Equation of State of dense matter for supernova

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  - Virial expansion for non-ideal gas
  - Relativistic mean field for solid (and liquid)
- Results & Discussion
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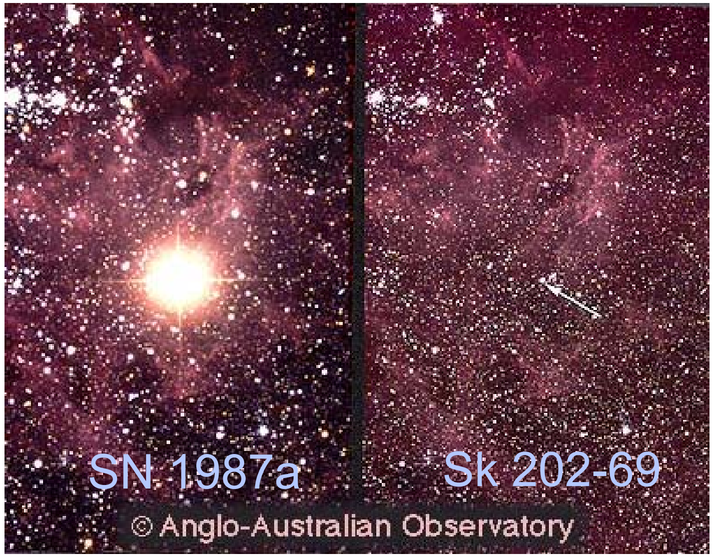
- Conclusions and Prospects

- In preparation
- Phys.Rev.C **78** (2008) 015801

INT 09-2A, Seattle, 2009

Introduction: SN

- Supernova: observed for over thousand year.
- Obstacles in modern studies of core collapse SN
  - Mutli-D hydrodynamics
  - Complex ν-matter, ν- ν interactions
  - Equation of State for dense matter



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1. pre SN star at the end of nuclear burning. Matter infalls under gravity

2 . outgoing shock due to incompressibility of nuclear matter

3. stalled shock due to energy loss at front

4. Neutrinos revived shock - explosion

Introduction: EOS

Baryon Density [fm^{-3}]	1E-9	1E-3	1E-1
Particle spacing [fm]	1E3	10	1
Phase	Gas	Solid	Liquid

❖ for low T: below critical point (gas-liquid)

- Efficient degrees of freedom: nucleon.
- Ingredients: n, p, e, (trapped neutrino).
- UNEDF $\longrightarrow$ NEDF

Introduction: existing EOS for SN

- Lattimer & Swesty (1991)

  Compressible liquid droplet model with a Skyrme density functional
- H. Shen et al (1998)

  Thomas-Fermi approximation with variational principle for a relativistic mean field density functional

new physics in our model

- Multi-component distribution in virial gas
- Properties of neutron rich matter in virial gas
- First microscopic relativistic mean field calculation for lattice at subnuclear density and finite T

Formalism 1: Virial expansion of non-ideal gas

- Grand partition function

Unknowns: z_n, z_p, r_c

$$\frac{\log Q}{V} = \frac{P}{T} = \frac{2}{\lambda_n^3}[z_n + z_p + (z_p^2 + z_n^2) b_n + 2 z_p z_n b_{pn}] + \frac{1}{\lambda_\alpha^3}[z_\alpha + z_\alpha^2 b_\alpha + 2 z_\alpha (z_n + z_p) b_{\alpha n}] + \sum_i \frac{1}{\lambda_i^3} z_i \Omega_i, \qquad (1)$$

- ➢ nucleon-nucleon [1]
- ➢ nucleon-alpha [1]
- ➢ nuclei [2]

1. Light species: nucleon and alpha (Horowitz & Schwenk 05)

   Second virial coefficients *b*: related to scattering phase shifts

2. Heavy species: nuclei (FRDM mass table: Moller et al 97)

Chemical equilibrium

$$\mu_i = Z\mu_p + N\mu_n, \quad z_i = \exp(\mu_i + E_i)/T = z_p^Z z_n^N e^{E_i/T}$$

Coulomb correction

$$E_i^C = \frac{3}{5}\frac{Z_i^2 \alpha}{r_A}[-\frac{3}{2}\frac{r_A}{r_c} + \frac{1}{2}(\frac{r_A}{r_c})^3].$$

