Spacelike pion form factor from AdS/CFT

\[ F_\pi(q^2) \]

\[ q^2(\text{GeV}^2) \]

Data Compilation
Baldini, Kloe and Volmer

One parameter - set by pion decay constant.

AdS/QCD and Novel QCD Phenomena

Stan Brodsky

INT
November 10, 2009
- Analytical continuation to time-like region $q^2 \rightarrow -q^2$  \[ M_\rho = 2\kappa = 750 \text{ MeV} \]

- Strongly coupled semiclassical gauge/gravity limit hadrons have zero widths (stable).

Space and time-like pion form factor for $\kappa = 0.375$ GeV in the SW model.

- Vector Mesons: Hong, Yoon and Strassler (2004); Grigoryan and Radyushkin (2007).
Note: Analytical Form of Hadronic Form Factor for Arbitrary Twist

- Form factor for a string mode with scaling dimension $\tau$, $\Phi_\tau$ in the SW model

$$F(Q^2) = \frac{\Gamma(\tau) \Gamma\left(1 + \frac{Q^2}{4\kappa^2}\right)}{\Gamma\left(\tau + \frac{Q^2}{4\kappa^2}\right)}.$$ 

- For $\tau = N$, $\Gamma(N + z) = (N - 1 + z)(N - 2 + z)\ldots(1 + z)\Gamma(1 + z)$.

- Form factor expressed as $N - 1$ product of poles

$$F(Q^2) = \begin{cases} 
1 & N = 2, \\
\frac{2}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)} & N = 3,
\end{cases}$$

$$\ldots$$

$$F(Q^2) = \frac{(N - 1)!}{\left(1 + \frac{Q^2}{4\kappa^2}\right)\left(2 + \frac{Q^2}{4\kappa^2}\right)\ldots\left(N - 1 + \frac{Q^2}{4\kappa^2}\right)}, \quad N.$$ 

- For large $Q^2$:

$$F(Q^2) \to (N - 1)! \left[\frac{4\kappa^2}{Q^2}\right]^{(N-1)}.$$
\[ \kappa = 0.534 \text{ GeV} \]

\[ Q^2 (\text{GeV}^2) \]

\[ |\pi> = \psi_{qq}|q\bar{q}> + \psi_{qqqq}|q\bar{q}q\bar{q}> \]

\[ \Gamma_\rho = 120 \text{ MeV}, \quad \Gamma'_\rho = 300 \text{ MeV} \]

\[ P_{qq\bar{q}\bar{q}} = 15\% \]
Light-Front Representation of Two-Body Meson Form Factor

• Drell-Yan-West form factor

\[ F(q^2) = \sum_q e_q \int_0^1 dx \int \frac{d^2 \vec{k}_\perp}{16\pi^3} \psi_P^*(x, \vec{k}_\perp - x\vec{q}_\perp) \psi_P(x, \vec{k}_\perp). \]

• Fourier transform to impact parameter space \( \vec{b}_\perp \)

\[ \psi(x, \vec{k}_\perp) = \sqrt{4\pi} \int d^2 \vec{b}_\perp e^{i\vec{b}_\perp \cdot \vec{k}_\perp} \tilde{\psi}(x, \vec{b}_\perp) \]

• Find \((b = |\vec{b}_\perp|)\):

\[ F(q^2) = \int_0^1 dx \int d^2 \vec{b}_\perp e^{ix \vec{b}_\perp \cdot \vec{q}_\perp} |\tilde{\psi}(x, b)|^2 \]

\[ = 2\pi \int_0^1 dx \int_0^\infty b \, db \, J_0(bq) |\tilde{\psi}(x, b)|^2, \]

Soper
Holographic Mapping of AdS Modes to QCD LFWFs

- Integrate Soper formula over angles:

\[ F(q^2) = 2\pi \int_0^1 dx \frac{(1-x)}{x} \int \zeta d\zeta J_0 \left( \zeta q \sqrt{1-x} \right) \tilde{\rho}(x, \zeta), \]

with \( \tilde{\rho}(x, \zeta) \) QCD effective transverse charge density.

