
Our results are free of end-point singularities, in both $\text{WW}$ approximation and full twist-3 order calculation:

- the flip contribution obviously does not have any end-point singularity because of the $k_0^2$ which regulates them

- the potential end-point singularity for the non-flip contribution is spurious since $\varphi_A^T(y), \varphi_1^T(y)$ vanishes at $y = 0, 1$ as well as $B(y_1, y_2)$ and $D(y_1, y_2)$. 
We have performed a full up to twist 3 computation of the $\gamma^* \rightarrow \rho$ impact factor, in the $t = t_{\text{min}}$ limit.

Our result respects gauge invariance. This is achieved only after including 2 and 3 body correlators.

It is free of end-point singularities (this should be contrasted with standard collinear treatment, at moderate $s$, where $k_T$-factorization is NOT applicable: see Mankiewicz-Piller).

Phenomenological applications will be done in the near future.

In this talk we relied on the Light-Cone Collinear approach (Ellis + Furmanski + Petronzio; Efremov + Teryaev; Anikin + Teryaev), which is non-covariant, but very efficient for practical computations.

This Light-Cone Collinear approach is systematic, and can be extended to any process, including higher twist effects (but does not preclude potential end-point singularities).
Conclusions

- Comparison with a fully **covariant approach** by Ball+Braun et al: The dictionary between the two approaches within a full twist 3 treatment is now established:

  \[
  B(y_1, y_2) = - \frac{V(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1},
  \]

  \[
  D(y_1, y_2) = - \frac{A(y_1, 1 - y_2, y_2 - y_1)}{y_2 - y_1},
  \]

  \[
  \varphi_1(y) = \phi_\parallel(y),
  \]

  \[
  \varphi_3(y) = g^{(v)}(y),
  \]

  \[
  \varphi_A(y) = - \frac{1}{4} \frac{\partial g^{(a)}(y)}{\partial y}
  \]

- We also performed calculations of the same impact factor within the **covariant approach** by Ball+Braun et al: calculations proceed in quite different way: eg. no $\varphi_{1,A}^T$—DAs but **Wilson** line effects are important!! We got a full agreement with our approach
Phenomenological prospects:

- We have all ingredients necessary to estimate:
  - $\frac{\sigma_L}{\sigma_T}$
  - elements of the density matrix
  - how important are $\bar{q}qq$ contributions compared to $\bar{q}q$ ones
  - generalizations for $t \neq 0$
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THANK YOU FOR ATTENTION