π⁺ Form Factor: Experimental Status and Prospects

Garth Huber

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The Pion Electric Charge Form Factor

\( F_\pi \) is a topic of fundamental importance to our understanding of hadronic structure, as the \( q\bar{q} \) valence structure of the \( \pi^+ \) is relatively simple.

In quantum field theory, \( F_\pi \) is the overlap integral: 

\[
F_\pi(Q^2) = \int \phi_\pi^*(p)\phi_\pi(p+q)dp
\]

The pion wave function can be separated into \( \phi_\pi^{\text{soft}} \) with only low momentum contributions \( (k<k_0) \) and a hard tail \( \phi_\pi^{\text{hard}} \).

While \( \phi_\pi^{\text{hard}} \) can be treated in pQCD, \( \phi_\pi^{\text{soft}} \) cannot.

**From a theoretical standpoint, the study of the \( Q^2 \)-dependence of \( F_\pi \) focuses on finding a description for the hard and soft contributions of the pion wave-function.**

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Determination of $F_\pi$ via Pion Electroproduction

At low $Q^2<0.3$ GeV$^2$, the $\pi^+$ form factor can be measured exactly using high energy $\pi^+$ scattering from atomic electrons.

$\Rightarrow$ 300 GeV pions at CERN SPS. [Amendolia et al., NP B277(1986)168]

$\Rightarrow$ Provides accurate measure of $\pi^+$ charge radius.

\[ r_\pi = 0.657 \pm 0.012 \text{ fm} \]

To access higher $Q^2$, one must employ the $p(e,e'\pi^+)n$ reaction.

- $t$-channel process dominates $\sigma_L$ at small $-t$.
- In the Born term model:

\[ \frac{d\sigma_L}{dt} \propto \frac{-tQ^2}{(t - m_\pi^2)^2} \; g^2_{\pi NN}(t) \; F_\pi^2(Q^2, t) \]
\[ 2\pi \frac{d^2 \sigma}{dtd\phi} = \varepsilon \frac{d\sigma_L}{dt} + \frac{d\sigma_T}{dt} + \sqrt{2\varepsilon (\varepsilon + 1)} \frac{d\sigma_{LT}}{dt} \cos \phi + \varepsilon \frac{d\sigma_{TT}}{dt} \cos 2\phi \]

Virtual-photon polarization:
\[ \varepsilon = \left( 1 + 2 \frac{(E_e - E_{e'})^2 + Q^2}{Q^2} \tan^2 \frac{\theta_e}{2} \right)^{-1} \]

1. Need to take data at smallest available \(-t\), so \(\sigma_L\) has maximum contribution from the \(\pi^+\) pole.
   - Increased reliability in the \(F_\pi\) extraction.
   - For given \(Q^2\), higher \(W\) allows smaller \(|t_{\text{min}}|\).

2. Extraction of \(F_\pi\) requires \(t\) dependence of \(\sigma_L\) to be known.
   - Only three of \(Q^2\), \(W\), \(t\), \(\theta_\pi\) are independent.
   - Keeping \(Q^2\), \(W\) fixed, vary \(\theta_\pi\) to measure \(t\) dependence.
   - Since non-parallel data needed, LT and TT must also be determined.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Chew-Low Method to determine Form Factor

$p(e,e'\pi^+)n$ data are obtained some distance from the $t=m_{\pi}^2$ pole.

→ “Chew Low” extrapolation method requires knowing the analytic dependence of $d\sigma_L/dt$ through the unphysical region.

Extrapolation method last used in 1972 by Devnish & Lyth [PRD 5,47].

- Very large systematic uncertainties.
- Failed to produce reliable result.
  → Different polynomial fits equally likely in physical region gave divergent form factor values when extrapolated to $t=m_{\pi}^2$.
- The method was subsequently abandoned.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Revisiting the Chew-Low Method

- Since 1972, the quality of the electroproduction data has improved immensely.

  \(\text{Can the Chew-Low method be used reliably now that better data are available?}\)

- Before trying this method on experimental data, we perform a simple test to see how reliably one can extrapolate to the pole.

  \(\text{Generate high precision } \sigma_L \text{ `pseudodata' versus } -t \text{ using input } F_\pi(Q^2) \text{ value.}\)

  \[ \text{Does } F_\pi^{\text{Chew-Low}} = F_\pi^{\text{input}}? \text{ (within fitting uncertainties)} \]

- Used VGL Regge Model to generate `pseudodata' in kinematic region where model describes experimental \(\sigma_L\) data well.

  \((Q^2=0.6-2.45 \text{ GeV}^2, W>2 \text{ GeV})\)

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Chew-Low Method Check with PseudoData

Plot  \[ F^2 = \frac{N}{4\hbar c g_{\pi NN}^2} \frac{(t-m_{\pi}^2)^2}{-Q^2 m_{\pi}^2} \frac{d\sigma_L}{dt} \] vs. \(-t\).

