Baryon resonance EM transition form factors in a light-cone model (and strong form factors)
M: Oh, this is futile!
A: No it isn't.
M: I came here for a good argument.
A: No you didn't; no, you came here for an argument.
M: An argument isn't just contradiction.
A: It can be.
M: No it can't. An argument is a connected series of statements intended to establish a proposition.
A: No it isn't.
M: Yes it is! It's not just contradiction.
A: Look, if I argue with you, I must take up a contrary position.
M: Yes, but that's not just saying 'No it isn't.'
A: Yes it is!
M: No it isn't!
A: Yes it is!
M: Argument is an intellectual process. Contradiction is just the automatic gainsaying of any statement the other person makes.

(short pause)

A: No it isn't.
Baryon resonance EM transition form factors in a light-cone model

- Calculations of EM transition form factors from N to N* (with Brad Keister)
  - Light-cone (relativistic) quark model fit to nucleon elastic form factors
  - Baryon wave functions found by solving a three-quark Hamiltonian
  - Calculate strong decay signs using pair-creation ($^3P_0$) model
EM transition form factors

- **Rigorous but difficult approaches:**
  - Schwinger-Dyson Bethe-Salpeter approach
    - Working on nucleon/Δ
  - Lattice QCD

- **Quark-model calculations:**
  - Most reliable use light-cone dynamics to improve one-body current approximation
    - Relativistic effects are large
      - Need to remove interaction dependence of boosts
      - Minimize effect of ignored two-body currents
    - Other groups using point-form
Light-cone model of EM form factors

- Construct baryon wave functions in baryon CM frame in terms of free-particle light-front spinors
  - Bakamjian-Thomas construction
- Evaluate matrix elements of one-body EM current using these wave functions
- Find helicity amplitudes for EM transitions in terms of reduced matrix elements
• Light-front Hamiltonian dynamics
  - Constituents are treated as particles rather than fields
  - Certain combinations of boosts and rotations are independent of interactions which govern the quark dynamics
    • Simplifies calculations of matrix elements in which composite baryons recoil with large momenta
  - Use complete orthonormal set of basis states composed of three constituent quarks which satisfy rotational covariance
Calculation scheme

• Bakamjian and Thomas scheme:
  - Three-body relativistic bound-state problem is solved for the wave functions of baryons with the assumption of three interacting constituent quarks
  - Wave functions used to calculate the matrix elements of one, two, and three-body electromagnetic current operators
Calculational details

- Expand in sets of free-particle states:

\[
\langle M' j; \tilde{P}' \mu' | I^+(0) | M j; \tilde{P} \mu \rangle = (2\pi)^{-18} \int d\tilde{p}'_1 \int d\tilde{p}'_2 \int d\tilde{p}'_3 \int d\tilde{p}_1 \int d\tilde{p}_2 \int d\tilde{p}_3 \sum 
\langle M' j'; \tilde{P}' \mu' | \tilde{P}'_1 \mu'_1 \tilde{P}'_2 \mu'_2 \tilde{P}'_3 \mu'_3 \rangle 
\times \langle \tilde{P}'_1 \mu'_1 \tilde{P}'_2 \mu'_2 \tilde{P}'_3 \mu'_3 | I^+(0) | \tilde{P}_1 \mu_1 \tilde{P}_2 \mu_2 \tilde{P}_3 \mu_3 \rangle 
\times \langle \tilde{P}_1 \mu_1 \tilde{P}_2 \mu_2 \tilde{P}_3 \mu_3 | M j; \tilde{P} \mu \rangle.
\]

