Connecting Quarks to the Cosmos
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Inflationary Cosmology II

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Successes of Standard Hot Big Bang Cosmology

- Big-bang nucleosynthesis
- Expansion of the Universe
- CMB
- Structure formation
- Tested account from $10^{-5}$ sec on
- Consistent age for the Universe (expansion, stars, radioactive elements)
Big Unanswered Questions

- Origin of baryons/large entropy per baryon
- Dark Matter
- Dark Energy, Cosmic Acceleration, \( \Lambda \)
- Dynamite behind the big bang
- Heat of the big bang
- Isotropy, homogeneity and flatness (not generic initial conditions)
- Origin of seed inhomogeneity
- Before the big bang

Just like the questions raised by the SM, not a logical inconsistency but clues about the grander theory

Addressed by Inflation
SPACE-TIME GEOMETRY

Our initial geometry

Smooth, small ripples

15 Gyr

Generic initial geometry

Black holes, anisotropy, ...

"A MESS"

Collins & Hawking '73

Not logical inconsistency!

Dilemma of initial data

(Why so special?)
"The Horizon Problems"

"Causal Limit": \( ct \sim H^{-1} \sim R \)

RD: \( n = 2 \)

MD: \( n = \frac{3}{2} \)
Said Another Way

- Horizon at last scattering subtends only 1 degree on the sky; what mechanism causes the temperature to be so uniform on scales » 1 degree?
- Galactic-sized masses entered the horizon about 1 year after the big bang; what causal physics created density perturbations that late in the history of Universe?
Cosmic Inflation

• Addresses isotropy, homogeneity & inhomogeneity, flatness, dynamite, and before the BB
• Paradigm, not a model
• Most important idea since big bang
• Strong evidence supporting inflation (CMB, large-scale structure)
Key Elements of Inflation

• Period of exponential expansion (constant Hubble Constant and horizon size)
• Tremendous entropy production (called reheating)
SOLVING THE HORIZON PROBLEMS

"CAUSAL LIMIT": \( ct \sim H^{-1} \sim R^n \) (RD: \( n = 2 \); MD: \( n = \frac{3}{2} \))

**Graph:**
- \( \ln(\text{SIZE}) \) vs. \( \ln(R) \)
- \( H = \text{const.} \) (\( R \sim e^{HT} \))
- Horizon Crossings:
  - Galaxy \( \propto R(t) \)
  - Hubble Volume \( \propto R(t) \)
- Causal Microphysics
- Size Current
- Today
Inflation Implemented as Scalar-field Dynamics

Theorists: When in doubt, just add a scalar field
Homogeneous Scalar-field is Just Like a Fluid

\[ \rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \quad p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \]

\[ w = \frac{-V(\phi)}{+V(\phi)} \approx -1 \]

\[ \ddot{\phi} + 3H\dot{\phi} + V'(\phi) [+\Gamma \dot{\phi}] = 0 \]

- Slow roll (flat part of potential): \( w \approx -1 \)
- Rapid oscillation: particle production and conversion of potential energy to particles (heat) aka decay of \( \phi \) particles
Entropy
Production/Reheating

Adiabatic: Constant Number of Photons per co-moving Volume, i.e., $RT = \text{const}$
Solving the Flatness, Horizon Problems

Tiny (<<1cm) bit of universe is flat & smooth (but too small to contain all we see today)

All that we can see today (still smooth & flat)
Quantum Fluctuations Seed Density Perturbations

Fluctuations: Micro to Macro

- Quantum fluctuations on subatomic scale
- "Lumpy" distribution of matter on macro scale

More matter → Light Yes
Less matter

The Horizon Problems

- Causal limit: $ct \sim H^{-1} - R$
- Size current: $\text{Hubble volume} \sim H^{-1}$
- Galaxy $\propto \Omega_{c}(R)$
- Horizon crossing

QM Fluctuations in the "Inflaton" $\phi$

$\Delta \phi = \frac{H}{2\pi}$

Energy Density Perturbations

$\delta \rho = V' \Delta \phi$

Density perturbations after reheating
Given scalar potential $V(\phi)$, can compute all observables in terms of $V$, $V'$ and $V''$. 
Q: Where did the almost perfectly smooth quark soup come from?

A: Decay of False Vacuum Energy!
Q: Where did the small lumps in the quark soup come from?