- Pure pole cross section gives straight line through origin, with value \(F_{\pi}^2(Q^2)\) at pole.

- Other contributions introduce non-linearities since don’t contain \((t-m_{\pi}^2)^2\) factor, but don’t influence \(F^2\) value at pole.
  - Do not know if behavior of \(F^2\) with \(-t\) is linear, quadratic, or higher order.

All fits missed the input \(F_{\pi}\).
  - no consistent trend on order of polynomial best able to reproduce input value
    (6-15% deviation, \(Q^2=0.6-2.45\text{ GeV}^2\)).

- Experimental data have only 4-6 \(t\)-bins and statistical and systematic uncertainties of 5-10%.
  - Extrapolation with real data will be even more uncertain.

For details see: G.M. Huber et al., PRC 78(2008)045203.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Only reliable approach is to use a model incorporating the $\pi^+$ production mechanism and the "spectator" nucleon to extract $F_\pi$ from $\sigma_L$.

- JLab $F_\pi$ experiments use the Vanderhaeghen-Guidal-Laget (VGL) Regge model as it has proven to give a reliable description of $\sigma_L$ across a wide kinematic domain.  

  [Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]

- More models would allow a better understanding of the model dependence of the $F_\pi$ result. There has been some recent interest:

Our philosophy remains to publish our experimentally measured $d\sigma_L/dt$, so that updated values of $F_\pi(Q^2)$ can be extracted as better models become available.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Extract $F_\pi(Q^2)$ from JLab $\sigma_L$ data via the VGL Regge Model

- Feynman propagator $\left( \frac{1}{t - m_{\pi}^2} \right)$ replaced by $\pi$ and $\rho$ Regge propagators.
- Represents the exchange of a series of particles, compared to a single particle.
- Model parameters fixed from pion photoproduction.
- Free parameters: $\Lambda_{\pi}$, $\Lambda_{\rho}$ (trajectory cutoff).

[Vanderhaeghen, Guidal, Laget, PRC 57(1998)1454]

\[
F_\pi = \frac{1}{1 + Q^2 / \Lambda_{\pi}^2}
\]

Fit to $\sigma_L$ to model gives $F_\pi$ at each $Q^2$.

Error bars indicate statistical and random (pt-pt) systematic uncertainties in quadrature. Yellow band indicates the correlated (scale) and partly correlated (t-corr) systematic uncertainties.

\[
\Lambda_{\pi}^2 = 0.513, 0.491 \text{ GeV}^2, \quad \Lambda_{\rho}^2 = 1.7 \text{ GeV}^2.
\]
### Issues with High $Q^2$ Cornell data

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>W (GeV)</th>
<th>$-t_{\text{min}}$ (GeV$^2$)</th>
<th>Comments</th>
</tr>
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<tbody>
<tr>
<td>1.20</td>
<td>3.08</td>
<td>0.019</td>
<td>High $\varepsilon$ unsep $\sigma$ only. $\pi^+\pi^-$ data on $^2$H used for isoscalar correction to unsep $d\sigma/dt$. PRD 13(1976)25.</td>
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<td>3.99</td>
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<td>High $\varepsilon$ unsep $\sigma$ only. $\pi^+\pi^-$ isoscalar correction from other $^2$H kinematics used. PRD 13(1976)25.</td>
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<td>1.99</td>
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<td>1.18</td>
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<td>0.069</td>
<td>High and low $\varepsilon$ from different expts used. Systematic error? PRL 37(1976)1326.</td>
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<td>1.94</td>
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<td>0.070</td>
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<td>3.33</td>
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<tr>
<td>6.30</td>
<td>2.66</td>
<td>0.43</td>
<td>Low $\varepsilon$ unsep $\sigma$ only. t-channel Born Term model used to extract $F_{\pi^-}$. Uncontrolled systematic errors! PRD 17(1978)1693.</td>
</tr>
<tr>
<td>9.77</td>
<td>2.63</td>
<td>0.87</td>
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</table>

- **Problematic L/T separation.**
  - High and low $\varepsilon$ from different expts used, or only low $\varepsilon$ setting taken.
  - In all cases, a model for $\sigma_T$ was used when extracting $\sigma_L$ and $F_{\pi^-}$.

- **Analysis based on assumptions with difficult to quantify systematic errors.**
  - Data taken far from pole, with $-t_{\text{min}}$ as high as $40 m_{\pi}^2$.

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“[we] question whether $F_{\pi^-}$ has been truly determined for large $Q^2$.”

Non-pole contributions to $\sigma_L$

In addition to Born terms, pQCD processes can also contribute to $\pi^+$ production.

