Nucleon-Nucleon correlations in inclusive and semi-inclusive measurements

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CONTENTS

1. Experimental evidence for NN correlations
2. Description of many-body nuclei using realistic wave functions
3. Monte Carlo generator of configurations in complex nuclei including correlations
4. Summary and conclusions
Mean Field vs Correlated $A(e,e'p)X$

Mean Field picture:

\[ k_1 + k_{A-1} = 0 \]

Two-Body Correlations picture:

\[ k_1 + k_2 + k_{A-1} = 0 \]

\[ k_1 \simeq -k_2 \]

\[ \downarrow \]

back-to-back nucleons

Ciofi, Simula, Frankfurt, Strikman \textit{PRC44 (1991)}

Ciofi, Simula \textit{PRC53 (1996)}

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Experimental Evidence for Two-Body Correlations

Triple coincidence $A(e, e'pp)X$ and $A(e, e'pn)X$ measurements:

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$A(e,e'pN)X$ reaction

incident electron

scattered electron

knocked-out proton

correlated partner

Subedi et al.

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\[ A(e,e'pN), \ N = p, \ n: \ \text{correlated pair measured with:} \]

**small** center of mass momentum, strong back to back correlation

\[ \downarrow \]

\[ \begin{array}{c}
\text{Counts} \\
\hline
40 \\
30 \\
20 \\
10 \\
0 \\
\end{array} \]

\[ \begin{array}{c}
\text{cos } \gamma \\
\hline
-1.00 \\
-0.98 \\
-0.96 \\
-0.94 \\
-0.92 \\
-0.90 \\
\end{array} \]

\[ \downarrow \]


\[ \downarrow \]

**large** relative momentum; strong \( pn \) dominance;

interpreted as *tensor* correlations in the ground state

Alvioli et al., *PRC*72 (2005)


\[ \downarrow \]

\[ \begin{array}{c}
\text{Mean Field} \\
\text{Central} \\
\text{Central} + \text{Tensor} \\
\end{array} \]

\[ \begin{array}{c}
\text{16 O} \\
\end{array} \]

\[ \begin{array}{c}
\text{VMC} \\
\end{array} \]

\[ \begin{array}{c}
\text{n(k) [fm}^3\text{]} \\
\hline
10^{-1} \\
10^{-2} \\
10^{-3} \\
10^{-4} \\
10^{-5} \\
10^{-6} \\
10^{-7} \\
\end{array} \]

\[ \begin{array}{c}
k [fm^{-1}] \\
\hline
0 \\
1 \\
2 \\
3 \\
4 \\
\end{array} \]
Tensor correlations induce strong $pn$ dominance

- Combined results of experiments on $^{12}\text{C}$ show that independent particle model accounts only for low-momentum nucleons.

- 20% of high-momentum nucleons are correlated.

- 18% of high-momentum nucleons are in a proton – neutron SRC pair!

- Calculations suggest the similar relative ratios due to tensor (spin and isospin dependent) correlations!

(Higinbotham, Piasetzky, Strikman, *CERN courier 49N1 (2009)*)
The Nuclear Many-Body Problem

- The nuclear many-body problem:
  \[
  \hat{H} \Psi_n = E_n \Psi_n, \quad \hat{H} = -\frac{\hbar^2}{2m} \sum_i \hat{\nabla}_i^2 + \frac{1}{2} \sum_{i<j} \hat{v}_{ij} + \ldots
  \]

- The ground state wave function obtained variationally using:
  \[
  \Psi_o = \hat{F} \phi_o \quad \longrightarrow \quad \hat{F} = \hat{S} \prod_{i<j} \hat{f}_{ij} = \hat{S} \prod_{i<j} \sum_n f^{(n)}(r_{ij}) \hat{O}^{(n)}_{ij}
  \]
  where
  \[
  \hat{v}_{ij} = \sum_n v^{(n)}(r_{ij}) \hat{O}^{(n)}_{ij}
  \]
  \[
  \hat{O}^{(n)}_{ij} = \left[ 1, \sigma_i \cdot \sigma_j, \hat{S}_{ij}, (L \cdot S)_{ij}, \ldots \right] \otimes \left[ 1, \tau_i \cdot \tau_j \right].
  \]

