T-odd PDFs in “a” Spectator Framework

Three-dimensional parton structure of the nucleon encoded in GPDs and TMDs

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also Piet Mulders and Asmita Mukherjee
• **Comments: Transverse Spin Effects & TSSAs**

• **Reaction Mechanisms**
  - Colinear-limit ETQS-Twist Three . . .
  - ISI/FSI-Twist Two

• **Gauge links-Color Gauge Invariance**
  “T-odd” TMDs-Gluonic poles

• **Transverse Distortion and TSSAs**

• **Rlnts. btwn leading twist T-odd PDFs and IP-GPDs?**

\[
f_{1T}^{\perp}(x, k_{\perp}^2) \quad \mathcal{E}(x, b_{\perp}^2)
\]
Transverse SPIN Observables SSA (TSSA) \( p^\uparrow p \rightarrow \pi X \)

- **Single Spin Asymmetry**
  \[ A_N = \frac{\sigma^\uparrow(x_F, p_\perp) - \sigma^\downarrow(x_F, -p_\perp)}{\sigma^\uparrow(x_F, P_\perp) + \sigma^\downarrow(x_F, -P_\perp)} \equiv \Delta \sigma \]

- **Rotational invariance**
  \[ \sigma^\downarrow(x_F, P_\perp) = \sigma^\uparrow(x_F, -P_\perp) \]
  \( \Rightarrow \) **Left-Right Asymmetry**

* Parity Conserving interactions: SSAs “Transverse” Scattering plane
  \( \Rightarrow \Delta \sigma \sim iS_T \cdot (P \times P_T) \)

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W.H. Dragoset et al.,
PRL 36, 929 (1976)

Argonne ZGS, \( P_{\text{beam}} = 12 \text{ GeV/c} \)

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200 GeV Beam
\( \sqrt{s} = 20 \text{ GeV} \)

- FNLAL-E704
- PLB261, 201 (1991)
- PLB264, 462 (1991)
Transverse SSA’s at $\sqrt{s} = 62.4$ & 200 GeV at RHIC

PRL 101, 042001 (2008)

see Talk of Les Bland
Sensitivity of $p_T \sim k_\perp \ll \sqrt{Q^2}$

**HERMES** $ep\uparrow \Rightarrow \pi X$

Compass-proton data 2007
comparison w/ HERMES-Collins

**see talk of Delia Hasch**
Unpolarized DRELL YAN–Azimuthal Asymmetry

\[ \pi^- + p \rightarrow \mu^+ + \mu^- + X \]

\[ \frac{dN}{d\Omega} = \left( \frac{d\sigma}{d^3q} \right)^{-1} \frac{d\sigma}{d^3q d\Omega} = \frac{3}{4\pi} \frac{1}{\lambda + 3} \left( 1 + \lambda \cos^2 \theta + \mu \sin^2 \theta \cos \phi + \nu \frac{\sin^2 \theta \cos 2\phi}{2} \right) \]

\[ \lambda, \mu, \nu \text{ depend on } s, x, m_{\mu\mu}, p_T \]

Lam-Tung relation (NLO QCD) \( 1 - \lambda - 2\nu \approx 0 \)

Unexpectedly large \( \cos 2\phi, \nu \approx 10 - 30\% \ AA \)

- Suggests a Non-Pertb. origin! Boer PRD 1999

E615, Conway et al. 1989, NA10, ZPC(1986)
TSSA requires relative phase between different helicity amps

\[ | \uparrow / \downarrow \rangle = (|+\rangle \pm i|-\rangle) \Rightarrow \hat{A}_N = \frac{\hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow}{\hat{\sigma}^\uparrow + \hat{\sigma}^\downarrow} \sim \frac{2 \text{Im} f^* f^\prime}{|f^+|^2 + |f^-|^2} \]

Co-linear factorized QCD-parton dynamics

\[ \Delta\sigma^{pp^\uparrow \to \pi X} \sim f_a \otimes f_b \otimes \Delta\hat{\sigma} \otimes D^{q^\to \pi} \]

Requires helicity flip-hard part \( \Delta\hat{\sigma} \equiv \hat{\sigma}^\uparrow - \hat{\sigma}^\downarrow \)

QCD interactions conserve helicity

\( m_q \to 0 \) and Born amplitudes real

\[ A_N \sim \frac{m_q \alpha_s}{E} \] Kane, Repko, PRL:1978

M. Anselmino hep-ph/0201150

“This makes single spin asymmetries in the partonic interactions entirely negligible”
Early Experiment-Δ Production $pp \rightarrow \Lambda^{\uparrow} X$


- Experiment *at odd with this result*

$P_\Lambda$ in $pp$ and $pBe$ scattering-Fermi Lab

$P_\Lambda = \frac{\sigma^{pp\rightarrow\Lambda^{\uparrow}X} - \sigma^{pp\rightarrow\Lambda^{\downarrow}X}}{\sigma^{pp\rightarrow\Lambda^{\uparrow}X} + \sigma^{pp\rightarrow\Lambda^{\downarrow}X}}$

**FIG. 3.** (a) $\Lambda^0$ polarization from this experiment compared to that from 300-GeV incident protons from Ref. 1 as a function of $p_T$. The number in parentheses is the average value of $x$ for that point. (b) $\Lambda^0$ and $\bar{\Lambda}^0$
QCD test-Λ Production $pp \rightarrow \Lambda^\uparrow X$

$$P_\Lambda = \frac{\sigma_{pp\rightarrow\Lambda^\uparrow X} - \sigma_{pp\rightarrow\Lambda^\downarrow X}}{\sigma_{pp\rightarrow\Lambda^\uparrow X} + \sigma_{pp\rightarrow\Lambda^\downarrow X}}$$

- Need strange quark to polarize a Λ

Interference of loops and tree level Phases in hard part $\Delta\hat{\sigma}$

FIG. 1. Feynman diagrams for gluon fusion, $g + g \rightarrow s + \bar{s}$. In the second order, only the diagrams which contribute to the imaginary amplitude are shown.