Carlson and Milana [PRL 65 (1990) 1717] calculated these contributions for Cornell kinematics.
→ Asymptotic form for $F_\pi$.
→ King-Sachrajda nucleon dist’n.

For $-t > 0.2 \text{ GeV}^2$, pQCD contributions grow rapidly.
→ Sets constraint on maximum $Q^2$ for given beam energy.
→ 11 GeV: $-t = 0.21 \text{ GeV}^2$ at $Q^2 = 7.6 \text{ GeV}^2$, $W = 3.6 \text{ GeV}$, $\varepsilon = 0.06$.

<table>
<thead>
<tr>
<th>$Q^2$ (GeV$^2$)</th>
<th>$W$ (GeV)</th>
<th>$-t$ (GeV$^2$)</th>
<th>$M_{\text{pQCD}}/M_{\text{pole}}$</th>
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<td>2.63</td>
<td>0.87</td>
<td>2.82</td>
</tr>
</tbody>
</table>

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Reliable $F_\pi$ results require appropriately-chosen kinematics

- Experiment must access small $-t$ to ensure $t$-channel dominance.

- Carlson and Milana [PRL 65(1990)1717] looked at competing non-pole QCD processes complicating the extraction of $F_\pi$ at large $Q^2$.
  - background ratio $M_{pQCD}/M_{pole}$ rises dramatically once $-t_{\text{min}}>0.20$.
  - “more reliable measurements of $F_\pi$ at high $Q^2$ require smaller $|t|$ and thus higher electron energy loss $\nu$.”

- 12 GeV upgrade and SHMS small angle capability are crucial for this task.
  - ⇒ large $\nu$ ⇒ large $W$ ⇒ smaller $|t_{\text{min}}|$.
  - ⇒ reduced model uncertainty in $F_\pi$ extraction.
  - ⇒ expected smaller background to $\pi$ pole diagram.
Expected $F_\pi$ Measurements with JLab Upgrade

- 10.9 GeV electron beam and SHMS $\theta=5.5^\circ$ capability will allow $F_\pi$ to be determined up to $Q^2=6.0$ GeV$^2$.
- Slightly lower than $Q^2=7.6$ GeV$^2$ theoretical upper limit for $E_{\text{beam}}=10.9$ GeV, $-t_{\text{min}}<0.21$ GeV$^2$ due to $\Delta\varepsilon>0.25$ needed for reliable L/T separation.
- Many model dependence tests also planned in E12-06-101.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
\[ F_\pi \text{ at Even Larger } Q^2? \]

*If* larger \(-t_{min}\) were usable, we could determine \(F_\pi\) up to \(Q^2=9 \text{ GeV}^2\) at 11 GeV.

Even at 6 GeV, data at \(Q^2=4 \text{ GeV}^2\) already exist.

**Needed:**
- More favorable L/T ratios than observed to date at higher \(Q^2, -t\).
  - Cornell data of too poor quality to provide useful L/T prediction.
- Model used to extract \(F_\pi\) from \(\sigma_L\) must pass reliability tests over the entire \(-t\) range of the data.
- Non-pole background studies, such as \(p(e,e^+\pi^0)p\), also important.

\(-t_{min} = 0.45 \text{ GeV}^2\)

\(Q^2=4 \text{ GeV}^2\) separated \(\pi^+\) cross sections

*T. Horn et al., PRC 78(2008)058201.*

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
A common criticism of the electroproduction technique is the difficulty to be certain one is measuring the “physical” form factor.

“What is at best measured in electroproduction is the transition amplitude between a mesonic state with an effective space-like mass $m^2=t<0$ and the physical pion. It is theoretically possible that the off-shell form factor $F_{\pi}(Q^2,t)$ is significantly larger than the physical form factor because of its bias towards more point-like $q\bar{q}$ valence configurations within its Fock state structure.”


What tests/studies can we do to give confidence in the result?

- Check consistency of model with data.
- Extract form factor at several values of $-t_{min}$ for fixed $Q^2$.
- Test that the pole diagram is really the dominant contribution to the reaction mechanism.
- Verify that electroproduction technique yields results consistent with $\pi$-e elastic scattering at same $Q^2$. 
VGL $p(e,e'\pi^+)n$ model check

- To check whether VGL Regge model properly accounts for:
  - $\pi^+$ production mechanism.
  - spectator nucleon.
  - other off-shell ($t$-dependent) effects.

  extract $F_\pi$ values for each $t$-bin separately, instead of one value from fit to all $t$-bins.

  Error band based on fit to all $t$-bins.

- Deficiencies in model may show up as $t$-dependence in extracted $F_\pi(Q^2)$ values.
- Resulting $F_\pi$ values are insensitive ($<2\%$) to $t$-bin used.
- Lends confidence in applicability of VGL model to the kinematical regime of the JLab data, and the validity of the extracted $F_\pi(Q^2)$ values.