- Set of correlation functions \( f^{(n)}(r) \) obtained variationally with \( AV8' + UIX \) within FHNC/SOC; we use \( n=1,\ldots,6 \) up to tensor \( \hat{S}_{ij} \)
Any one- or two-body quantity can be calculated using:

\[ \rho^{(1)}(\mathbf{r}) = A \int \prod_{j=2}^{A} d\mathbf{r}_j \psi_o^\dagger(\mathbf{r}, \mathbf{r}_2, ..., \mathbf{r}_A) \psi_o(\mathbf{r}, \mathbf{r}_2, ..., \mathbf{r}_A) \]

to be compared with electron scattering data;

\[ \rho^{(1)}(\mathbf{r}, \mathbf{r}') = A \int \prod_{j=2}^{A} d\mathbf{r}_j \psi_o^\dagger(\mathbf{r}, \mathbf{r}_2, ..., \mathbf{r}_A) \psi_o(\mathbf{r}', \mathbf{r}_2, ..., \mathbf{r}_A) \]

used to calculate: \( \langle T \rangle, \ n^{(1)}(k) \);

\[ \rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{A(A-1)}{2} \int \prod_{j=3}^{A} d\mathbf{r}_j \psi_o^\dagger(\mathbf{r}_1, ..., \mathbf{r}_A) \hat{O}^{(n)}_{12} \psi_o(\mathbf{r}_1, ..., \mathbf{r}_A) \]

to calculate \( \langle V \rangle = \sum n \langle v(n) \rho^{(2)}_{(n)} \rangle \); exactly satisfies \( \int d\mathbf{r}_2 \rho^{(2)}_{(n=1)}(\mathbf{r}_1, \mathbf{r}_2) = \frac{A-1}{2} \rho^{(1)}(\mathbf{r}_1) \)

\[ \rho^{(2)}(\mathbf{r}_1, \mathbf{r}_2; \mathbf{r}_1', \mathbf{r}_2') = \frac{A(A-1)}{2} \int \prod_{j=3}^{A} d\mathbf{r}_j \psi_o^\dagger(\mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_A) \psi_o(\mathbf{r}_1', \mathbf{r}_2', \mathbf{r}_3, ..., \mathbf{r}_A) \]

used to calculate \( n^{(2)}(k_1, k_2) \);

\( (M. \ Alvioli, \ C. \ Ciofi \ degli \ Atti, \ H. \ Morita, \ PRC72 \ (2005)) \)
$^4\text{He: comparison with VMC}$

\[ n_{pN}(k_{rel}) = \int dK_{CM} n_{pN}(k_{rel}, K_{CM}) \]

\[ n_{pN}(k_{rel}, K_{CM} = 0) \]

- good agreement with VMC calculations
- \( n_{pn}(k_{rel}, 0)/n_{pp}(k_{rel}, 0) \) peak location ok →

(AV18: Schiavilla at al. PRL98 (2007))

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\( ^4\text{He}: \) comparison with VMC and many-body contributions

\[
n_{pN}(k_{rel}) = \int d\mathbf{K}_{CM} n_{pN}(k_{rel}, \mathbf{K}_{CM})
\]

\( ^4\text{He} - \text{pp} \)

\( ^4\text{He} - \text{pn} \)

\((\text{AV18}: \text{Schiavilla at al.} \ PRL_{98} \ (2007))\)

\begin{itemize}
  \item Shell Model
  \item two-body
  \item three-body
  \item four-body
\end{itemize}

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Back-to-Back nucleons: \( pn \) to \( pp \) ratio

\[ R_{pN} = \frac{n_{pN}^{\text{tensor}}(k_{\text{rel}}, 0)}{n_{pN}^{\text{central}}(k_{\text{rel}}, 0)} \]

\( R_{pN} \) largely enhanced in the correlation region

(M. Alvioli, C. Ciofi degli Atti, H. Morita, *TAM '07* (Bologna))

Back-to-Back nucleons: $pn$ and $pp$ probabilities

\[ P_{pN} = \frac{\int_{a}^{b} dk_{rel} k_{rel}^{2} n_{pN}(k_{rel}, 0)}{\int_{a}^{b} dk_{rel} k_{rel}^{2} \left( n_{pp}(k_{rel}, 0) + n_{pn}(k_{rel}, 0) \right)} ; \quad 0 < P_{pN} < 1 \]

- integration over the whole $k_{rel}$ range: $(a, b) = [0, \infty]$

<table>
<thead>
<tr>
<th>$A$</th>
<th>4</th>
<th>12</th>
<th>16</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{pp}$ (%)</td>
<td>19.7</td>
<td>30.6</td>
<td>29.5</td>
<td>31.0</td>
</tr>
<tr>
<td>$P_{pn}$ (%)</td>
<td>81.3</td>
<td>69.4</td>
<td>70.5</td>
<td>69.0</td>
</tr>
</tbody>
</table>