- Transversality variable

\[ \zeta = \sqrt{x(1-x)b^2_\perp} \]

- Compare AdS and QCD expressions of FFs for arbitrary \( Q \) using identity:

\[ \int_0^1 dx J_0 \left( \zeta Q \sqrt{1-x} \right) = \zeta Q K_1(\zeta Q), \]

the solution for \( J(Q, \zeta) = \zeta Q K_1(\zeta Q) \)!
• Electromagnetic form-factor in AdS space:

\[ F_{\pi^+}(Q^2) = R^3 \int \frac{dz}{z^3} J(Q^2, z) |\Phi_{\pi^+}(z)|^2, \]

where \( J(Q^2, z) = zQ K_1(zQ) \).

• Use integral representation for \( J(Q^2, z) \)

\[ J(Q^2, z) = \int_0^1 dx J_0 \left( \xi Q \sqrt{\frac{1-x}{x}} \right) \]

• Write the AdS electromagnetic form-factor as

\[ F_{\pi^+}(Q^2) = R^3 \int_0^1 dx \int \frac{dz}{z^3} J_0 \left( zQ \sqrt{\frac{1-x}{x}} \right) |\Phi_{\pi^+}(z)|^2 \]

• Compare with electromagnetic form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} \frac{x(1-x) |\Phi_{\pi}(\zeta)|^2}{\zeta^4} \]

with \( \zeta = z, 0 \leq \zeta \leq \Lambda_{\text{QCD}} \)
Light-Front Holography: Unique mapping derived from equality of LF and AdS formula for current matrix elements
Hadronic gravitational form-factor in AdS space

\[ A_\pi(Q^2) = R^3 \int \frac{dz}{z^3} H(Q^2, z) |\Phi_\pi(z)|^2, \]

where \( H(Q^2, z) = \frac{1}{2} Q^2 z^2 K_2(zQ) \)

Use integral representation for \( H(Q^2, z) \)

\[ H(Q^2, z) = 2 \int_0^1 x dx J_0(zQ \sqrt{1 - x/x}) \]

Write the AdS gravitational form-factor as

\[ A_\pi(Q^2) = 2R^3 \int_0^1 x dx \int \frac{dz}{z^3} J_0(zQ \sqrt{1 - x/x}) |\Phi_\pi(z)|^2 \]

Compare with gravitational form-factor in light-front QCD for arbitrary \( Q \)

\[ \left| \tilde{\psi}_{q\bar{q}/\pi}(x, \zeta) \right|^2 = \frac{R^3}{2\pi} x(1 - x) \frac{|\Phi_\pi(\zeta)|^2}{\zeta^4}, \]

Identical to LF Holography obtained from electromagnetic current
Light-Front Holography:  
Map AdS/CFT to 3+1 LF Theory

Relativistic LF radial equation!

\[
\left[ -\frac{d^2}{d\zeta^2} + \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right] \phi(\zeta) = \mathcal{M}^2 \phi(\zeta)
\]

\[
\zeta^2 = x(1 - x)b_\perp^2.
\]

\[
U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)
\]

G. de Teramond, sjb

Soft wall confining potential:
Derivation of the Light-Front Radial Schrodinger Equation directly from LF QCD

\[
\mathcal{M}^2 = \int_0^1 dx \int \frac{d^2 k}{16\pi^3} \frac{k^2}{x(1-x)} \left| \psi(x, k) \right|^2 + \text{interactions}
\]

\[
= \int_0^1 \frac{dx}{x(1-x)} \int d^2 b \psi^*(x, b) \left( -\nabla^2 b \right) \psi(x, b) + \text{interactions}.
\]

**Change variables**

\((\zeta, \varphi), \zeta = \sqrt{x(1-x)b}: \nabla^2 = \frac{1}{\zeta} \frac{d}{d\zeta} \left( \zeta \frac{d}{d\zeta} \right) + \frac{1}{\zeta^2} \frac{\partial^2}{\partial \varphi^2}\)

\[
\mathcal{M}^2 = \int d\zeta \phi^*(\zeta) \sqrt{\zeta} \left( -\frac{d^2}{d\zeta^2} - \frac{1}{\zeta} \frac{d}{d\zeta} + \frac{L^2}{\zeta^2} \right) \frac{\phi(\zeta)}{\sqrt{\zeta}} \left( -\frac{d^2}{d\zeta^2} - \frac{1 - 4L^2}{4\zeta^2} + U(\zeta) \right) \phi(\zeta)
\]
\[ H_{QED} \]

\[
(H_0 + H_{int}) |\Psi\rangle \geq E |\Psi\rangle
\]

\[
\left[-\frac{\Delta^2}{2m_{\text{red}}} + V_{\text{eff}}(\vec{S}, \vec{r})\right] \psi(\vec{r}) = E \psi(\vec{r})
\]

\[
\left[-\frac{1}{2m_{\text{red}}} \frac{d^2}{dr^2} + \frac{1}{2m_{\text{red}}} \frac{\ell(\ell + 1)}{r^2} + V_{\text{eff}}(r, S, \ell)\right] \psi(r) = E \psi(r)
\]

\[ V_{\text{eff}} \rightarrow V_C(r) = -\frac{\alpha}{r} \]

