The GPD $E$, single spin asymmetries and Ji’s sum rule

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Outline:

• What do we know about $E$?
• Ji’s sum rule
• Single spin asymmetries in electroproduction of vector mesons
• Summary

based on work done in collaboration with S. Goloskokov arXiv:0809.4126
The $\gamma^* p \to VB$ amplitudes

consider large $Q^2$, $W$ and small $t$; kinematics fixes skewness: $\xi \simeq \frac{x_{Bj}}{2-x_{Bj}}[1 + m_V^2/Q^2] \simeq x_{Bj}/2 + \text{m.m.c.}$

$$
\mathcal{M}_{\mu^+,\mu^+}(V) = \frac{e_0}{2} \left\{ \sum_a e_a C_V^{aa} \langle H_{\text{eff}}^g \rangle_{V\mu} + \sum_{ab} C_V^{ab} \langle H_{\text{eff}}^{ab} \rangle_{V\mu} \right\},
$$

$$
\mathcal{M}_{\mu^-,\mu^+}(V) = -\frac{e_0}{2} \frac{\sqrt{-t}}{M+m} \left\{ \sum_a e_a C_V^{aa} \langle E^g \rangle_{V\mu} + \sum_{ab} C_V^{ab} \langle E^{ab} \rangle_{V\mu} \right\},
$$

$C_V^{ab}$ flavor factors, $M(m)$ mass of $B(p)$, $H_{\text{eff}} = H - \frac{\xi^2}{1-\xi^2} E$

contributions from $\tilde{H}$ to T-T amplitude not shown

electroproduction with unpolarized protons at small $\xi$:

$E$ not much larger than $H$ (see below) $\implies H_{\text{eff}} \to H$ for small $\xi$

$|M_{\mu^-,\mu^+}|^2 \propto t/m^2$ neglected $\implies$ probes $H$ (exception $\rho^+$)

trans. polarized target: probes $I m[\langle E \rangle \langle H \rangle^*]$ interference

polarized beam and target: probes $R e[\langle H \rangle \langle \tilde{H} \rangle^*]$ interference
Subprocess amplitudes

\[ F = H, E \quad \lambda \text{ parton helicities} \]

\[ \langle F \rangle_{V\mu}^{ab(g)} = \sum_{\lambda} \int d\bar{x} \mathcal{H}_{\mu\lambda,\mu\lambda}^{Vab(g)}(\bar{x}, \xi, Q^2, t = 0) F^{ab(g)}(\bar{x}, \xi, t) \]

\[ F^{aa} = F^a, \quad F^{ab} = F^a - F^b \quad (a \neq b) \quad (\text{with flavor symmetry}) \]

\[ \mathcal{H}_{\mu\lambda,\mu\lambda}^{Vab} = \int d\tau d^2b \hat{\Psi}_{V\mu}(\tau, -\vec{b}) \exp[-S(\tau, \vec{b}, Q^2)] \times \hat{F}_{\mu\lambda,\mu\lambda}^{ab}(\bar{x}, \xi, \tau, Q^2, \vec{b}) \]

LO pQCD

+ quark trans. mom.
+ Sudakov supp.

\[ \Rightarrow \text{lead. twist for } Q^2 \to \infty \]

Sudakov factor (Sterman et al)

\[ S \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{QCD})}{-\ln(b\Lambda_{QCD})} + \text{NLL} \]

\[ \hat{F} \quad \text{FT of hard scattering kernel} \]

\[ \text{e.g. FT of } \propto e_a/[k^2_\perp + \tau(\bar{x} + \xi)Q^2/(2\xi)] \]

regularizes also TT amplitude

in collinear appr:

TT : \[ \int_0^1 d\tau \frac{\Phi_V(\tau)}{\tau} \frac{1}{(1-\tau)t + \tau Q^2} \]

IR singular for large \( Q^2 \)

regular for large \( -t \) (wide-angles)
Double distributions

integral representation (i= valence, sea quarks, gluons)

\[ H_i(\bar{x}, \xi, t') = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi \alpha - \bar{x}) f_i(\beta, \alpha, t') + D_i \Theta(\xi^2 - \bar{x}^2) \]