Nuclear partition function and
Level density
(Fowler, Engelbrecht & Woosley, 78)

$$\Omega_i = (2J_0 + 1) + \int_{E_d}^{E_t} dE \rho(E) \exp(-E/T),$$

$$\rho(E) = \frac{\sqrt{\pi}}{12}\frac{\exp(2\sqrt{aU})}{a^{1/4}U^{5/4}},$$

Formalism 2: NEDF-RMF for solid/liquid

RMF Lagrangian

NL3

$$\mathcal{L}_{\mathrm{RMF}} = \bar{\psi}\left[i\partial^{\mu}\gamma_{\mu} - M - g_{\sigma}\sigma - g_{\omega}\gamma^{\mu}\omega_{\mu} - g_{\rho}\gamma^{\mu}\vec{\tau}\cdot\vec{\rho}_{\mu} - e\gamma^{\mu}\frac{1+\tau_3}{2}A_{\mu}\right]\psi$$
$$+\frac{1}{2}\partial^{\mu}\sigma\partial_{\mu}\sigma - \frac{1}{2}m_{\sigma}^{2}\sigma^{2} - \frac{1}{3}g_{2}\sigma^{3} - \frac{1}{4}g_{3}\sigma^{4} - \frac{1}{4}\omega^{\mu\nu}\omega_{\mu\nu} + \frac{1}{2}m_{\omega}^{2}\omega^{\mu}\omega_{\mu}$$
$$+\frac{1}{4}c_{3}(\omega^{\mu}\omega_{\mu})^{2} - \frac{1}{4}\vec{\rho}^{\mu\nu}\vec{\rho}_{\mu\nu} + \frac{1}{2}m_{\rho}^{2}\vec{\rho}^{\mu}\vec{\rho}_{\mu} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

EoMs:

Nucleon:

$$\left[\alpha\cdot\mathbf{p}+V(\mathbf{r})+\beta\big(M+S(\mathbf{r})\big)\right]\psi_i=\varepsilon_i\psi_i$$

Hartree Mean fields:

$$\begin{cases} V(\mathbf{r})=g_{\omega}\omega_{0}(\mathbf{r})+g_{\rho}\tau_{3}\rho_{0}(\mathbf{r})+e\dfrac{1-\tau_3}{2}A_{0}(\mathbf{r}) \\ S(r)=g_{\sigma}\sigma(\mathbf{r}) \end{cases}$$

Mesons and Photons:

$$\begin{cases} (-\Delta+\partial_{\sigma}U(\sigma))\sigma(\mathbf{r})=-g_{\sigma}\rho_{s}(\mathbf{r}) \\ (-\Delta+m_{\omega}^{2})\omega_{0}(\mathbf{r})=g_{\omega}\rho_{v}(\mathbf{r}) \\ (-\Delta+m_{\rho}^{2})\rho_{0}(\mathbf{r})=g_{\rho}\rho_{3}(\mathbf{r}) \\ -\Delta A_{0}(\mathbf{r})=e(\rho_{c}(\mathbf{r})-\rho_{e}) \end{cases}$$

Finite temperature n_i: Fermi-Dirac stat.

$$\begin{cases} \rho_s(\mathbf{r})=\sum_{i=1}^{A}\bar{\psi}_i(\mathbf{r})\psi_i(\mathbf{r})n_i,\ \rho_3(\mathbf{r})=\sum_{i=1}^{A}\psi_i^{+}(\mathbf{r})\tau_3\psi_i(\mathbf{r})n_i \\ \rho_v(\mathbf{r})=\sum_{i=1}^{A}\psi_i^{+}(\mathbf{r})\psi_i(\mathbf{r})n_i,\ \rho_c(\mathbf{r})=\sum_{i=1}^{A}\psi_i^{+}(\mathbf{r})\dfrac{1-\tau_3}{2}\psi_i(\mathbf{r})n_i \end{cases}$$

Solutions of Hartree mean field

- Subnuclear density non-uniform matter has lattice structure, whose unit is treated in Wigner-Seitz approximation as spherical cell.