Dharmartna & Goldstein PRD 1990

FIG. 4. Strange-quark polarization in the proton c.m. frame, $P_{c.m.} = 14$ GeV/c (400-GeV beam), after the convolution for the initial state gluons. $x_F$ is the Feynman $x$ for the strange quark. Dashed curve corresponds to $P_{c.m.} = 30.6$ GeV/c.
• Non-perturbative origin many theorists. . .

\( Q \sim P_T \gg \Lambda_{\text{qcd}} \) Co-linear Twist Three Mechanism

Phases in soft poles of propagator in hard subprocess Efremov & Teryaev: PLB 1982

\[ \frac{1}{x_{\pm} x_{\mp}} = P \left( \frac{1}{x_{\pm}} \right) \mp \pi \delta(x_{\pm}) \]

\[ \Delta \sigma \sim f_a \otimes T_F \otimes H_{\text{ETQS}} \otimes D^q \rightarrow \pi \]


Get helicity flips and phases \( m_q \rightarrow \sim M_H \)

Transversity in pp Koike 2002

Transverse SPIN Observables SSA (TSSA)

\[ \pm \frac{1}{2} \sigma \]

\[ \Delta \sigma \sim \delta q(x) f(x') \otimes \hat{H}(z_1, z_2) \otimes \hat{\sigma} \]

see talk of Feng Yuan
Sensitivity to $p_T \sim k_\perp << \sqrt{Q^2}$ TSSAs thru “T-Odd” TMD

- **Sivers PRD: 1990** TSSA is associated with correlation transverse spin and momenta in initial state hadron

- **Collins NPB: 1993** TSSA is associated with momenta

\[
\Delta \sigma_{pp}^{\uparrow \rightarrow \pi X} \sim D \otimes f \otimes \Delta f^\perp \otimes \hat{\sigma}_{\text{Born}} \quad \Rightarrow \quad \Delta f^\perp(x, k_\perp) = i S_T \cdot (P \times k_\perp) f_{1T}^\perp(x, k_\perp)
\]

- **Collins NPB: 1993** TSSA is associated with transverse spin of fragmenting quark and transverse momentum of final state hadron

\[
\Delta \sigma_{ep}^{\uparrow \rightarrow e\pi X} \sim \Delta D^\perp \otimes f \otimes \hat{\sigma}_{\text{Born}} \quad \Rightarrow \quad \Delta D^\perp(x, p_\perp) = i s_T \cdot (P \times p_\perp) H_1^\perp(x, p_\perp)
\]
**T-ODD Transverse Spin Transverse Momentum Correlations**

Boer, Mulders PRD: 1998

Correlation of transversely polarized *quark spin* with intrinsic $k_\perp$

$$i s_T \cdot (k_\perp \times P) \rightarrow h_{1T}^+(x, k_\perp)$$

$$i S_T \cdot (k_\perp \times P) \rightarrow f_{1T}^+(x, k_\perp),$$
Mechanism FSI produce phase in TSSAs - Leading Twist

Brodsky, Hwang, Schmidt PLB: 2002
SIDIS w/ transverse polarized nucleon target

\[ e \uparrow p \rightarrow e \pi X \]

Ji, Yuan PLB: 2002 - Sivers fnct. FSI emerge from Color Gauge-links

L.G & Goldstein 2002, 2003 Boer-Mulders Fnct,

Collins PLB 2002 - Gauge link Sivers function doesn’t vanish

Ji, Ma, Yuan: PLB, PRD 2004, 2005 Extend factorization of CS-NPB: 81
T-Odd Effects From Color Gauge Inv. via Wilson Line

see talk Piet Mulders

Amsterdam grp Boer Bomhof Mulders Pijlman, et al. 2003 - 2008 ....

- Sub-class of interactions of colinear & transverse gluons re-summed to render physical process color gauge invariant

- Gauge link emerges from resummation of gluon ISI and FSI btw. active quark and hadron remnants

\[ U_{[\eta, \xi]}^{[C]} = \mathcal{P} \exp(-ig \int_C ds^\mu A_\mu) \]

etc . . .

- The path \([C]\) is fixed by hard subprocess within hadronic process.
**T-Odd Effects** From Color Gauge Inv. via Wilson Line

**Fund. Prediction of QCD**
\[ f_{1T_{SIDIS}}(x, k_T) = - f_{1T_{DY}}(x, k_T) \]

**Process Dependence**
Collins PLB 02, Brodsky, Hwang, Schmidt NPB 02

\[ d\sigma = L_{\mu\nu} W^{\mu\nu} \Rightarrow \]

SIDIS Hadronic Tensor

Drell-Yan Hadronic Tensor

\[ \Phi[+]^*(x, p_T) = i \gamma^1 \gamma^3 \Phi[-](x, p_T) i \gamma^1 \gamma^3 \]
“Observables” and TMD
Correlators in SIDIS

\[ \Phi(x, p_T) = \frac{1}{2} \left\{ f_1(x, p_T) \frac{[\psi_T, p]}{2M} + i h_1^T(x, p_T) \frac{[\psi_T, p]}{2M} - f_1^T(x, p_T) \frac{\epsilon_{ij} p_{Ti} S_{Tj}}{M} p \cdots \right\} \]

\[ \Delta(z, k_T) = \frac{1}{4} \left\{ z D_1(z, k_T) p_h + i z H_1^T(z, k_T) \frac{[k_T, p_h]}{2M_h} - z D_{1T}^T(z, k_T) \frac{\epsilon_{ij} k_{Ti} S_{Tj}}{M_h} p_h + \cdots \right\} \]