- correlation region: $(a, b) = [1.5, 3.0] \text{ fm}^{-1}$

<table>
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<th>16</th>
<th>40</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{pp}$ (%)</td>
<td>2.9</td>
<td>13.3</td>
<td>10.8</td>
<td>24.0</td>
</tr>
<tr>
<td>$P_{pn}$ (%)</td>
<td>97.1</td>
<td>86.7</td>
<td>89.2</td>
<td>76.0</td>
</tr>
</tbody>
</table>

$P_{pN}^{A=4}$ in agreement with Schiavilla et al., PRL98 (2007)
(extracted from published figures, AV18: $P_{pp} \simeq 3\%$, $P_{pn} \simeq 97\%$)

$P_{pp} \simeq 10 - 13\%$ consistent with Shneor et al., PRL99 (2007)
(extracted from $^{12}C(e, e'pp)X / ^{12}C(e, e'p)X$)

(Alvioli, Ciofi degli Atti, Morita PRL100 (2008))
center of mass of the pair $\neq 0$

$$n_{pn}^{(2)}(k_{rel}, K_{CM}) = n_{pn}^{(2)} \left( \frac{k_1}{2} - \frac{k_2}{2}, k_1 + k_2 \right)$$

$pn$ pairs - $\theta_{k_1k_2} = 180^\circ$ - $k_1 \neq k_2$ - $K_{CM} \parallel k_{rel}$

$^{12}$C preliminary - calculations in progress for $K_{CM} \perp k_{rel}$
correlated vs. random (parallel) momenta

\[ n_n^{(1)}(k_1^{(1)}, k_2^{(1)}) / n_{pn}^{(2)}(k_1, k_2) \]

\[ \theta_{k_1k_2} = 180^\circ \]

\[ K_{CM} = 0 \]

\[ K_{CM} = k_1 + k_2 \neq 0 \]

\[ k_{rel} = \frac{1}{2}(k_1 - k_2) \]

\[ K_{CM} \parallel k_{rel} \]

\[ K_{CM} = 0.0 \]

\[ k_1 \quad k_2 \]

\[ k_{rel} \]

\[ K_{CM} = 0.5 \]

\[ K_{CM} = 1.0 \]

\[ K_{CM} = 1.5 \]

\[ 12C \text{ preliminary - calculations in progress for } K_{CM} \perp k_{rel} \]

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modified $pn/pp$ relative probabilities (at $K_{CM} = 0$)

We define a modified $pn/pp$ ratio

$$R(k_{rel}) = \frac{n^{(2)}_{pn}(k_{rel}, K_{CM}=0)}{n^{(2)}_{pp}(k_{rel}, K_{CM}=0)} \left(1 - \frac{n^{(1)}_{p}(k_{rel})n^{(2)}_{n}(k_{rel})}{n^{(2)}_{pn}(k_{rel}, K_{CM}=0)}\right) = \frac{n^{mod}_{pn}(k_{rel}, K_{CM}=0)}{n^{mod}_{pp}(k_{rel}, K_{CM}=0)}$$

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modified $pp$ relative probabilities (at $K_{CM} = 0$)

$$P_{pp}^{mod} = \frac{\int_a^b dk_{rel} k_{rel}^2 n_{pp}^{mod}(k_{rel}, 0)}{\int_a^b dk_{rel} k_{rel}^2 \left( n_{pp}^{mod}(k_{rel}, 0) + n_{pn}^{mod}(k_{rel}, 0) \right)} ; \quad 0 < P_{pN} < 1$$
three-body correlations?

We can easily evaluate within the cluster expansion the three-body density

\[ \rho^{(3)}(r_1, r_2, r_3; r'_1, r'_2, r'_3) \]

and calculate, for given values of \( k_1, k_2 \) and \( k_3 \)

\[
n(k_1, k_2, k_3) = \frac{1}{(2\pi)^9} \int \prod_{i=1}^{3} dr_i dr'_i e^{i \sum_{j=1}^{3} k_j \cdot (r_j - r'_j)} \rho^{(3)}(r_1, r_2, r_3; r'_1, r'_2, r'_3) \]

the random “noise” to be subtracted:

\[
n^{(1)}(k_1)n^{(2)}(k_2, k_3) + \]
\[
+ n^{(1)}(k_2)n^{(2)}(k_1, k_3) + \]
\[
+ n^{(1)}(k_3)n^{(2)}(k_1, k_2) + \]
\[
+ n^{(1)}(k_1)n^{(1)}(k_2)n^{(1)}(k_3) \]
A Monte Carlo generator for nucleon configurations

- Configurations generated according to the independent particle model contain overlapping nucleons.