\[
\langle \tilde{P}_1 \mu_1 \tilde{P}_2 \mu_2 \tilde{P}_3 \mu_3 | M j; \tilde{P} \mu \rangle = \left| \frac{\partial(\tilde{P}_1, \tilde{P}_2, \tilde{P}_3)}{\partial(\tilde{P}, k_1, k_2)} \right|^{-\frac{1}{2}} (2\pi)^{3}\delta(\tilde{p}_1 + \tilde{p}_2 + \tilde{p}_3 - \tilde{P}) 
\times \langle \frac{1}{2} \tilde{\mu}_1 \frac{1}{2} \tilde{\mu}_2 | s_{12} \mu_{12} \rangle \langle s_{12} \mu_{12} \frac{1}{2} \tilde{\mu}_3 | s \mu_s \rangle 
\times \langle l_{\rho \mu_\rho} l_{\lambda \mu_\lambda} | L_{\mu L} \rangle \langle L_{\mu L} s \mu_s | j \mu \rangle 
\times Y_{l_{\rho \mu_\rho}} (k_\rho) Y_{l_{\lambda \mu_\lambda}} (K_\lambda) \Phi (k_\rho, K_\lambda) 
\times D_{\mu_1 \mu_1}^{(\frac{1}{2})\dagger} [R_{cf}(k_1)] D_{\mu_2 \mu_2}^{(\frac{1}{2})\dagger} [R_{cf}(k_2)] 
\times D_{\mu_3 \mu_3}^{(\frac{1}{2})\dagger} [R_{cf}(k_3)].
\]
Calculational details...

- **Cluster expansion of electromagnetic current operator**
  \[ I^\mu(x) = \sum_j I_j^\mu(x) + \sum_{j<k} I_{jk}^\mu(x) + \cdots. \]

- We evaluate only one-body matrix elements and assume struck quark has EM current of free Dirac particle
  \[ \langle \vec{p}'\mu' | I^+(0) | \vec{p}\mu \rangle = 1\delta_{\mu'\mu}. \]

- Result is a 6D integral that we evaluate using numerical techniques [quasi-random number (Sobol) sequences]
Light-cone model...

- Wave functions expanded in h.o. basis up to N=6 or 7
  - e.g. 50 components for N and Roper
- Requires simultaneous calculation of strong-decay amplitudes
  - Calculate Nπ sign using $^3P_0$ model using identical wave functions
- Fit quark EM form factors to nucleon EM form factors (moments and $Q^2$ dependence)
  - Similar calculations performed by Rome group (Cardarelli, Pace, Salme, Simula)
proton and neutron $G^M \kappa = 0.036, \kappa_d = -0.125$

$F^u_1 = 1/(1+Q^2/1.22 \text{ GeV}^2), \quad F^d_2 = 1/(1+Q^2/1.22 \text{ GeV}^2)^2$
proton and neutron $G_E \kappa_u = 0.036$, $\kappa_d = -0.125$

$F_1^q = 1/(1+Q^2/1.22 \text{ GeV}^2)$, $F_2^q = 1/(1+Q^2/1.22 \text{ GeV}^2)^2$
Roper resonance transverse amplitude

- point-like constituent quarks
- + anomalous moments fit to nucleon moments
- + quark form factors
- .... with relativistic effects turned off

Roper resonance scalar amplitude

- point-like constituent quarks, single h.o. wave fns
- anomalous moments fit to nucleon moments + quark form factors
$\Delta(1232)$ transition form factors

$\Delta(1232)$ transition form factors with $N=6$ mixed wave functions.

- $A^p_{3/2}$ point-like quarks
- $A^p_{1/2}$ point-like quarks
- $A^p_{3/2} \kappa_u = +0.036, \kappa_d = -0.125$, form factors
- $A^p_{1/2} \kappa_u = +0.036, \kappa_d = -0.125$, form factors
- Photocoupling data
N(1535)S_{11} resonance transverse amplitude

N=6 or 7 mixed wave functions

\[ A^P_{1/2}(Q^2) \left[ 10^{-3} \text{ GeV}^{-1/2} \right] \]

\[ Q^2 (\text{GeV}^2) \]

- \( \kappa_u = +0.036, \kappa_d = -0.125 \), quark form factors
- Point-like quarks
Rotational covariance

- States with higher $J$
  - Rotations are dynamical in light-front QM
  - It is possible to quantify the violation of rotational covariance by forming a linear combination of light-front spin matrix elements which should be zero
    - For $\Delta(1232)$ there is one such combination which becomes comparable to $A_{p\frac{3}{2}}, A_{p\frac{1}{2}}$ only at higher $Q^2$
      - Calculation believable at $Q^2$ below roughly 2 GeV$^2$
    - Non-zero because we are truncating our calculation at one-body currents
Rotational covariance…

• For states with $J=5/2$ there are three linear combinations which should be zero
  - For $N5/2^+(1680)$ these may not be small at $1 \text{ GeV}^2$
• Some authors claim to have a work around for $J=1/2$
  - Evaluate light-front matrix elements of other components of the EM current, take linear combinations to eliminate matrix elements which must be zero
  - But there is no free lunch for higher $J$!
    • If use other components of $I$, don’t have minimal set of matrix elements which transform into each other under boosts
Quark mass dynamically generated