Semiclassical first approximation to QED

QED atoms: positronium and muonium

Coupled Fock states

Effective two-particle equation

Includes Lamb Shift, quantum corrections

Spherical Basis \( r, \theta, \phi \)

Coulomb potential

Bohr Spectrum
\[
H_{QCD}^{LF} \\
(H_{LF}^0 + H_{LF}^I)|\Psi> \geq M^2|\Psi> \\
[\frac{\vec{k}_2^2 + m^2}{x(1-x)} + V^{LF}_{\text{eff}}] \psi_{LF}(x, \vec{k}_2) = M^2 \psi_{LF}(x, \vec{k}_2) \\
[-\frac{d^2}{d\zeta^2} + \frac{-1 + 4L^2}{\zeta^2} + U(\zeta, S, L)] \psi_{LF}(\zeta) = M^2 \psi_{LF}(\zeta) \\
U(\zeta, S, L) = \kappa^2 \zeta^2 + \kappa^2 (L + S - 1/2) \\
\text{Semiclassical first approximation to QCD}
\]
Prediction from AdS/CFT: Meson LFWF

\[ \psi_M(x, k^2_{\perp}) = \frac{4\pi}{\kappa \sqrt{x(1-x)}} e^{-\frac{k^2_{\perp}}{2\kappa^2 x(1-x)}}, \]

\[ \phi_M(x, Q_0) \propto \sqrt{x(1-x)} \]

"Soft Wall" model

\[ \kappa = 0.375 \text{ GeV} \]

massless quarks

Note coupling \( k^2_{\perp}, x \)

Connection of Confinement to TMDs

INT

November 10, 2009

AdS/QCD and Novel QCD Phenomena

Stan Brodsky

SLAC
Hadron Distribution Amplitudes

$$\phi_H(x_i, Q)$$

$$\sum_i x_i = 1$$

Fixed $$\tau = t + z/c$$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

- Evolution Equations from PQCD, OPE, Conformal Invariance

- Compute from valence light-front wavefunction in light-cone gauge

$$\phi_M(x, Q) = \int^Q d^2 \vec{k} \, \psi_{q\bar{q}}(x, \vec{k}_\perp)$$

Lepage, sjb
Efremov, Radyushkin
Sachrajda, Frishman Lepage, sjb
Braun, Gardi

$$k^2_\perp < Q^2$$
Second Moment of Pion Distribution Amplitude

\[
< \xi^2 > = \int_{-1}^{1} d\xi \; \xi^2 \phi(\xi)
\]

\[\xi = 1 - 2x\]

\[< \xi^2 >_{\pi} = 1/5 = 0.20\]

\[< \xi^2 >_{\pi} = 1/4 = 0.25\]

\[\phi_{asympt} \propto x(1 - x)\]

\[\phi_{AdS/QCD} \propto \sqrt{x(1 - x)}\]

Lattice (I) \[< \xi^2 >_{\pi} = 0.28 \pm 0.03\]

Lattice (II) \[< \xi^2 >_{\pi} = 0.269 \pm 0.039\]

Donnellan et al.