Only statistical and $t$-uncorrelated systematic uncertainties shown.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Form Factor Extraction at different $-t_{\text{min}}$

Does the VGL model handle the “off-shellness” of the pion appropriately?

Test by extracting $F_\pi$ at different distances from pole.

Expt: $F_\pi^2$, $-t_{\text{min}}=0.093 \text{ GeV}^2$

$W=2.22 \text{ GeV}$.

$F_\pi^2$, $-t_{\text{min}}=0.15 \text{ GeV}^2$

$W=1.95 \text{ GeV}$.

$W=2.22$ point 30% closer to pole.  
→ Agreement ~4%.

Additional data after 12 GeV upgrade will allow further tests:

$Q^2=1.6 \text{ GeV}^2$  
$-t_{\text{min}}=0.029 \text{ GeV}^2$, $W=3.00 \text{ GeV}$.

$Q^2=2.45 \text{ GeV}^2$  
$-t_{\text{min}}=0.048 \text{ GeV}^2$, $W=3.20 \text{ GeV}$.
Experimental verification that $\sigma_L$ is dominated by the $\pi$ $t$-channel process

- $\pi^+$ $t$-channel diagram is purely isovector (G-parity conservation).
- $^2\text{H}$ target L/T separations

\[
R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \left|\frac{A_V - A_S}{A_V + A_S}\right|^2
\]

- Isoscalar backgrounds (such as $b_3(1235)$ contributions to $t$-channel) will dilute ratio.

Because one of the many problems encountered by the historical data was isoscalar contamination, this test is performed to increase the confidence in the extraction of $F_\pi(Q^2)$ from the $\sigma_L$ data.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
**Experimental $\sigma_L[\pi^-]/\sigma_L[\pi^+]$ Ratios**

\[
R_L = \frac{\sigma_L[n(e,e'\pi^-)p]}{\sigma_L[p(e,e'\pi^+)n]} = \frac{|A_V - A_S|^2}{|A_V + A_S|^2}
\]

- **$R_L$ values consistent with pion-pole dominance of $\sigma_L$ data at low $-t$ ($|A_y/A_V| < 5\%$).**

- **Further tests planned in E12-06-101 at $Q^2=1.60$ and 3.50 GeV$^2$.**

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**Notes:**

Error bars indicate statistical and estimated random (pt-pt) systematic uncertainties in quadrature. Correlated (scale) and partly correlated (t-corr) systematic uncertainties are not yet added.

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Electroproduction Method Check

Directly compare $F_\pi(Q^2)$ values extracted from very low $-t$ electroproduction with the exact values measured in elastic $e-\pi$ scattering.

METHOD PASSES CHECKS:

• $Q^2=0.35 \text{ GeV}^2$ data from DESY consistent with limit of elastic scattering data within uncertainties.

  [H. Ackermann, et al., NP B137(1978)294]

• A much better check is planned in E12-06-101 by taking $Q^2=0.30 \text{ GeV}^2$ data at 50% lower $-t$ (0.005 GeV$^2$).

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Question: Two $\gamma$ exchange?

- In the Rosenbluth separation of the proton electric form factor, $2\gamma$ contributions may be important because one is trying to separate a small cross section (electric) from a much larger (magnetic) one.

- $2\gamma$ exchange is not expected to be a significant issue in the extraction of $\sigma_L$ in pion electroproduction.

$Q^2=6$ GeV$^2$ calculation performed by Tjon and Melnitchouk. 
Correction=$\delta_{\text{FULL}}-\delta_{\text{Mo&Tsai}}$

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.
Summary

- **Measurement of the charged pion form factor at large $Q^2$** requires quality $p(e,e'\pi^+)n\sigma_L$ data close to the pole.
  - Cornell high $Q^2$ results are difficult to interpret because of problematic L/T separations and large distance from pole.

- **Use best available model(s) for $\sigma_L$ to extract $F_{\pi^*}$**.
  - JLab 6 GeV data provided useful tests of the validity of the model used ($t$-dependence, extractions at different $-t_{min}$).
  - 12 GeV will allow even more checks.

- **Can test electroproduction method by taking data near overlap with elastic $e-\pi$ results.**
  - Direct comparison of DESY $Q^2=0.35$ GeV$^2$ data to elastic $\pi$-$e$ data.
  - Better test planned for 12 GeV.

- **Can take $\pi^\pm$ data on deuterium target to investigate isoscalar backgrounds.**

12 GeV upgrade + SHMS controlled systematics and small angle capability present a unique opportunity to explore $F_{\pi}(Q^2)$. 

Dr. Garth Huber, Dept. of Physics, Univ. of Regina, Regina, SK S4S0A2, Canada.