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

- Lattice, model predictions (from a talk by Craig Roberts)

Rapid acquisition of mass is effect of gluon cloud

Simon Capstick, Florida State University

hadron spectroscopy workshop@INT 11/13/09
Constituent quark form factors

- Can estimate the strong form factor for u,d quarks from this [fit to $\exp(-q^2/\sigma^2)$, $\sigma \sim 0.8$ GeV]
  - Quarks are extended objects, “size” $\sim 0.25$ fm
  - If smear inter-quark coordinate $r_{ij}$ with form factor $\exp(-\sigma_{ij}^2r_{ij}^2)$, then $\sigma_{ij} \sim 0.6$ GeV
  - Godfrey-Isgur and SC-Isgur used $\sigma_{ij} = 1.8$ GeV
    - Builds large high-momentum components into the wave functions of states with negative contact interactions (e.g. nucleon, Roper)
Constituent quark form factors...

• Fit to nucleon form factors at larger $Q^2$ requires quark EM form factors
  - Large high-momentum components require an EM quark form factor that falls off rapidly in $Q^2$
  - Incommensurate with quark strong form factor used in spectrum calculation
Strong form factors

• Calculation of re-scattering effects is necessary for unitary description of meson production reactions

• Requires description of vertices (in c.m.)
  \[ B(0) \leftrightarrow B'(-k) \ M(k) \]
  - For \( k = |k| \) given by kinematics of decay process
    (\( \sim \) coupling constants \( \times \) phase space) gives \( B \) partial width into \( B'M \) channel
  - Need all values of \( k \) for loops
Form factors in $^3P_0$ model

- Calculation with Danielle Morel (Stetson U.)
- Pair creation at a point?
- Geiger & Isgur, Silvestre-Brac & Gignoux give pair-creation operator a form factor
  \[ f^2 = 3.0 \text{ GeV}^{-2} \]
  \[ e^{-f^2(p_q-p_{q^-})^2} \]
  - Gives pair-creation vertex a size of \(~0.35 \text{ fm}\)
  - Softens form factors
- We will use \( f=1.27 \text{ GeV}^{-1}, \sim 0.25 \text{ fm} \)
Universal form factors?

- Often when modeling reactions we use universal form factors at the strong vertices,
  - Or $k^L F(k^2)$ for partial-wave $L$
- Convenient for EM gauge invariance
- Ignores internal structure of hadrons coupling at the vertex
- What is the structure dependence of these vertices?
Structure dependence of strong vertices

- Koniuk and Isgur PRD21, 1868 (1980)
  - Calculate strong decays using elementary-meson emission model
    - Nonrelativistic
    - Mesons emitted directly from (third) quark
  - Transition operator:
    - $K$ is decay three momentum
    - Spin of quark $\sigma_3$
    - $X^M_3$ is a Gell-Mann flavor matrix (different for each flavor of $q_3$ and meson $M$)
Structure dependence…

• Calculate strong decay widths
  - Harmonic oscillator basis to N=2
  - Calculate form factors in terms of couplings $g$ and $h$
    • Evaluate to $O(k^2)$ (drop $k^4$ terms)
  - Apart from spin-flavor [SU(6)] dependent factors:
    • Amplitudes reduce to a small set of universal partial-wave amplitudes
    • Compare to results of a relativistic model
      - Feynman, Kislinger and Ravndal, PRD3, 2706 (1971)
TABLE I. The universal partial-wave amplitudes for pseudoscalar emission to unmerged gound states. The full amplitudes denoted by the symbols in column two are obtained by multiplying column three by $\alpha [(K/\pi)(E'/M_{R})]^{1/2} \exp[-\frac{1}{6}(K^2/\alpha^2)]$. For the definitions of $G$, $G'$, $G''$, $H'$, and $H''$ compare to Ref. 22.