Braun et al.
Photon-to-pion transition form factor

\[ Q^2 F_{\gamma \rightarrow \pi^0}(Q^2) \]

F. Cao, GdT, sjb (preliminary)
ERBL Evolution of Pion Distribution Amplitude

\[ x(1 - x) \]

\[ Q^2 = 100 \text{ GeV}^2 \]

\[ \sqrt{x(1 - x)} \]

\[ Q^2 = 2 \text{ GeV}^2 \]

\( \phi(x, Q^2)/f_\pi \)

F. Cao, GdT, sjb (preliminary)
Photon-to-pion transition form factor with ERBL evolution

\[ Q^2 F_{\gamma\rightarrow\pi^0} (Q^2) \]

F. Cao, GdT, sjb (preliminary)
Baryons in Ads/CFT

- Baryons Spectrum in "bottom-up" holographic QCD


- Action for massive fermionic modes on AdS$_{d+1}$:

\[ S[\bar{\Psi}, \Psi] = \int d^{d+1}x \sqrt{g} \bar{\Psi}(x, z) \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z). \]

- Equation of motion:

\[ \left( i\Gamma^\ell D_\ell - \mu \right) \Psi(x, z) = 0 \]

\[ \left[ i \left( z\eta^{\ell m} \Gamma_\ell \partial_m + \frac{d}{2} \Gamma_z \right) + \mu R \right] \Psi(x^\ell) = 0. \]
Baryons

Holographic Light-Front Integrable Form and Spectrum

• In the conformal limit fermionic spin-$\frac{1}{2}$ modes $\psi(\zeta)$ and spin-$\frac{3}{2}$ modes $\psi_\mu(\zeta)$ are two-component spinor solutions of the Dirac light-front equation

$$\alpha \Pi(\zeta) \psi(\zeta) = M \psi(\zeta),$$

where $H_{LF} = \alpha \Pi$ and the operator

$$\Pi_L(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{L + \frac{1}{2}}{\zeta} \gamma_5 \right),$$

and its adjoint $\Pi^\dagger_L(\zeta)$ satisfy the commutation relations

$$\left[ \Pi_L(\zeta), \Pi^\dagger_L(\zeta) \right] = \frac{2L + 1}{\zeta^2} \gamma_5.$$
• Note: in the Weyl representation \((i\alpha = \gamma_5\beta)\)

\[
i\alpha = \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}, \quad \beta = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}.
\]

• Baryon: twist-dimension \(3 + L\) \((\nu = L + 1)\)

\[
\mathcal{O}_{3+L} = \psi \{D_{\ell_1} \ldots D_{\ell_q} \psi D_{\ell_{q+1}} \ldots D_{\ell_m}\} \psi, \quad L = \sum_{i=1}^{m} \ell_i.
\]

• Solution to Dirac eigenvalue equation with UV matching boundary conditions

\[
\psi(\zeta) = C \sqrt{\zeta} \left[ J_{L+1}(\zeta M) u_+ + J_{L+2}(\zeta M) u_- \right].
\]

Baryonic modes propagating in AdS space have two components: orbital \(L\) and \(L + 1\).

• Hadronic mass spectrum determined from IR boundary conditions

\[
\psi_{\pm}(\zeta = 1/\Lambda_{\text{QCD}}) = 0,
\]

given by

\[
\mathcal{M}^+_{\nu,k} = \beta_{\nu,k} \Lambda_{\text{QCD}}, \quad \mathcal{M}^-_{\nu,k} = \beta_{\nu+1,k} \Lambda_{\text{QCD}},
\]

with a scale independent mass ratio.
Fig: Light baryon orbital spectrum for $\Lambda_{QCD} = 0.25$ GeV in the HW model. The 56 trajectory corresponds to $L$ even $P = +$ states, and the 70 to $L$ odd $P = -$ states.
Non-Conformal Extension of Algebraic Structure (Soft Wall Model)

• We write the Dirac equation

\[(\alpha \Pi(\zeta) - \mathcal{M}) \psi(\zeta) = 0,\]

in terms of the matrix-valued operator \(\Pi\)

\[\Pi_\nu(\zeta) = -i \left( \frac{d}{d\zeta} - \frac{\nu + \frac{1}{2}}{\zeta} \gamma_5 - \kappa^2 \zeta \gamma_5 \right),\]

and its adjoint \(\Pi^\dagger\), with commutation relations

\[\left[ \Pi_\nu(\zeta), \Pi^\dagger_\nu(\zeta) \right] = \left( \frac{2\nu + 1}{\zeta^2} - 2\kappa^2 \right) \gamma_5.\]

• Solutions to the Dirac equation

\[\psi_+(\zeta) \sim z^{\frac{1}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_n^\nu(\kappa^2 \zeta^2),\]
\[\psi_-(\zeta) \sim z^{\frac{3}{2} + \nu} e^{-\kappa^2 \zeta^2 / 2} L_{n+1}^\nu(\kappa^2 \zeta^2).\]