SIDIS cross section

\[ d\sigma_{\ell N \rightarrow \ell \pi X}^{\{\lambda, \Lambda\}} \propto f_1 \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes D_1 + \frac{k_{\perp}}{Q} f_1 \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes D_1 \cdot \cos \phi \]

\[ + \left[ \frac{k_{\perp}^2}{Q^2} f_1 \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes D_1 + h_1^T \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes H_1^T \right] \cdot \cos 2\phi \]

\[ + |S_T| \cdot h_1 \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes H_1^T \cdot \sin(\phi + \phi_S) \quad \text{Collins} \]

\[ + |S_T| \cdot f_{1T} \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes D_1 \cdot \sin(\phi - \phi_S) \quad \text{Sivers} \]

\[ + |S_L| \cdot h_{1L} \otimes d\hat{\sigma}^{\ell q \rightarrow \ell q} \otimes H_1^T \cdot \sin(2\phi) \quad \text{Kotzinian–MuldersPLB} \]
Spect. model workbench ISI/FSI in AA & TMDs $h_1^+, f_{1T}^+, H_1^+$ gluonic poles

- \( \Phi[U[C]](x, p_T) = \int \frac{d\xi^- d^2\xi_T}{2(2\pi)^3} e^{ip \cdot \xi} \langle P|\overline{\psi}(0)U[C]_\xi \psi(\xi^-, \xi_T)|P\rangle|_{\xi^+ = 0} \)

- Use Spectator Framework Develop a QFT to explore and estimates these effects with gauge links
  - BHS FSI/ISI Sivers fnct, -PLB 2002, NPB 2002
  - Ji, Yuan PLB 2002 - Sivers Function
  - Metz PLB 2002 - Collins Function
  - L.G. Goldstein, 2002 ICHEP- Boer Mulders Function
  - L.G. Goldstein, Oganessyan TSSA & AAS PRD 2003-SIDIS
  - Boer Brodsky Hwang PRD 2003-Drell Yan Boer Mulders
  - Bacchetta Jang Schafer 2004- PLB, Flavor-Sivers, Boer Mulders
  - Lu Ma Schmidt PLB, PRD, 2004/2005 Pion Boer Mulders
  - L.G. Goldstein DY and higher twist, PLB 2007
  - LG, Goldstein, Schlegel PRD 2008-Flavor dep. Boer Mulders $\cos 2\phi$ SIDIS
  - Conti, Bacchetta, Radici, Ellis, Hwang, Kotzinian 2008 hep-ph . . . !

- Spectator Model “Field Theoretic” used study Universality of T-odd Fragmentation $\Delta_{ij}$
  - Metz PLB 2002, Collins Metz PRL 2004
  - Bacchetta, Metz, Yang, PBL 2003, Amrath, Bacchetta, Metz 2005,
  - Bacchetta, L.G. Goldstein, Mukherjee, PLB 2008
  - Collins Qui, Collins PRD 2007,2008
  - Yuan 2-loop Collins function PRL 2008
  - L.G., Mulders, Mukherjee Gluonic Poles PRD 2008
Many other model calculations of TMDs

Pasquini, Schweitzer et al.
Bacchetta Conti Radici
Liuti and co-workers
Hwang and Mueller
many more......
By using the Ward Identity:

same Collins fun.

See Feng Yuan’s Talk
Spectral Analysis Gluonic Poles-Fragmentation

L.G., Mukherjee, Mulders PRD 08

• In this approach rather than integrating over the longitudinal component of the “loop momenta” we look at the limit of a zero gluon momentum in quark-gluon-quark matrix element

• That is considering the multi-parton correlators $\Phi_G(k, k - k_1)$ and $\Delta_G(k, k - k_1)$ in light-cone gauge
Taking the limit $x_1 \rightarrow 0$ we get the gluonic pole correlators, for distribution functions ($0 \leq x \leq 1$),

$$\Phi_G(x, x) = - \int d^2k_T d^2k_{1T} \frac{(1 - x)F_1(x, 0, k_T, k_{1T})\theta(1 - x)}{(\mu^2 - k_T^2)(xB_1 + (1 - x)A_2)A_1},$$

and for fragmentation functions ($x = 1/z \geq 1$)

$$\Delta_G(x, x) = 0$$
Collins Qiu 2007 PRD Breakdown of factorization
see talk of J.W Qiu and Ted Rogers.....

\[ H_1 + H_2 \rightarrow H_3 + H_4 + X \]

**FIG. 8:** The exchange of two extra gluons, as in this graph, will tend to give non-factorization in unpolarized cross sections.
Mechanisms explored thru T-odd Contribution SIDIS and Drell Yan

Impacts pre and “post”-dictions at COMPASS, HERMES, JLAB 6 & 12 GeV FAIR, RHIC, JPARC

\( \cos 2\phi \) Asymmetry in SIDIS- “Boer Mulders Effect”

- Early wk. in spectator framework  

\[
 h_1^{(s)}(x, k_{\perp}) = f_{1T}^{(s)}(x, k_{\perp})
\]