A: Quantum Fluctuations!
Inflation in the Universe

Early epoch of tremendous expansion driven by vacuum energy

Accounts for:
- Smoothness, heat of Big Bang & absence of monopoles

& Predicts:
- "Flat universe" ($\Omega_0 = \frac{\text{rho}_0}{\text{H}^2} \approx 1.0$)
- Nearly scale-invariant density perturbations
- Nearly scale-invariant gravity waves
Important Facts About Inflation

1. Paradigm, no standard model, many viable models (new, chaotic, …)

2. Key predictions
   - Flat Universe: $\Omega_0 = 1.000$
   - Almost scale-invariant adiabatic, almost power-law, Gaussian adiabatic fluctuations
     - $|n-1| \sim 0.1$, $|dn/dlnk| \sim 10^{-3}$
   - Almost scale-invariant spectrum of gravitational waves
     - $n_T \sim 0$ to -0.1 (i.e., negative)

3. Consistency relation: $T/S = -5n_T$
   - Unfortunately, $T/S$ not related to $n$
4. Measuring GWs immediately gives scale of inflation!

\[ H_I^{-1} = \frac{2 \times 10^{-39} \text{sec}}{\sqrt{T/S}} \]

\[ V^{1/4} = 3 \times 10^{16} \text{ GeV} (T/S)^{1/4} \]

5. But, no robust prediction for T/S (=r)

6. Inflationary perturbations + Cold Dark Matter + “Λ” = ΛCDM scenario for structure formation (another test)
T/S > 0.001 if n > 0.9?

Hoffman/Turner, PRD 64, 02350 (2001)
Slow-roll approximation (neglect $\ddot{\phi}$) → $3H \dot{\phi} + V' = 0$, requires

\[
m_{\text{Pl}} V'/V < \sqrt{48\pi} \quad m_{\text{Pl}}^2 V''/V < \sqrt{24\pi}
\]

Worked example: $V(\phi) = m^2\phi^2/2$

**Slow-roll condition:** $\phi/m_{\text{Pl}} > 1/12\pi \quad \Rightarrow \quad \text{equation of motion:}

\[
3H \dot{\phi} = -V' \quad \Rightarrow \quad \frac{8\pi}{m_{\text{Pl}}^2} \int \frac{V d\phi}{V'} = \int H dt \int d\ln R \equiv N
\]

\[
N(\phi_i) = 2\pi (\phi_i/m_{\text{Pl}})^2 \sim 60 \Rightarrow \phi_i > 5m_{\text{Pl}}
\]

**Density perturbations:** $(\delta \rho/\rho)_{\text{HOR}} \simeq H^2/\dot{\phi} \simeq 8\pi \sqrt{24\pi}/3 (V^{3/2} m_{\text{Pl}}^3 V')$

\[
(\delta \rho/\rho)_{\text{HOR}} \simeq \frac{4\pi}{3} \sqrt{12\pi} N^{1/2} (m/m_{\text{Pl}}) \simeq 10^{-5} \quad \text{for} \quad N \sim 50 \quad \Rightarrow \quad m \simeq 10^{-6} m_{\text{Pl}}
\]

**Gravity waves:** $h \simeq H/m_{\text{Pl}}$

\[
h \simeq \sqrt{\frac{8\pi}{3}} \left( \frac{m\phi}{m_{\text{Pl}}^2} \right) \simeq 10^{-5} \quad T/S \simeq 0.3 \left( \frac{m_{\text{Pl}} V'}{V} \right)^2 \simeq 0.3
\]
Comments about models

1. Many viable models, non-compelling
2. All based upon a weakly, coupled scalar field with small parameter (e.g., $m \sim 10^{-6} m_{Pl}$ or $\lambda \sim 10^{-14}$)
3. Weakly coupled $\rightarrow$ re-heating is a challenge, $\rightarrow$ perturbations almost Gaussian
4. Very flat potentials
5. No hint about cosmological constant
"ripples" in the microwave echo
Temperature variation ~ 30 μK
Evidence for "primordial lumpiness" that seeded structure (stars, galaxies, clusters of galaxies, superclusters, voids, walls,...)

S. Hawking: "Greatest discovery of all time"

Only a slight overstatement

COBE
23 April 1992

\[ \frac{\Delta T}{T} \approx 10^{-5} \]
\[ \theta \approx 7^\circ-90^\circ \]

WOW!
Serious Testing of Inflation
Began with WMAP and SDSS
CMB anisotropy is a non-trivial map of density inhomogeneity to temperature fluctuations: Mapping depends upon cosmological parameters (good news!)
Serious testing of Inflation has begun

**Key Predictions**

- Flat Universe
- *Almost* scale-invariant, Gaussian perturbations: \(|(n-1)| \sim 0.1\) and \(|dn/d\ln k| \sim 0.001\)
- Gravity waves: spectrum, but not amplitude
- Cold Dark Matter Scenario

**Key Results**

- \(\Omega_0 = 1.0 \pm 0.006\)
- \((n-1) = -0.04 \pm 0.014^*; \ dn/d\ln k = -0.032 \pm 0.02;\) no evidence for non-Gaussianity
- \(r < 0.2\) (95% cl)*