• Eigenvalues

\[\mathcal{M}^2 = 4\kappa^2 (n + \nu + 1).\]
\[ \mathcal{M}^2 \]

\[ \begin{align*}
4\kappa^2 & \text{ for } \Delta n = 1 \\
4\kappa^2 & \text{ for } \Delta L = 1 \\
2\kappa^2 & \text{ for } \Delta S = 1
\end{align*} \]

Parent and daughter 56 Regge trajectories for the \( N \) and \( \Delta \) baryon families for \( \kappa = 0.5 \text{ GeV} \)
E. Klempt et al.: $\Delta^*$ resonances, quark models, chiral symmetry and AdS/QCD

<table>
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<tr>
<th>$SU(6)$</th>
<th>$S$</th>
<th>$L$</th>
<th>Baryon State</th>
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<td>$N^{\frac{1}{2}-}(1650)$ $N^{\frac{3}{2}-}(1700)$ $N^{\frac{5}{2}-}(1675)$</td>
</tr>
<tr>
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<td>$\Delta^{\frac{1}{2}-}(1620)$ $\Delta^{\frac{3}{2}-}(1700)$</td>
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<td>$N^{\frac{3}{2}+}(1720)$ $N^{\frac{5}{2}+}(1680)$</td>
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<td>$\Delta^{\frac{1}{2}+}(1910)$ $\Delta^{\frac{3}{2}+}(1920)$ $\Delta^{\frac{5}{2}+}(1905)$ $\Delta^{\frac{7}{2}+}(1950)$</td>
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<tr>
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<td>$N^{\frac{7}{2}-}$ $N^{\frac{9}{2}^{-}}$ $N^{\frac{11}{2}^{-}}$ $N^{\frac{13}{2}^{-}}$</td>
</tr>
</tbody>
</table>
Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

\[ F_+(Q^2) = g_+ \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_-(Q^2) = g_- \int d\zeta J(Q, \zeta) |\psi_-(\zeta)|^2, \]

where the effective charges \( g_+ \) and \( g_- \) are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have \( S^z = +1/2 \). The two AdS solutions \( \psi_+(\zeta) \) and \( \psi_-(\zeta) \) correspond to nucleons with \( J^z = +1/2 \) and \(-1/2\).

- For \( SU(6) \) spin-flavor symmetry

\[ F_1^p(Q^2) = \int d\zeta J(Q, \zeta) |\psi_+(\zeta)|^2, \]
\[ F_1^n(Q^2) = -\frac{1}{3} \int d\zeta J(Q, \zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right], \]

where \( F_1^p(0) = 1, \ F_1^n(0) = 0. \)
Scaling behavior for large $Q^2$: $Q^4 F_1^p(Q^2) \rightarrow \text{constant}$  

Proton $\tau = 3$

• Scaling behavior for large $Q^2$: $Q^4 F^m_1(Q^2) \rightarrow$ constant

\[ \text{Neutron } \tau = 3 \]

Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs

Harmonic Oscillator Confinement
Normalized to anomalous moment

$F^p_2(Q^2) = 1 + O\left(\frac{Q^2}{m_\pi m_p}\right)$
in chiral perturbation theory

$\kappa = 0.49$ GeV

G. de Teramond, sjb

AdS/QCD No chiral divergence!
Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

Five dimensional action in presence of dilaton background

\[ S = -\frac{1}{4} \int d^4x dz \sqrt{g} \ e^{\phi(z)} \frac{1}{g_5^2} G^2 \quad \text{where} \quad \sqrt{g} = \left(\frac{R}{z}\right)^5 \quad \text{and} \quad \phi(z) = +\kappa^2 z^2 \]

Define an effective coupling \( g_5(z) \)

\[ S = -\frac{1}{4} \int d^4x dz \sqrt{g} \frac{1}{g_5^2(z)} G^2 \]

Thus \( \frac{1}{g_5^2(z)} = e^{\phi(z)} \frac{1}{g_5^2(0)} \) or \( g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0) \)

Light-Front Holography: \( z \rightarrow \zeta = b_\perp \sqrt{x(1-x)} \)

\[ \alpha_s(q^2) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q) \alpha_s(\zeta) \quad \text{where} \quad \alpha_s(z) = e^{-\kappa^2 z^2} \alpha_s(0) \]
Running Coupling from AdS/QCD

\[ \frac{\alpha_s(Q)}{\pi} = e^{-Q^2/4\kappa^2} \]