- Collins, Sivers and Boer Mulders Asymmetries with Gaussian Distribution in \( k_{\perp} \)  

\[
 h_1(x, k_{\perp}) = \alpha_s N_s \frac{M(m + xM)(1 - x)}{k_{\perp}^2 \Lambda(k_{\perp}^2)} R(k_{\perp}^2, x)
\]
Spectator Framework: BM $h_{1}^{+}(1/2)$, Sivers $f_{1T}^{+}(1/2)$, $f_1(x)$ and $h_{1L}^{+}$

- **Quark-Quark Correlator**

$$
\Phi_{ij}(x, \vec{p}_T) = \sum_{\lambda} \int \frac{dz d^2 z_T}{(2\pi)^3} e^{ixP^+z^- - i\vec{p}_T \cdot \vec{z}_T} \langle P, S | \bar{\psi}_j(0) U[0, \infty^-] | X \rangle \langle X | U[\infty^-, z] \psi_i(z) | P, S \rangle
$$

- **Diquark-model:** $|X\rangle \longrightarrow |dq; q, \lambda\rangle$ **one particle-state**
  Thomas and Melnitchouk 1994, Mulders Rodrigues 1997

- **Two kinds of diquarks:** Scalar (spin $0^+$) and Axial-vector (spin $1^+$)

**Specification of Nucleon-Diquark-Quark vertex:**

- **Ingredients**, sufficient for T-even PDFs, e.g. $f_1^{(u)}$ and $f_1^{(d)}$

$$
\gamma_{sc}^{\mu} = g(p^2) \quad \gamma_{ax}^{\mu} = \frac{g(p^2)}{\sqrt{3}} \gamma_5 \left( \gamma^{\mu} - R g \frac{P^{\mu}}{M} \right)
$$
• **T-odd PDFs:** consequence of Gauge link \( \rightarrow \) 1 Gluon exchange approximation

\[
\mathcal{W} \longrightarrow \quad \text{will describe} \quad ...
\]

\[
\epsilon_T^{ij} k_T^i S_T^j \frac{1}{f_{TT}(x, k_T^2)} = -\frac{M}{8(2\pi)^3 (1-x) P^+} \left[ \mathcal{W} \gamma^+ W \right]_{S_T} - \mathcal{W} \gamma^+ W \left]_{-S_T} \right.
\]

• **gauge boson-axial vector diquark coupling**

\[
\Gamma_{a,x}^{\mu \nu_1 \nu_2} = -ie_d q [g^{\nu_1 \nu_2} (p_1 + p_2)^\mu + (1 + \kappa) (g^{\mu \nu_2} (p_2 + q)^\nu_1 + g^{\mu \nu_1} (p_1 - q)^\nu_2)]
\]

• **axial-vector diquark and scalar diquark propagator:**

\[
\mathcal{D}_{ax}^{\mu \nu}(P - p - l) = \frac{-i (g^{\mu \nu} - \frac{(P-p-l)^\mu (P-p-l)^\nu}{m_s^2})}{(P-p-l)^2 - m_s^2 + i0}; \quad \mathcal{D}_{sc}(P - p - l) = \frac{-i}{(P-p-l)^2 - m_s^2 + i0}
\]
\[
W_i(P, k, S) = -ie_q^*\delta_{dq}\int \frac{d^4l}{(2\pi)^4} \frac{g_{ax}(p + l)^2}{\sqrt{3}} \epsilon^*_{\sigma}(P - p, \lambda) D_{\rho\eta}(P - p - l)
\]
\[
\times \left[ g^{\sigma\rho} v \cdot (2P - 2p - l) + (1 + \kappa)(v^\sigma(P - p + l)^\rho + \nu^\rho(P - p - 2l)^\sigma) \right]
\]
\[
\times \left[ (\bar{P} + l + m_q)\gamma_5 \left( \gamma^\eta - \frac{R_g}{M} u(P, S) \right) \right].
\]

\[(T\text{-odd}) \text{TMDs:}\]
\[
\epsilon^{ij}_T k^i_T S^j_T f^\perp_{1T}(x, \vec{k}_T^2) = -\frac{M}{8(2\pi)^3(1 - x)P^+} \left( \bar{W}\gamma^+ W \bigg|_{S_T^+} - \bar{W}\gamma^+ W \bigg|_{-S_T^+} \right)
\]
Technical Issues (that do not arise in a quark target model !!!)

**Loop-Integral (axial-vector diquark)**

- **Simplification the numerator** → sort by powers of loop-momentum $l$

  $$\rightarrow J^{(i)\alpha_1...\alpha_i} = \int \frac{d^4l}{(2\pi)^4} \frac{g((l+p)^2)g(p^2)l^{\alpha_1...\alpha_i}}{[(v\cdot l)+i0][l^2+i0][(l+p-P)^2-m_s^2+i0][(l+p)^2-m_q^2+i0]}$$

- $v = \left[1^- , 0^+ , \vec{0}_T \right]$, $l^+ \rightarrow 0$, $l^- \rightarrow \infty$, when $\alpha_k = -$  
  $\Rightarrow$ *Light cone rapidity divergence*

- $\int dl^+ \delta(l^+) \Theta(l^+)$
• **Regularization procedure** Collins:NPB 1982 , Ji, Ma, Yuan PLB: 2004 :

1) (LG, Hwang, Metz, Schlegel, PLB 2006)

*Introduction of Wilson lines off the light cone*, $v = \left[ 1^{-}, \lambda^{+}, \vec{0}_{T} \right]$, 

$$\rightarrow h_{1,ax}^{\perp}(x, p_{T}^{2}, v) \propto \ln \left( \frac{v^{2}}{v \cdot P} \right)$$

2) **Phenomenological procedure**: Form factor $g(p^{2})$

$$g(p^{2}) = N^{2n} \frac{[p^{2} - m_{q}^{2}] F(p^{2})}{[p^{2} - \Lambda^{2} + i0]^{n}}$$