<table>
<thead>
<tr>
<th>Multiplet</th>
<th>Amplitude</th>
<th>Nonrelativistic model</th>
<th>Relativistic model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(56, 0^+)$</td>
<td>$P_0$</td>
<td>$<a href="%5Cfrac%7BK%7D%7B%5Calpha%7D">g - \frac{1}{3}h</a>$</td>
<td>$G$</td>
</tr>
<tr>
<td>$(70, 1^-)$</td>
<td>$S$</td>
<td>$[(g - \frac{1}{3}h)(\frac{K}{\alpha})^2 + 3h]$</td>
<td>$G' - 3H'$</td>
</tr>
<tr>
<td>$(70, 1^-)$</td>
<td>$D$</td>
<td>$<a href="%5Cfrac%7BK%7D%7B%5Calpha%7D">g - \frac{1}{3}h</a>^2$</td>
<td>$G'$</td>
</tr>
<tr>
<td>$(56', 0^+)$ and $(70, 0^+)$</td>
<td>$P'_0$</td>
<td>$<a href="%5Cfrac%7BK%7D%7B%5Calpha%7D">(g - \frac{1}{3}h)(\frac{K}{\alpha})^2 + 2h</a>$</td>
<td>$G'' - 2H''$</td>
</tr>
<tr>
<td>$(56', 0^+)$ and $(70, 0^+)$</td>
<td>$F'_0$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(56, 2^+)$ and $(70, 2^+)$</td>
<td>$P$</td>
<td>$<a href="%5Cfrac%7BK%7D%7B%5Calpha%7D">(g - \frac{1}{3}h)(\frac{K}{\alpha})^2 + 5h</a>$</td>
<td>$G'' - 5H''$</td>
</tr>
<tr>
<td>$(56, 2^+)$ and $(70, 2^+)$</td>
<td>$F$</td>
<td>$<a href="%5Cfrac%7BK%7D%7B%5Calpha%7D">g - \frac{1}{3}h</a>^3$</td>
<td>$G''$</td>
</tr>
<tr>
<td>$(20, 1^+)$</td>
<td>$P'$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(20, 1^+)$</td>
<td>$F'$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Form factors in $^3P_0$ model...

- $P$-wave transition $N \leftrightarrow N\pi$
  - Amplitude proportional to $k^L$, here $L=1$

\[ k = K_0 \text{ (GeV)} \]
Form factors in $^3P_0$ model…

- P-wave transition $N \leftrightarrow N\pi$

Divide by $k \ (K_0)$

Normalize: $F(k^2=0) = 1$
Form factors in $^3P_0$ model...

- **P-wave transition** $N \leftrightarrow N\pi$
  - Compare normalized $F(k^2)/k^L$ to $\exp(-k^2/\Lambda^2)$, $\Lambda = 1$ GeV
Form factors in $^3P_0$ model...

- $P$-wave transitions $\Delta \leftrightarrow N\pi, N \leftrightarrow N\pi$
Form factors in $^3P_0$ model...

- Radial excitation in initial state adds a node:

$$N \leftrightarrow N\pi$$

$$N(1440)P_{11} \leftrightarrow N\pi$$

$$N(1710)P_{11} \leftrightarrow N\pi$$
Form factors in $^3P_0$ model…

- Orbital excitations which decay in $S$-waves have node:
  - $N(1535)S_{11} \leftrightarrow N\pi$
  - $N(1650)S_{11} \leftrightarrow N\pi$
  - $N \leftrightarrow N\pi$
Form factors in $^3P_0$ model...

- Orbital excitations which decay in $D$ (-ve parity) and $F$-waves (+ve parity) have no node:

- $N(1520)D_{13} \leftrightarrow N\pi$
- $N(1680)F_{15} \leftrightarrow N\pi$
- $N \leftrightarrow N\pi$
Conclusions/Outlook

• Calculation of EM transition form factors for low J reliable using light cone dynamics
  - Can estimate uncertainties at higher $Q^2$ from lack of rotational covariance
  - We have made simple fit to nucleon form factors extracted from polarization data
  - We have looked at nucleon, $\Delta(1232)P_{33}$, $N(1440)P_{11}$, $N(1535)S_{11}$
    • Results similar to those of Rome group
  - Model can be applied to any state
• Working on transitions to $N^*$ with higher J
Conclusions/Outlook…

• Structure dependence of strong form factors should not be ignored
  – Calculation in NR models is available

• Need to develop relativistic model of strong decays
  – Some work published
    • Quality of fits to decay partial widths is degraded relative to $^3P_0$
    • Requires development