- Simple excluded volume models rejecting Monte-Carlo generated overlapping nucleons do not reproduce the nucleus density used as an input.

- We developed a Metropolis code to include NN correlations in a way which is consistent with the input one-body density and with a realistic two-body density (M. Alvioli, H.-J. Drescher, M. Strikman PLB 680(2009)).
We used $|\Psi|^2$ as a Metropolis weight function

$$\Psi(r_1, ..., r_A) = \prod_{i<j}^A \hat{f}(r_{ij}) \Phi(r_1, ..., r_A)$$

where $\Phi$ is given by the independent particle model.

$$C(r) = 1 - \rho^{(2)}(r)/\rho^{(1)}(r_1)\rho^{(1)}(r_2)$$

• new result: with correlation functions from variational calculations!

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hadron-nucleus collisions

• probability of interaction with nucleon $i$: $P(b, b_i) = 1 - [1 - \Gamma(b - b_i)]^2$

• $\Gamma(b)$ is the usual Glauber profile

• probability of interaction with $N$ nucleons, vs impact parameter $b \rightarrow$

given by:

$$P_N(b) = \sum_{i_1, \ldots, i_N} P(b, b_{i_1}) \cdots \cdots P(b, b_{i_N}) \prod_{j \neq i_1, \ldots, i_N}^{A-N} \left[ 1 - P(b, b_j) \right]$$
• average number of single and double collisions:

\[ \langle N \rangle = \sum N P_N(b) \]

\[ \langle N(N - 1) \rangle = \sum \left( N^2 - N \right) P_N(b) \]

• variance of \( \langle N \rangle \):

\[ D(b) = \frac{\langle N^2 \rangle - [\langle N \rangle]^2}{\langle N \rangle} \]

• correlations give a non-negligible effect.

(M. Alvioli, H.-J. Drescher, M. Strikman

Nucleus - Nucleus collisions; RHIC at low energies

- soft nucleon emitted at large angles
- for $b \sim R_A$ fast nucleons emitted at definite angles originate from correlations
- trigger efficiency altered by fast nucleons emission
- excitation energy of the spectator system: $- \sum_{s,w} \langle V_{sw} \rangle$
  $(s=$ spectator nucleons $w=$ wounded nucleons $)$
largely affected by correlations

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Potential energy: \( pn \) and \( pp \) contributions

\[
\langle V \rangle_{pN} = \sum_j \int d\mathbf{r}_{12} \, v_p^{(j)}(r_{12}) \rho_p^{(2)(j)}(r_{12}), \quad v^{(j)}(r_{12}) \in AV8'
\]

\( ^{16}\text{O} - \text{NN} \) \hspace{2cm} \( ^{16}\text{O} - \text{pp} \) \hspace{2cm} \( ^{16}\text{O} - \text{pn} \)

\[
A \quad \langle V \rangle_{pp}(= \langle V \rangle_{nn}) \quad \langle V \rangle_{pn}
\]
\[
\begin{array}{c|cc}
16 & 8\% & 83\% \\
40 & 9\% & 82\%
\end{array}
\]

no tensor correlations \( \rightarrow \) proportionality to number of pairs restored

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Nucleons marked in green were correlated with one interacting nucleon.

Large energy released by disrupting correlated pairs: mostly from pn pairs!

Nucleons with large momentum are emitted by disrupting correlated pairs!

Configurations: [http://www.phys.psu.edu/~malvioli/eventgenerator](http://www.phys.psu.edu/~malvioli/eventgenerator)
spectator neutrons in Lead - Lead

new result: large $N_1/N_2$ dispersion at medium impact parameter

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Summary

- Many-body calculations can be reliably performed within a convergent cluster expansion method: any one and two-body quantity can be calculated.
- The contributions of different isospin pairs to two-body momentum distributions and to binding energy of nuclei have been calculated.
- High-energy processes are also affected by NN correlations (Claudio’s talk and Alvioli, Ciofi, Morita, Palli PRC78 2008 and Alvioli, Ciofi, Kopelevich, Potashnikova, Schmidth to appear)
- Inclusion of NN correlations in calculations of nuclear reactions on an event-by-event basis can be made using correlated nucleon configurations generated by Monte Carlo in a consistent way; MC-generated configurations at: http://www.phys.psu.edu/~malvioli/eventgenerator
- Three-body correlations? Calculations for neutron stars?
- New experiments are scheduled on $^4He$ and $^{40}Ca/^{48}Ca$ to investigate SRC effects as a function of A.