$\rightarrow$ additional pole produces additional factor $[l^{+}]^{n}$ in numerator $\rightarrow$ Regularization.
Flavor Dependence: Results & Phenomenology

Flavor-dependent PDFs from diquark models: \( u = \frac{3}{2}s + \frac{1}{2}a, \; d = a, \)

moments: \( h_1^{(1/2)}(x) = \int d^2 \vec{p}_T \frac{1}{M} h_1^{(1/2)}(x, \vec{p}_T) \)

L.G. Goldstein, Schlegel PRD 2008

- Comparison to \( f_1^{(u,d)} \) (Glück, Reya, Vogt) → parameters of the model,

- Boer Mulders up and down are negative is spectator model and \( f_{1T}^{(u)} \sim h_1^{(u)} \)

\[ f_{1T} = \frac{1}{2} \left( f^{(u)} - f^{(d)} \right) \]
Supports

- Lg $N_C$ arguments Pobylitsa hep-ph/0301236
- Bag Model calculation Yuan PLB 2003
- Implications $\cos 2\phi$ phenomenology in SIDIS & Drell Yan
- Lattice QCDSF/UKQCD, Hägler et al... calculations of matrix elements on the lattice

Preliminary results for the spin densities

Ph. Hägler, QCDN'06
Bacchetta, L.G., Goldstein, Mukherjee Re-analysis and Kaons

\[ u \rightarrow \pi^+ \]

- BGGM \( Q^2 = 0.4 \text{ (GeV/c)}^2 \)
- BGGM \( Q^2 = 110 \text{ (GeV/c)}^2 \)
- EGS Error

\[ Z \]

\[ H_1 \]

(a) + (b) + (c) + (d) + H.c.
Present State of Pheno . . . Pion Fragmentation Function

Bacchetta, L.G., Goldstein, Mukherjee PLB 2008 $Q_0^2 = 0.4$ GeV$^2$

Normalized to Kretzer, PRD: 2000
In particular, the number of pions in this case has an azimuthal dependence [34]. Asymmetries in the Collins function (20) thus evolve as a non-singlet. An alternative choice could be to assume the positivity bound (35) would be violated at large $x$. Where standard NLO calculations give a reasonable agreement with the phenomenology, we choose a value of the strong coupling constant $\alpha_s$.

Obviously, also the mass of the hadron changes: we take $m = 78$ GeV for the pions and $m = 135$ GeV for the kaons. We use the following definition of the Collins function [12] for the fragmentation into a pion contributing to the Collins function in the eikonal approximation. “H.c.” stands for the Hermitian conjugate diagrams which are not shown.

Fig. 3. Single gluon-loop corrections to the fragmentation of a quark into a pion contributing to the Collins function in the eikonal approximation. “H.c.” stands for the Hermitian conjugate diagrams which are not shown.

Fig. 4. Half moment of the Collins function for $u \to \pi^+$ in our model. (a) $H_1^{1/2}$ at the model scale (solid line) and at a different scale under the assumption in Eq. (37) (dot-dashed line), compared with the error band from the extraction of Ref. [6]. (b) $H_1^{1/2}/D_1$ at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38). The error band from the extraction of Ref. [7] is shown for comparison.
Scaling-”evolution”

\[
\frac{H_1^{(1/2)}}{D_1} \bigg|_{Q_0^2} = \frac{H_1^{(1/2)}}{D_1} \bigg|_{Q^2},
\]

Fig. 5. Half moment of the Collins function for \( u \rightarrow K^+ \) in our model. (a) \( H_1^{(1/2)} \) at the model scale of 0.4 GeV\(^2\), (b) \( H_1^{(1/2)}/D_1 \) at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38).

Fig. 6. Half moment of the Collins function for \( \bar{s} \rightarrow K^+ \) in our model. (a) \( H_1^{(1/2)} \) at the model scale of 0.4 GeV\(^2\), (b) \( H_1^{(1/2)}/D_1 \) at the model scale (solid line) and at two other scales (dashed and dot-dashed lines) under the assumption in Eq. (38).
Fig. 7. Azimuthal asymmetry $A_{12}(z_1, z_2)$ for the production of two pions as a function of $z_2$ and integrated in bins of $z_1$ at $Q^2 = 110.7$ GeV$^2$. Dashed lines are obtained assuming Eq. (37), solid lines assuming Eq. (38). Note that the last $z_1$ bin in our calculation is narrower than in the corresponding experimental measurement.

Fig. 8. Azimuthal asymmetry $A_{12}(z_1, z_2)$ for the production of two kaons as a function of $z_2$ and integrated in bins of $z_1$ at $Q^2 = 110.7$ GeV$^2$. Dashed lines are obtained assuming Eq. (37), solid lines assuming Eq. (38).

\[ A_{UU}^{\cos(2\phi_h)} \propto \sum_a e_a^2 \int d^2 p_T d^2 k_T \delta^{(2)} \left( \vec{p}_T - \vec{k}_T - \frac{\vec{p}_{h\perp}}{z_h} \right) \frac{\vec{k}_T \cdot \vec{p}_T - 2(\vec{h} \cdot \vec{k}_T)(\vec{h} \cdot \vec{p}_T)}{M m_{\pi}} h_1^{(a)}(a) H_1^{(a)} \]

Model assumption:

Dis-favored fragmentation

\[ H_1^{\perp(d \rightarrow \pi^+)} = -H_1^{\perp(u \rightarrow \pi^+)} \]

\[ \sum_h \int_0^1 dz H_1^{(1)(q \rightarrow h)} = 0 \]  
Schäfer and Teryaev, PRD 2000
Theory issues twist-4 Cahn terms and gluon brem/higher twist


and talk of Alexei Prokudin
<cos(2φ_h)>: Model 1

Gamberg et al.