*Depends significantly upon the priors assumed*
CDM explains all the structure that exists in the Universe today and all measurements of it – only theory that does (circumstantial proof)
INFLATION SCORECARD

**Predictions**

**Flat Universe**
- $\Omega_0 = 1.000$

**Density Perturbations from**
- Adiabatic
- Nearly Scale-Invariant ($n-1 \sim 0(\pm 0.1)$)
- Nearly Power-Law $dn/d\ln k \sim 10^{-3}$
- Gaussian

**QM Fluc**
- $33$ acoustic peaks
- $n = 1.05 \pm 0.09$
- $dn/d\ln k = -0.03 \pm 0.02$
- No evidence against

**CDM**
- "Has much of the truth"

**Grav Waves from QM Metric Fluc**
- $T/S > 10^{-3}$ (?)
- Nearly scale invariant $n = -1/3$

**NOW**
- $n = 1.03 \pm 0.03$

**GRADE**
- $\pm 0.001$
- $\pm 0.004$
- $\pm 10^{-3}$

**Goal**
- $10^{-3}/10^{-4}$
STATUS OF INFLATION:
EXCELLENT!
The Largest Things in the Universe Began from Subatomic Quantum Fluctuations!
Quantum World Projected Across the Sky by the Expansion of the Universe

< one billionth the size of a proton
GWs: The Smokin’ Gun Test of Inflation

- Directly reveals epoch of inflation
- Spectrum provides consistency check: $T/S = -5n_T$
- Reconstruction of scalar potential

\[
\begin{align*}
  V_{50} & = 1.65 T (1 - 1.2n_T); \\
  V'_{50} & = 5.01 \sqrt{-n_T} \left(\frac{V_{50}}{m_{Pl}}\right); \\
  V''_{50} & = 4\pi \left[ (n - 1) - 3n_T \right] \left(\frac{V_{50}}{m_{Pl}^2}\right).
\end{align*}
\]

NB: $n_T = -(T/S)/5$

- Direct detection:
  - LIGO & LISA unlikely
  - “Big Bang Observer” (NASA Beyond Einstein concept)

- B mode of CMB polarization
Reconstruction

FIG. 5. The four generic inflationary potentials: (a) $V(\phi) = 10^{10}M^4 \phi^4$, (b) $V(\phi) = 10^{10}M^4 \phi^2$, (c) $V(\phi) = 10^{10}M^4 \phi$, and (d) $V(\phi) = 10^{10}M^4$. Here $M = 10^{10}$ GeV and $\phi_{\text{fo}} = 10^{3}$ m$_{\text{pl}}$. The cases (a) and (c) correspond to $V(\phi)$ large and small, respectively, whereas cases (b) and (d) correspond to $V(\phi)$ large and small, respectively.
CMB Anisotropy from Gravity Waves

- $\Theta \Theta = \text{GW temp}$
- $\text{EE} = \text{E mode}$ (scalar)
- $g$ lensing: gravlensing of EE
- $\text{BB}/g$ waves = GW B-mode
DETECTION OF GRAVITY WAVES

Promised Land of Slow Roll

$\Omega_{_{GW}(H^2)} h^2$

$T/S$

Very Challenging!

"Double Detection" $\Rightarrow \eta_t \rightarrow \pm 0.03$

NB:

LIGO: $10^{-8}$

LISA: $10^{-12}$
Inflation: The Challenges

- Precision testing: measure and $\Omega_\phi$ & $n-1$ (to ±0.001), $dn/d\ln k$ (±0.001), search for nonGaussianity
- Detection of GW: T/S (B modes or directly)
- Measure $n_T$ (need direct direction)
- Fundamental theory of inflation: Who is $\phi$?
- Successor to inflation (… it seems like duct tape)
- Laboratory test of inflation (e.g., produce a $\phi$)
Summary

- Inflation is a central part of the consensus cosmology ("most important idea since the big bang itself")
- Strong, but not compelling evidence for inflation
- Important tests and challenges remain
Inflation might be the dynamite of the Big Bang,
But what happened before the Big Bang?
neat & tidy!

... but Einstein’s theory does not incorporate quantum mechanics.

... and the conditions at the beginning are precisely where quantum effects should be critical!
Einstein got the right answer for the wrong reason!

Einstein's Big Bang

= Emergence of space and time

No before the Big Bang
INFLATIONARY MULTIVERSE

INFINITE NUMBER OF BEGINNINGS

Tremendous burst of expansion "inflation"

"Cosmic river of time"
THE MULTIVERSE
IS IT SCIENCE IF IT IS NOT TESTABLE?
We Can Test Whether or Not “Our Piece of the Multiverse” Originated From Inflation

... well on our way to doing so