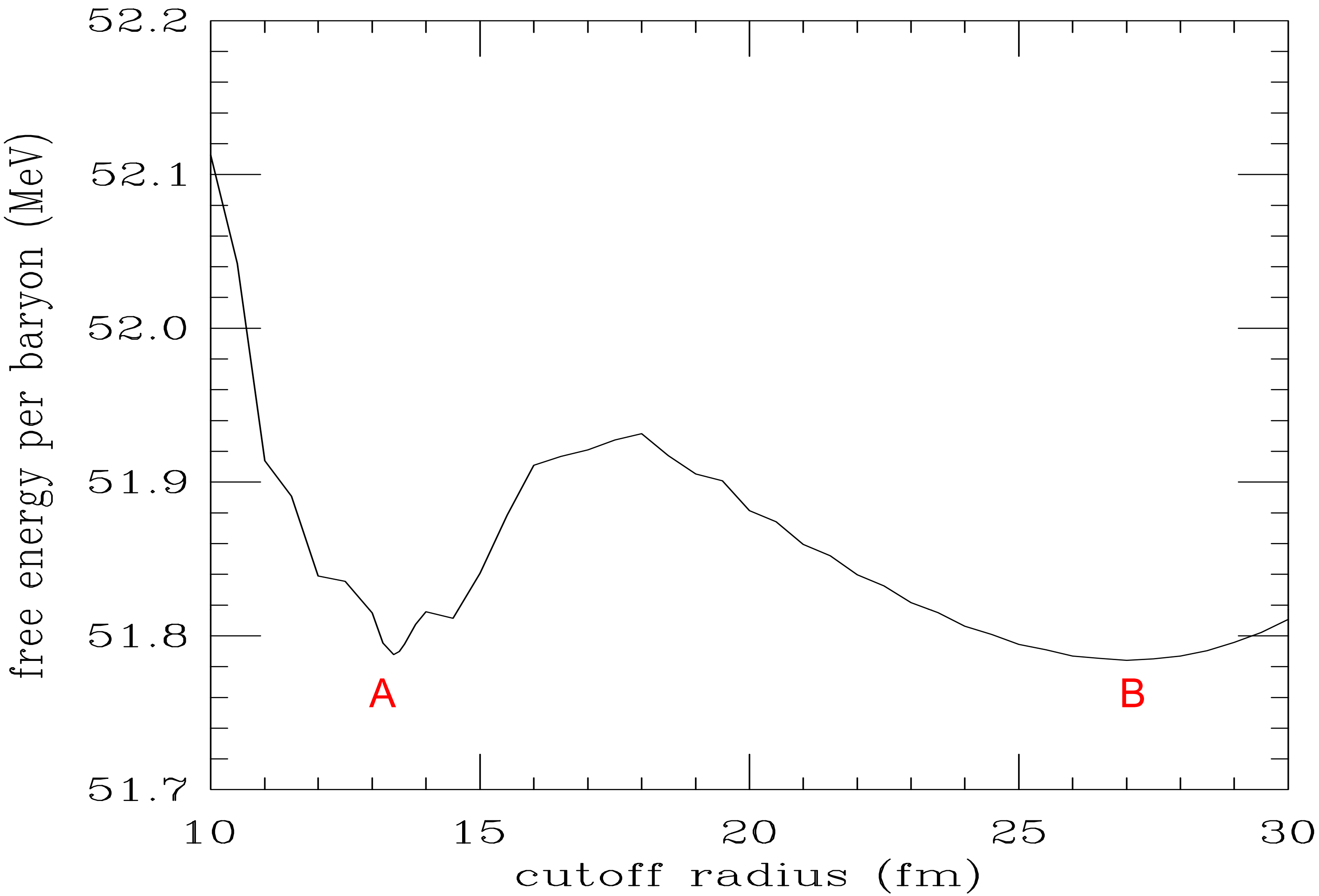
- For each ρ (fix Y_p and T), minimize free energy per baryon to determine the cell radius R_c, where nucleon number $A = 4/3\pi R_c^3 \rho$.

- The mean fields would have ~1000 levels for hundreds of nucleons $\Longrightarrow$ time consuming to get self-consistent mean field solution.

- Liquid: uniform matter.