Diquark spectator model does well... without Cahn term
Beyond the One-Loop Approximation

**So far:** Most phenomenological approaches to T-odd TMDs
→ Final state interactions modeled by a **one-gluon exchange**

- e.g. Diquark-model, MIT-Bag model etc.
- Sivers-effect ~5%,
  \[ f_{1T}^{(1)} \sim \pm 0.05 \]
- \( \alpha_s \simeq 0.2 - 0.3 \) “strength of FSI”

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BHS (02)
Ji-Yuan (02)
LG, G. Goldstein (02,03..)
LG, GRG, Schlegel (08)
Bacchetta et al. (04, 08)
Lu Schmidt (05, 06)
Can we do better?

• Still work within **spectator framework**, but *non-perturbative model of FSI*.

\[ W \sim \]

\[ \epsilon^{ij} k^i_T S^j_T f^{+\perp}_{1T}(x, \vec{k}_T^2) = - \frac{M}{8(2\pi)^3(1-x)P^+} \left( \bar{W} \gamma^+ W \Big|_{S_T} - \bar{W} \gamma^+ W \Big|_{-S_T} \right) \]
• Relativistic Eikonal models: Treat FSI non-perturbatively.

\[
\int \frac{d^4 q}{(2\pi)^4} g_N [(P - q)^2] \frac{[(P - \not q + m_q)u(P, S)]_i \mathcal{M}_{bc}^{ab}(q, P - k)}{[n \cdot (P - k - q) + i\varepsilon][(P - q)^2 + m_q^2 + i\varepsilon][q^2 - m_s^2 + i\varepsilon]}
\]

• Only diagrams that reflect the ”naive picture”.

• **Step 1: Integration over q−**: Assume no q− & q+ poles in M.

q− - poles at one loop for higher twist T-odd TMDs [Gamberg, Hwang, Metz, MS, PLB 639, 508]

• **Step 2: Integration over q+**: Fix the q+ - pole emphasizes a ”natural” picture of FSI equivalent to Cutkosky cut, assumptions of Step 1 valid in Eikonal models
Azimuthal asymm. corresponds to transv. moments of correlator
Boer, Mulders, Pijlman, Bomhof NPB ... 2003-2008

\[ \Phi_{\alpha}^{\mu}(x) = \int d^2k_T k_\perp^{\alpha} \Phi^{[\mu]}(x, k_T). \]

Decomposes

\[ \Phi_{\alpha}^{\mu}(x) = \tilde{\Phi}_{\alpha}^{\mu}(x) + C_{G}^{[\mu]} \pi \Phi_{G}^{\alpha}(x, x), \]

T-even \hspace{1cm} T-odd

\[ \epsilon_{T}^{ij}k_\perp S_{T}^{j}f_{1T}^{\perp}(x, k_\perp) \sim \int d^2k_T k_\perp^{i} \frac{1}{2} \left[ \text{Tr}[\gamma^{+}\Phi(\vec{S}_T)] - \text{Tr}[\gamma^{+}\Phi(-\vec{S}_T)] \right] \]

\[ 2\pi \Phi_{G}^{\alpha}(x, x) = (ih^{\perp(1)}_{1} \frac{1}{2} [\Phi, \gamma^{\alpha}] + \epsilon_{T}^{\alpha} S_{T} f_{1T}^{\perp}(x) \Phi) \]

Weighted Cross Sections contain ETQS Functions \hspace{0.5cm} LINK BTW \hspace{0.5cm} TWO Pictures!
Intuitive picture of the Sivers asymmetry:
Spatial distortion in the transverse plane due to polarization

Spatial distortion + FSI lead to observable net effect
→ non-zero Left-Right (Sivers) asymmetry

\[ < k^\perp_i(x) >_{UT} = \int d^2 k_T \, k_T^i \frac{1}{2} \left[ \text{Tr}[\gamma^+ \Phi(\vec{S}_T)] - \text{Tr}[\gamma^+ \Phi(-\vec{S}_T)] \right] \sim \epsilon_{ij} k^i T S_T^j f_{1T}^{\perp (1)}(x) \]
In other words, the correlator is distorted. In fact, one can show in a model-independent way that for a nucleon target the correlator has a large distortion, where the effect is seen in semi-inclusive reactions.

In Ref. [1], a non-trivial relation was proposed for the first time—a connection between ordinary parton distributions on the one hand and GPDs on the other. An example is given by the average transverse momentum

\[ \langle \mathbf{T} \rangle = \frac{1}{M} \int d^2 \mathbf{k}_T \Phi^q(x, \mathbf{k}_T^2; S) \]

in Eq. (3) can be interpreted as the probability density of finding an unpolarized quark inside a transversely polarized nucleon.

Comparing the respective structures of the correlators in Eqs. (3) and (4), they are identical after exchanging the impact parameter \( \mathbf{b} \) for \( \mathbf{k}_T \) and the transverse parton distributions on the other. An example is given by

\[ \rho_{ij}(x, \mathbf{b}_T^2; S) = \frac{\epsilon_{ij} b_T^i S_T^j}{M} \left( \mathcal{H}^q(x, b_T^2) + \frac{i j}{M} \left( \mathcal{E}^q(x, b_T^2) \right) \right). \]

Not conjugates and

\[ f_{1T}^q(x, \mathbf{k}_T^2) \quad \text{Naive T-odd} \]

\[ \mathcal{E}(x, \mathbf{b}_T^2) \quad \text{“...-even”} \]

FSIs needed.... How do we test this further?
Beyond the One-Loop Approximation

So far:
- Most phenomenological approaches to TMDs
- Final state interactions modeled by a single gluon exchange
  - Sivers effect $\sim q e^g$
  - Diquark model, MIT bag model, etc.