\( f_i \) double distributions Mueller et al (94), Radyushkin (99)

advantage - polynomiality automatically satisfied

\( D_i(\bar{x}, t) \) (i = gluon, sea) additional free function, support \(-\xi < \bar{x} < \xi\)

useful ansatz with relation to PDFs (reduction formula respected)

\[ f_i(\beta, \alpha, t') = h_i(\beta) \exp[(\beta_i + \alpha' \ln(1/\beta))t] \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1 - |\beta|)^2 - \alpha^2]^{n_i}}{(1 - |\beta|)^{2n_i+1}} \]

\[ h_g(t = 0) = |\beta|g(|\beta|), \quad n_g = 2 \quad \alpha'_g = 0.15 \text{ GeV}^{-2} \]

\[ h_{\text{sea}}(t = 0) = q(|\beta|) \text{sign}(\beta), \quad n_{\text{sea}} = 2 \quad \alpha_{\text{sea}} = \alpha_g \]

\[ h_{\text{val}}(t = 0) = q_{\text{val}}(\beta) \Theta(\beta), \quad n_{\text{val}} = 1 \quad \alpha'_v = 0.9 \text{ GeV}^{-2} \]

sea quarks mix with gluons under evolution
assume $H$ is fairly well-known at small $\xi$ and $x \lesssim 0.6$
from analysis of cross sections and spin density matrix elements
for $\rho^0$ and $\phi$ electroproduction

Goloskokov-K. 06, 07, 08
data taken from HERMES, COMPASS, E665, H1, ZEUS

$H$ constructed from CTEQ6 PDFs through the double distr. ansatz
($D = 0$, sum rules and positivity bounds checked numerically)
What do we know about $E_v$?

analysis of Pauli FF for proton and neutron at $\xi = 0$ Diehl et al (04):

$$F_2^{p(n)} = \int_0^1 dx \left[ e_u(d) E_v^u(x, \xi = 0, t) + e_d(u) E_v^d(x, \xi = 0, t) \right]$$

ansatz for small $-t$: $E_v^a = e_v^a(x) \exp \left\{ t(\alpha_v^0 \ln(1/x) + b_v^a) \right\}$

forward limit: $e_v^a = N_a x^{-\alpha_v^0} (1 - x)^{\beta_v^a}$ (analogously to PDFs)

$N_a$ fixed from $\kappa_a = \int_0^1 dx E_v^a(x, \xi = 0, t = 0)$

fitting FF data provides: $\beta_v^u = 4$, $\beta_v^d = 5.6$

(other powers not excluded in 04 analysis)

new JLab data on $G_{E,M}^n$ up to 3.5(5.0) GeV$^2$ favor $\beta_v^u < \beta_v^d$

Input to double distribution model
$E$ for gluons and sea quarks

sum rule (Ji's s.r. and momentum s.r. of DIS) at $t = \xi = 0$

$$\int_0^1 dx x e_g(x) = e_{20}^g = -\sum e_{20}^{a_v} - 2\sum e_{20}^{\bar{a}}$$

valence term very small, in particular if $\beta^u_{v} \leq \beta^d_{v}$

$\Rightarrow$ gluon and sea quark moments cancel each other almost completely

parameterization (flavor symm. sea for $E$ assumed)

$$e^i = N_i x^{-\alpha_g(0)} (1 - x)^{\beta^i}$$

and exponential $t$ dependence:

$$\propto \exp \left\{ t (\alpha'_i \ln(1/x) + b^e_i) \right\}$$

input to double distribution model
Positivity bounds for $E$

Fourier transform of zero-skewness GPDs ($f_i^e = \alpha_i' \ln(1/x) + b_i^e$)

$$e_i(x, b) = \frac{1}{4\pi} \frac{e_i(x)}{f_i^e(x)} \exp\left[-\frac{b^2}{4f_i^e(x)}\right]$$