AdS/QCD
LF Holography

AdS/QCD with \( \alpha_{s,g1} \) extrapolation

\( \alpha_{s,g1}/\pi \) (pQCD)
\( \alpha_{s,g1}/\pi \) world data
\( \alpha_{s,F3}/\pi \)

GDH limit

\( \alpha_{s,\tau}/\pi \) OPAL

JLab CLAS PLB 665 249

Hall A/CLAS PLB 650 4 244

Lattice QCD

Deur, de Teramond, sjb, (preliminary)
-2.25
-2
-1.75
-1.5
-1.25
-1
-0.75
-0.5
-0.25
0

\( \frac{d\alpha_s}{d\log(Q^2)} \)

Bjorken sum rule
GDH sum rule
Lattice QCD
AdS/QCD with \( \alpha_{s,g1} \) extrapolation
AdS/QCD LF Holography

\( \alpha_{s,F3}/\pi \)

\( \alpha_{s,g1} \)

Hall A/CLAS PLB 650 4 244
JLab CLAS PLB 665 249

Deur, de Teramond, sjb, (preliminary)
AdS/QCD LF Holography
AdS/QCD with \( \alpha_{s,g1} \) extrapolation
Empirical \( \alpha_{s,g1} \)
Applications of Nonperturbative Running Coupling from AdS/QCD

• Sivers Effect in SIDIS, Drell-Yan
• Double Boer-Mulders Effect in DY
• Diffractive DIS
• Heavy Quark Production at Threshold

All involve gluon exchange at small momentum transfer
Features of Soft-Wall AdS/QCD

- Single-variable frame-independent radial Schrödinger equation
- Massless pion ($m_q = 0$)
- Regge Trajectories: universal slope in $n$ and $L$
- Valid for all integer $J$ & $S$. Spectrum is independent of $S$
- Dimensional Counting Rules for Hard Exclusive Processes
- Phenomenology: Space-like and Time-like Form Factors
- LF Holography: LFWFs; broad distribution amplitude
- No large $N_c$ limit
- Add quark masses to LF kinetic energy
- Systematically improvable -- diagonalize $H_{LF}$ on AdS basis
Features of AdS/QCD LF Holography

- Based on Conformal Scaling of Infrared QCD Fixed Point
- Conformal template: Use isometries of AdS5
- Interpolating operator of hadrons based on twist, superfield dimensions
- Finite \( N_c = 3 \): Baryons built on 3 quarks -- Large \( N_c \) limit not required
- Break Conformal symmetry with dilaton
- Dilaton introduces confinement -- positive exponent
- Origin of Linear and HO potentials: Stochastic arguments (Glazek); General ‘classical’ potential for Dirac Equation (Hoyer)
- Effective Charge from AdS/QCD
- Conformal Dimensional Counting Rules for Hard Exclusive Processes
- Use CRF (LF Constituent Rest Frame) to reconstruct 3D Image of Hadrons (Glazek, de Teramond, sjb)
String Theory

AdS/CFT

AdS/QCD

Semi-Classical QCD / Wave Equations

Boost Invariant 3+1 Light-Front Wave Equations

Hadron Spectra, Wavefunctions, Dynamics

Mapping of Poincare’ and Conformal SO(4,2) symmetries of 3+1 space to AdS5 space

Conformal behavior at short distances + Confinement at large distance

Holography

Integrable!

Goal: First Approximant to QCD
Counting rules for Hard Exclusive Scattering Regge Trajectories

QCD at the Amplitude Level

J = 0, 1, 1/2, 3/2 plus L

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November 10, 2009

AdS/QCD and Novel QCD Phenomena

Stan Brodsky
SLAC

99
Heisenberg Matrix Formulation

Light-Front QCD

\[ L_{QCD} \rightarrow H^{QCD}_{LF} \]

\[ H^{QCD}_{LF} = \sum_{i} \left[ \frac{m^2 + k^2}{x} \right]_i + H^{int}_{LF} \]

\( H^{int}_{LF} \): Matrix in Fock Space

Eigenvalues and Eigensolutions give Hadron Spectrum and Light-Front wavefunctions

Physical gauge: \( A^+ = 0 \)
\[ H_{QCD}^{LC} |\Psi_h\rangle = M_h^2 |\Psi_h\rangle \]