Can we do better? Can we learn about the quality of the relations?
- [L. Gamberg, M.S., in preparation]
- Still work within spectator framework, but non-perturbative model of FSI.
- In order to separate out GPDs, “cut” the diagram → “natural” picture of FSI.

\[ f_{1T}^{u(1)}(x) = -\frac{1}{2(1-x)M^2} \int \frac{d^2p_T}{(2\pi)^2} p_T^y I^y(x, |\vec{p}_T|) E^u(x, 0, -\frac{\vec{p}_T^2}{(1-x)^2}) \]
• Calculate the amplitude $M$ in a relativistic eikonal model: [1970's: Fried, Quiros, Levy, Sucher, Zuber, etc....]

**Exact 4-point function for quark-diquark scattering:**

$$T = -e^{-iL_{12}} \left[ (e^{-\frac{i}{2}L_{11}} G^{-1}(x_2, x_1 | \bar{A}_1)e^{\frac{1}{2}Tr \ln G(\bar{A}_1)}) \times (e^{-\frac{i}{2}L_{22}} \mathcal{K}^{-1}(y_2, y_1 | \bar{A}_2)e^{-\frac{1}{2}Tr \ln \mathcal{K}(\bar{A}_2)}) \right]_{\bar{A}_1 = \bar{A}_2 = 0}$$

**Linkage operator:**

$$L_{ij} = -\int d^4z_1 d^4z_2 \frac{\delta}{\delta A_i(z_1)} \mathcal{D}^{-1}(Z_1 - z_2) \frac{\delta}{\delta A_j(z_2)}$$

**Eikonal approximation:**

$$L_{ii} G^{-1} = 0$$
• **Eikonal Propagator:**

*idea:* highly energetic particle looses spin information

\[
\mathcal{G}^{-1}(x, y | \bar{A}) = \int_0^\infty ds \ e^{-is(m_q-i0)} \delta(4)(x - y - sv) \ e^{ie_q \int_0^s d\alpha v \cdot A(y+\alpha v)}
\]

Abarabanel Itzykson PRL 69
Gamberg Milton PRD 1999
Fried et al. 2000

\[
G^{ab}_{eik}(x, y | A) = -i \int_0^\infty ds \ e^{-is(m_q-i0)} \delta(4)(x - y - sv) \left( e^{-ig \int_0^s d\beta v \cdot A^\alpha(y+\beta v) t^\alpha} \right)^{ab}
\]

Trick to disentangle the A-field and the color matrices t: Functional FT

\[
\left( e^{-ig \int_0^s d\beta v \cdot A^\alpha(y+\beta v) t^\alpha} \right)^{ab} = N' \int D\alpha \int Du e^{i \int d\tau A^\alpha(\tau) u^\beta(\tau)} e^{ig \int d\tau \alpha^\beta(\tau) v \cdot A^\beta(y+\tau v)} \left( e^{i \int_0^\infty d\tau t^\beta u^\alpha(\tau)} \right)^{ab}
\]

• **Eikonal Amplitude and Coulomb Phase:**

\[
M^{eik}(|\vec{q_T}|) \propto \int_0^\infty dz \ z J_0(z |\vec{q_T}|)(e^{i\chi(z)} - 1)
\]

\[
\chi(z) = -2C_F\alpha_s \int_0^\infty dr r J_0(rz) n_+ n_- D^{-1}_{\mu\nu}(-r^2)
\]
Further steps (in words) [Fried, Gabellini, Avan, Eur.Phys.J. C13, 699]:

• Use scale invariance of the gauge vector n: ”Quasi-Abelian limit”

• Separate off quadratic gluon terms

• Neglect Fermion loops and gluon self-interactions (Ladder approximation)

Final result for the eikonal quark-antiquark scattering amplitude:

\[ M_{ab}^{\text{eik}}(qT) = \frac{2(1-x)P^+}{m_s} \int d^2z_T e^{-iq\cdot z_T} \left[ \int d^{N^2-1}\alpha \int \frac{d^{N^2-1}u}{(2\pi)^{N^2-1}} \left( e^{i\chi(|z_T|)}t\cdot\alpha \right)_{ac} (e^{i\alpha\cdot u})_{cb} e^{-i\alpha\cdot u} - 1 \right] \]

Abelian U(1)-theory: recover Coulomb phase

\[ \left[ e^{i\chi(|z_T|)} - 1 \right] \]

SU(2): analytical evaluation of color effects:

\[ e^{i\frac{\sigma\cdot u}{2}} = \cos\left(\frac{|u|}{2}\right) + i\frac{\sigma\cdot u}{|u|} \sin\left(\frac{|u|}{2}\right) \]

SU(3): (?) numerical evaluation, in process...
The Eikonal Phase

Recall: \[ \chi(|z_T|) = g^2 \int_{-\infty}^{\infty} \, d\tau_1 \int_{-\infty}^{\infty} \, d\tau_2 \, n^\mu \tilde{n}^\nu D_{\mu\nu}(z + \tau_1 n - \tau_2 \tilde{n}) = \frac{\alpha}{\pi} \int d^2 k_T n^\mu \tilde{n}^\nu \tilde{D}_{\mu\nu}(k_T) e^{i k_T \cdot \tilde{z}_T} \]

For a free gluon propagator: \[ \chi(|z_T|) = 2\alpha K_0(\mu |z_T|) \]

- **Good news:** Recover exactly old perturbative results in the one gluon exchange.
  [Burkardt, Hwang; Meißner, Metz, Goeke]

  e.g.: scalar diquark: \[ I^i(x, \vec{b}_T) = -\alpha (1 - x) \frac{b^i_T}{b^2_T}, \quad f^{\perp 1}_{1T}(x, \vec{k}^2_T) = -\frac{g^2}{2(2\pi)^3} \frac{\alpha (1-x) M(x M + m_q)}{k^2_T (k^2_T + \tilde{m}^2)} \ln \left( \frac{k^2_T + \tilde{m}^2}{\tilde{m}^2} \right) \]

- **Bad news:** Dependence on IR-regulator.