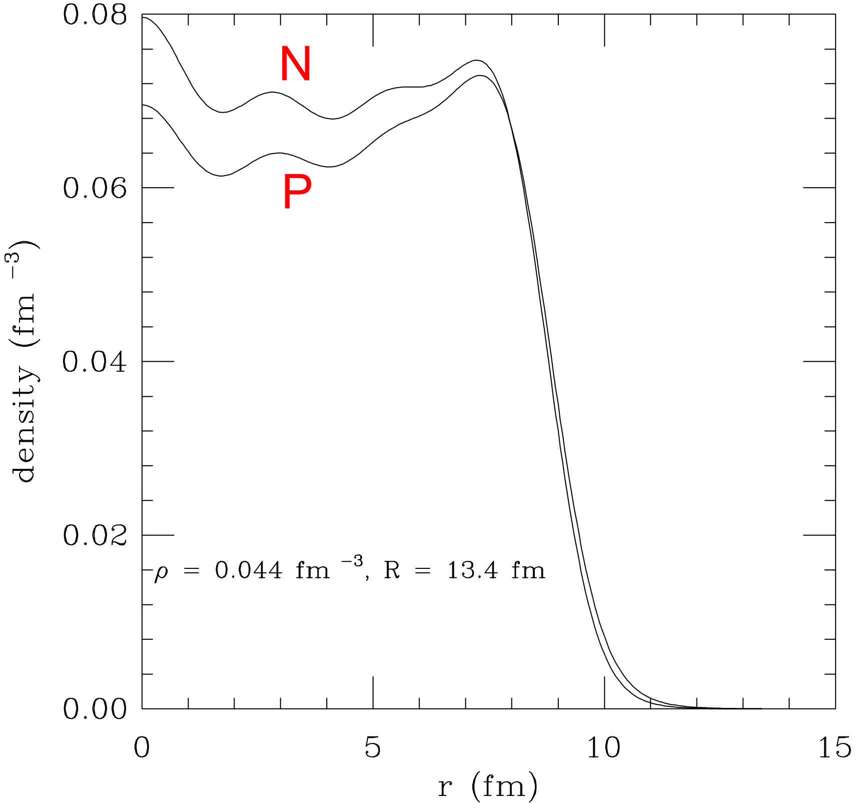
minimization in Hartree

$T = 1$ MeV, $Y_p = 0.5$, $\rho = 0.044$ fm^{-3}.



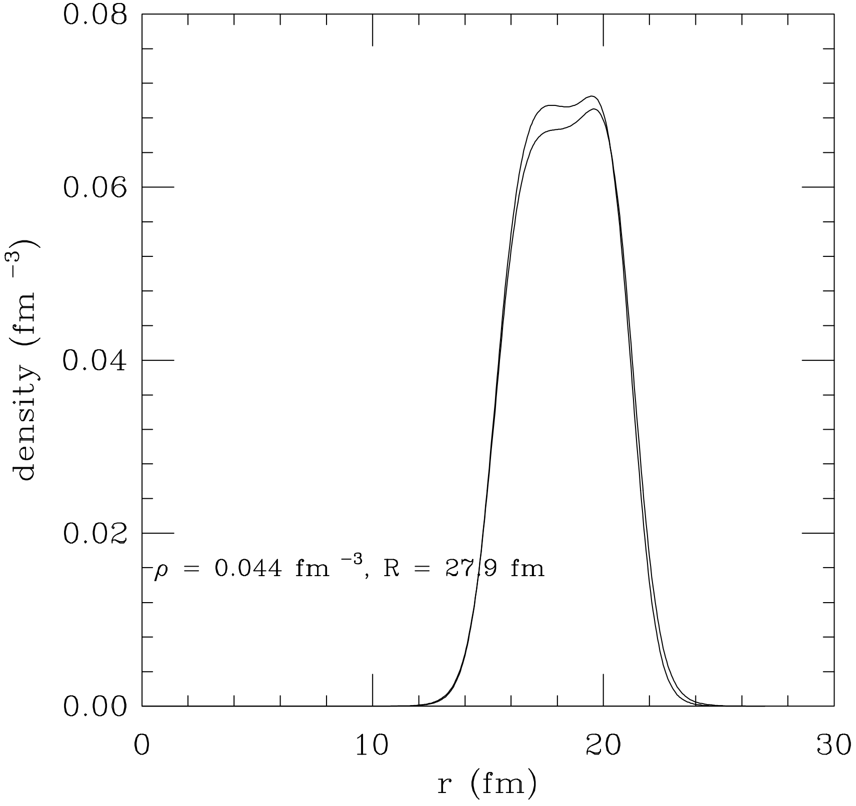
Minimization of free energy per baryon versus cell radius