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Use AdS/QCD basis functions

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AdS/QCD and Novel QCD Phenomena

Stan Brodsky
SLAC
Use AdS/CFT orthonormal LFWFs as a basis for diagonalizing the QCD LF Hamiltonian

- Good initial approximant
- Better than plane wave basis
- DLCQ discretization -- highly successful 1+1
- Use independent HO LFWFs, remove CM motion
- Similar to Shell Model calculations

Pauli, Hornbostel, Hiller, McCartor, sjb

Vary, Harinandrath, Maris, sjb
Formation of Relativistic Anti-Hydrogen

Measured at CERN-LEAR and FermiLab

Munger, Schmidt, sjb

Coalescence of off-shell co-moving positron and antiproton.

Wavefunction maximal at small impact separation and equal rapidity

“Hadronization” at the Amplitude Level
Hadronization at the Amplitude Level

Construct helicity amplitude using Light-Front Perturbation theory; coalesce quarks via LFWFs
Features of LF T-Matrix Formalism

“Event Amplitude Generator”

- Coalesce color-singlet cluster to hadronic state if

\[ \mathcal{M}^2_n = \sum_{i=1}^{n} \frac{k_{\perp i}^2 + m_i^2}{x_i} < \Lambda_{QCD}^2 \]

- The coalescence probability amplitude is the LF wavefunction

\[ \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \]

- No IR divergences: Maximal gluon and quark wavelength from confinement

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i} \]

\[ P^+ = P^0 + P^z \]
Features of LF $T$-Matrix Formalism

“Event Amplitude Generator”

If $M^2_n \geq \Lambda^2_{QCD}$ use PQCD hard gluon exchange

- Generates PQCD Hard Tail of LFWF at high $x$ and high transverse momentum

- Dimensional Counting rules and Color Transparency for Hard Exclusive Channels

- Counting rules for structure functions and fragmentation functions at large $x$ and $z$:

  $$(1 - x)^{2n_{spect} - 1}, (1 - z)^{2n_{spect} - 1}$$

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp \]

\[ P^+ = P^0 + P^z \]
Features of LF  T-Matrix Formalism

“Event Amplitude Generator”

If $M^2_n \geq \Lambda^2_{QCD}$ use PQCD hard gluon exchange

- DGLAP and ERBL Evolution from gluon emission and exchange
- Factorization Scale for structure functions and fragmentation functions set:
  \[ \mu_{fact} = \Lambda_{QCD} \]

\[ x_i P^+, x_i \vec{P}_\perp + \vec{k}_\perp i \]

\[ P^+ = P^0 + P^z \]
Features of LF T-Matrix Formalism

“Event Amplitude Generator”

• Same principle as antihydrogen production: off-shell coalescence

• Coalescence to hadron favored at equal rapidity, small transverse momenta

• Leading heavy hadron production: D and B mesons produced at large $z$

• Hadron helicity conservation if hadron LFWF has $L_z = 0$

• Baryon AdS/QCD LFWF has aligned and anti-aligned quark spin

$$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$$

$$P^+ = P^0 + P^z$$
Features of LF T-Matrix Formalism

- Only positive + momenta; no backward time-ordered diagrams
- Frame-independent! Independent of $P^+$ and $P^z$
- LC gauge: No ghosts; physical helicity
- $J^z = L^z + S^z$ conservation at every vertex
- Sum all amplitudes with same initial-and final-state helicity, then square to get rate
- Renormalize each UV-divergent amplitude using “alternating denominator” method
- Multiple renormalization scales (BLM)
- Cluster Decomposition; Unitary Cuts; `History’
Deep Inelastic Electron-Proton Scattering

Struck quark is virtual

\[ k^2 \sim -\frac{k_{\perp}^2}{1-x} \rightarrow -\infty \text{ at } x \rightarrow 1 \]

Off-shell Effect: Breakdown of DGLAP at \( x \sim 1 \) !