  Exchanged gluons are soft → free propagator? What is the value of \( \alpha_s \) in the IR-limit?

→ **Input from Dyson-Schwinger Equations.**


- Calculations in Landau gauge + Euclidean space → applicable here.
- IR-finite gluon propagator \[ D(p^2 \to 0) \sim (p^2)^{2\kappa - 1} \]
- Renorm. Point at \( \mu_R = m_Z \) → here: \( \mu_R = 1 \text{GeV} \)
- DSE: IR-limit of \[ \alpha_s(p^2 \to 0) \sim 2.972 \]
- Use running of \( \alpha_s \) as vertex factor.
**Lensing Function**

Express Lensing Function in terms of Eikonal Phase:

\[
\bar{I}^{i}_{(N=1)}(x, \bar{b}_T) = \frac{1}{4} \frac{b_T^i}{|\bar{b}_T|} \chi'(\frac{|\bar{b}_T|}{1-x}) \left[ 1 + \cos \left( \frac{|\bar{b}_T|}{1-x} \right) \right]
\]

\[
\bar{I}^{i}_{(N=2)}(x, \bar{b}_T) = \frac{1}{4} \frac{b_T^i}{|\bar{b}_T|} \chi'(\frac{|\bar{b}_T|}{1-x}) \left[ 1 + \cos \left( \frac{x}{4} \right) + \frac{\chi}{8} \left( \frac{x}{4} - \sin \frac{x}{4} \right) \right] (\frac{|\bar{b}_T|}{1-x})
\]

\[
\bar{I}^{i}_{(N=3)}(x, \bar{b}_T) = ???
\]

---

**Final State Interactions remain negative (even in SU(2))**
The GPD $E$ in the diquark model

The remaining piece of the puzzle: GPD $E$

- **Need to fit the model parameters:**
  - GPD-limits:
    - Form Factors $F1, F2 \rightarrow t$-dependence
    - Valence $u(x)$ (GRV) $\rightarrow x$-dependence

- Difficult to fit all limits for diquark only
  - $\rightarrow$ Form Factor fit preferred.

- Other GPD model:
  - [Diehl et al., EPJ C39 (2005) 1-39]

$E^u_v(x, 0, t) = (N^u u \kappa^u x^{-\alpha} (1 - x)\beta) e^{tg^u(x)}$
Results for the u-quark Sivers function

- Relation produces a Sivers effect $\sim 0.04 - 0.08$, Torino - Extraction $\sim 0.05$.
- SU(2) color factors seem to increase the effect. Effect of SU(3) ???
- Relation overshoots extraction, but correct sign and order of magnitude...
Boer Mulders for Pion

\[ h_{1,\pi}^{T} (\mu) \]

\[ \mu^2 = 2 \text{ GeV}^2 \]
Summary:

- Relation Sivers – E via separation of FSI + spatial distortion of parton dist.
- Relation is not rigorous, model-dependent. Holds for lowest order spectator models.
- Relation reproduces the right order of magnitude of the Sivers effect.

Outlook:

- Apply formalism for other soft objects: Boer-Mulders, Collins function, soft factor...
- Improve the diquark model through axial-vectors.
- SU(3) color factors
- Other Dyson-Schwinger extractions of Gluon-Prop. / running coupling.
They Have the same Mothers

\[ F_{1,n}^e = F_{1,n}^e (x, \xi, \vec{k}_T^2, \vec{k}_T \cdot \vec{\Delta}_T, \vec{\Delta}_T^2) \]

\[ H(x, \xi, t) = \int d^2 \vec{k}_T \left[ F_{1,1}^e + 2\xi^2 \left( \frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} F_{1,2}^e + F_{1,3}^e \right) \right] \]

\[ f_1(x, \vec{k}_T^2) = F_{1,1}^e (x, 0, \vec{k}_T^2, 0, 0) \]

Metz, Meissner Schlegel Goeke to appear....

Trivial Relations are well-known:

\[ f_1(x) = H(x, 0, 0) = \int d^2 k_T f_1(x, \vec{k}_T^2) = \int d^2 b_T H(x, \vec{b}_T^2) \]

\[ g_1(x) = \tilde{H}(x, 0, 0) = \int d^2 k_T g_1 L(x, \vec{k}_T^2) \]

\[ h_1(x) = H_T(x, 0, 0) = \int d^2 k_T h_1(x, \vec{k}_T^2) \]

model-independent, integrated relations
They Have Different Mothers

\[ f_{1T}^{\perp}(x, \vec{k}_T^2; \eta) = -F_{1,2}^o(x, 0, \vec{k}_T^2, 0, 0; \eta) \]

\[ E(x, \xi, t) = \int d^2\vec{k}_T \left[ -F_{1,1}^e + 2(1 - \xi^2) \left( \frac{\vec{k}_T \cdot \vec{\Delta}_T}{\vec{\Delta}_T^2} F_{1,2}^e + F_{1,3}^e \right) \right] \]