$H_i \rightarrow q_i(x, b) \quad \tilde{H}_i \rightarrow \Delta q_i(x, b)$

bound (Pobylitsa(02), Burkardt(04))

$$\frac{b^2}{m^2} \left( \frac{\partial}{\partial b^2} e_i(x, b) \right)^2 \leq \left[ q_i^2(x, b) - \Delta q_i^2(x, b) \right]$$

multiplication with $b^2$ and integration over $b$ leads for exponential $t$ dependence with assumption $\tilde{f}_i = f_i$ to: (Diehl et al(04))

$$\left[ \frac{e_i(x)}{q_i^2(x) - \Delta q_i^2(x)} \right] \leq 8em^2 \left[ \frac{f_i^e(x)}{f_i(x)} \right]^3 \left[ f_i(x) - f_i^e(x) \right]$$

positivity bound forbids large sea $\Rightarrow$ gluon small too
**Solutions for $E$ and Ji’s sum rule**

\[
\langle J^a \rangle = \frac{1}{2} \left[ q_{20}^a + e_{20}^a \right] \quad \langle J^g \rangle = \frac{1}{2} \left[ g_{20} + e_{20}^g \right] \\
(\xi = 0) \quad \langle J \rangle \text{ means average value of three component of } J
\]

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<th>var.</th>
<th>$\beta^u_{\text{val}}$</th>
<th>$\beta^d_{\text{val}}$</th>
<th>$\beta^g$</th>
<th>$\beta^s$</th>
<th>$N_g$</th>
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<th>$J^d$</th>
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$J^i$ quoted at scale $4 \text{ GeV}^2$  
(spread indicates uncertainties of present knowledge)

$\sum J^i = 1/2$, the spin of the proton

characteristic, stable pattern: for all variants $J^u$ and $J^g$ are large, others small
\[ \langle J^{u\nu} \rangle = 0.211(17) \quad \langle J^{d\nu} \rangle = 0.000(19) \quad \text{at scale } 4 \text{ GeV}^2 \]

Lattice (Hägler et al (07)): \( \langle J^u \rangle = 0.214(27) \), \( \langle J^d \rangle = -0.001(27) \)

at \( m_\pi(\text{phys}) \)  
sea quark contributions seem to be small

orbital angular momenta: subtract contribution from spin

\[ \langle L^i \rangle := \langle J^i \rangle - \Delta q^i \]

valence quarks (for variants 1, 2, 3):  
\( \langle L^{u\nu} \rangle \simeq -0.241, \langle L^{d\nu} \rangle \simeq 0.155, \)

\( \Delta g \) very small:  
\( \langle L^g \rangle \simeq \langle J^g \rangle \)

(note: for gluons no gauge inv. separation into \( L \) and \( S \))

\( \langle L^g \rangle \) defined as  
\( \langle J^g \rangle - \int dx \Delta g(x) \quad \text{Ji (96), Burkardt-BC (08)} \)
The first moments at $t = 0$

for orientation

(assuming that the GPDs have no nodes and similar $t$ dependence)

<table>
<thead>
<tr>
<th></th>
<th>$H$</th>
<th>$E$</th>
<th>$\tilde{H}$</th>
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<td>$u_v$</td>
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<td>$\kappa_u = 1.67$</td>
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<tr>
<td>$d_v$</td>
<td>1</td>
<td>$\kappa_d = -2.03$</td>
<td>-0.34</td>
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Results for $A_{UT}(V)$

Goloskokov-K (08)

$W = 5 \text{ GeV} \quad Q^2 = 3 \text{ GeV}^2$

variant 1, 2, 3, 4

$t$ dependence controlled by trivial factor $\sqrt{-t'}$

except for $\rho^+$: since $H^u_v - H^d_v$ small and $E^u_v - E^d_v$ large

$E$ non-negligible in cross section, contribution from helicity flip ampl. $\propto t'$

more data on $\rho^0, \omega, \phi$ from HERMES and COMPASS will come
Preliminary COMPASS result

variant 1
The $\phi$ cross section

at $Q^2 = 3.8 \text{ GeV}^2$  
HERMES ($\bullet$), ZEUS ($\square$), H1 ($\blacksquare$), CLAS ($\circ$)

Goloskokov-K (09)  
(calculation with $E$ neglected, hints at small $E^9$)

JLAB12 may measure $\phi$ cross section at $W = 3, 4 \text{ GeV}$
Summary

- an improved form factor analysis which takes into account the new JLAB data on $G_E^m$, $G_M^m$ and $G_E^p/G_M^p$, will improve knowledge of $E_{val}$

- HERMES and COMPASS measurements of $A_{UT}$ for $\rho^0$, $\phi$ and $\omega$ and ??? will probe $E_{val}$ further and may tell us whether $E_g$ and $E_{sea}$ are indeed small

- measurement of $\phi$ cross section at JLAB12 will also probe $E_g$ and $E_{sea}$