Off-shell Effect: Breakdown of DGLAP at \( z \sim 1 \) !
Hadronization at the Amplitude Level

\[ \tau = x^+ \]

Higher Fock State Coalescence  \[ |uuds\bar{s}\bar{s}> \]

Asymmetric Hadronization!  \[ D_{s\rightarrow p}(z) \neq D_{s\rightarrow \bar{p}}(z) \]

B-Q Ma, sjb

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AdS/QCD and Novel QCD Phenomena

Stan Brodsky
SLAC
\[ D_{s \rightarrow p}(z) \neq D_{s \rightarrow \bar{p}}(z) \]

\[ A_{sp}^{pp}(z) = \frac{D_{s \rightarrow p}(z) - D_{s \rightarrow \bar{p}}(z)}{D_{s \rightarrow p}(z) + D_{s \rightarrow \bar{p}}(z)} \]

Consequence of \( s_p(x) \neq \bar{s}_p(x) \quad |uuuds\bar{s}| > \sim |K^+ \Lambda| \)
Chiral symmetry breaking effect in AdS/QCD depends on weighted $z^2$ distribution, not constant condensate

$$\delta M^2 = -2m_q < \bar{\psi} \psi > \times \int dz \ \phi^2(z) z^2$$

- $z^2$ weighting consistent with higher Fock states at periphery of hadron wavefunction
- AdS/QCD: confined condensate
- Suggests “In-Hadron” Condensates

Erlich et al.

de Teramond, Shrock, sbj
“One of the gravest puzzles of theoretical physics”

DARK ENERGY AND
THE COSMOLOGICAL CONSTANT PARADOX

A. ZEE

Department of Physics, University of California, Santa Barbara, CA 93106, USA
Kavil Institute for Theoretical Physics, University of California,
Santa Barbara, CA 93106, USA
zee@kitp.ucsb.edu

\[
(\Omega_\Lambda)_{QCD} \sim 10^{45}
\]

\[
(\Omega_\Lambda)_{EW} \sim 10^{56}
\]

\[
\Omega_\Lambda = 0.76 (expt)
\]

QCD Problem Solved if Quark and Gluon condensates reside within hadrons, not LF vacuum

Shrock, sjb
Chiral magnetism (or magnetohadrochirronics)

Aharon Casher and Leonard Susskind
Tel Aviv University Ramat Aviv, Tel-Aviv, Israel
(Received 20 March 1973)

I. INTRODUCTION

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.\(^1\) Because of an instability of the chirally invariant vacuum, the real vacuum is “aligned” into a chirally asymmetric configuration.

On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.\(^2\) A number of investigations have indicated that in this frame the vacuum may be regarded as the structureless Fock-space vacuum. Hadrons may be described as nonrelativistic collections of constituents (partons). In this framework the spontaneous symmetry breakdown must be attributed to the properties of the hadron’s wave function and not to the vacuum.\(^3\)
Davier et al.

Geshkenbein, Ioffe, Zyablyuk

Ioffe, Zyablyuk

Consistent with zero vacuum condensate
Use Dyson-Schwinger Equation for bound-state quark propagator:

\[ < \bar{b} | \bar{q}q | \bar{b} > \text{ not } < 0 | \bar{q}q | 0 > \]
Pion mass and decay constant.

e-Print: nucl-th/9707003

Pi- and K meson Bethe-Salpeter amplitudes.

e-Print: nucl-th/9708029

Concerning the quark condensate.

e-Print: nucl-th/0301024

“\textit{In-Meson Condensate}”

\[- \langle \bar{q}q \rangle_\pi^\zeta = f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle.\]

Valid even for \(m_q \to 0\)

\(f_\pi \) nonzero
Quark and Gluon condensates reside within hadrons, not LF vacuum

- Bound-State Dyson-Schwinger Equations
- Spontaneous Chiral Symmetry Breaking within infinite-component LFWFs
- Finite size phase transition - infinite # Fock constituents
- AdS/QCD Description -- CSB is in-hadron Effect
- Analogous to finite-size superconductor!
- Phase change observed at RHIC within a single-nucleus-nucleus collisions -- quark gluon plasma!
- Implications for cosmological constant -- reduction by 45 orders of magnitude!

“Confined QCD Condensates”

Shrock, sjb

Maris, Roberts, Tandy

Casher Susskind
• Color Confinement: Maximum Wavelength of Quark and Gluons

• Conformal symmetry of QCD coupling in IR

• Conformal Template (BLM, CSR, ...)

• Motivation for AdS/QCD

• QCD Condensates inside of hadronic LFWFs

• Technicolor: confined condensates inside of technihadrons -- alternative to Higgs

• Simple physical solution to cosmological constant conflict with Standard Model