Normal nuclei vs. shell nuclei



N=Z=222

A



N=Z=1814

B

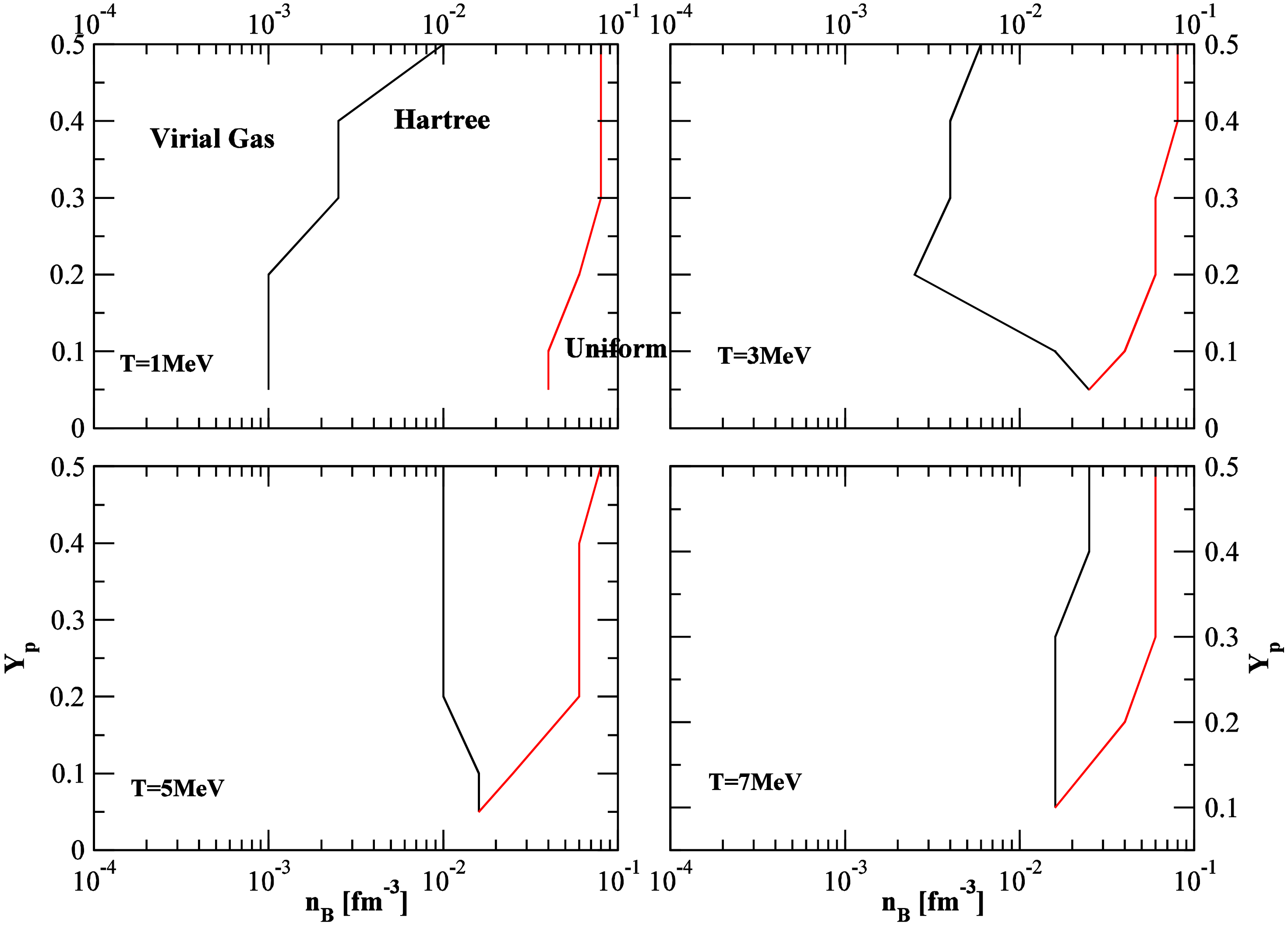
T = 1 MeV, Y_p = 0.5, ρ = 0.044 fm^{-3}.

3-D Phase spaces (T, **ρ**,Y_p) for gas-solid-liquid

	Virial Gas	Hartree	Uniform matter
Temperature [MeV]	1~20	1~10	1~20
Density [fm^{-3}]	1E-9~1E-1	1E-3~1E-1	1E-2~1
Proton fraction	0.05~0.5	0.05~0.5	0.05~0.5
# points in phase space	40896	9471	10626
CPU time (hr)	1E4	1E5	1E2

- Parallel computation: MPI
- Computer clusters at IU

R&D: phase interfaces



R&D: solid-liquid cross and Finite temperature RPA

- Use real time formalism for quantum Hadrodynamic theory at finite temperature:

  same Lagrangian as before, but all D.O.F are included.

- Dyson's equation for the proper polarization matrices:

$$\Pi^{(ab)}(q) = \Pi^{0(ab)}(q) + \Pi^{(ac)}(q) D^{0(cd)}(q) \Pi^{0(db)}(q),$$

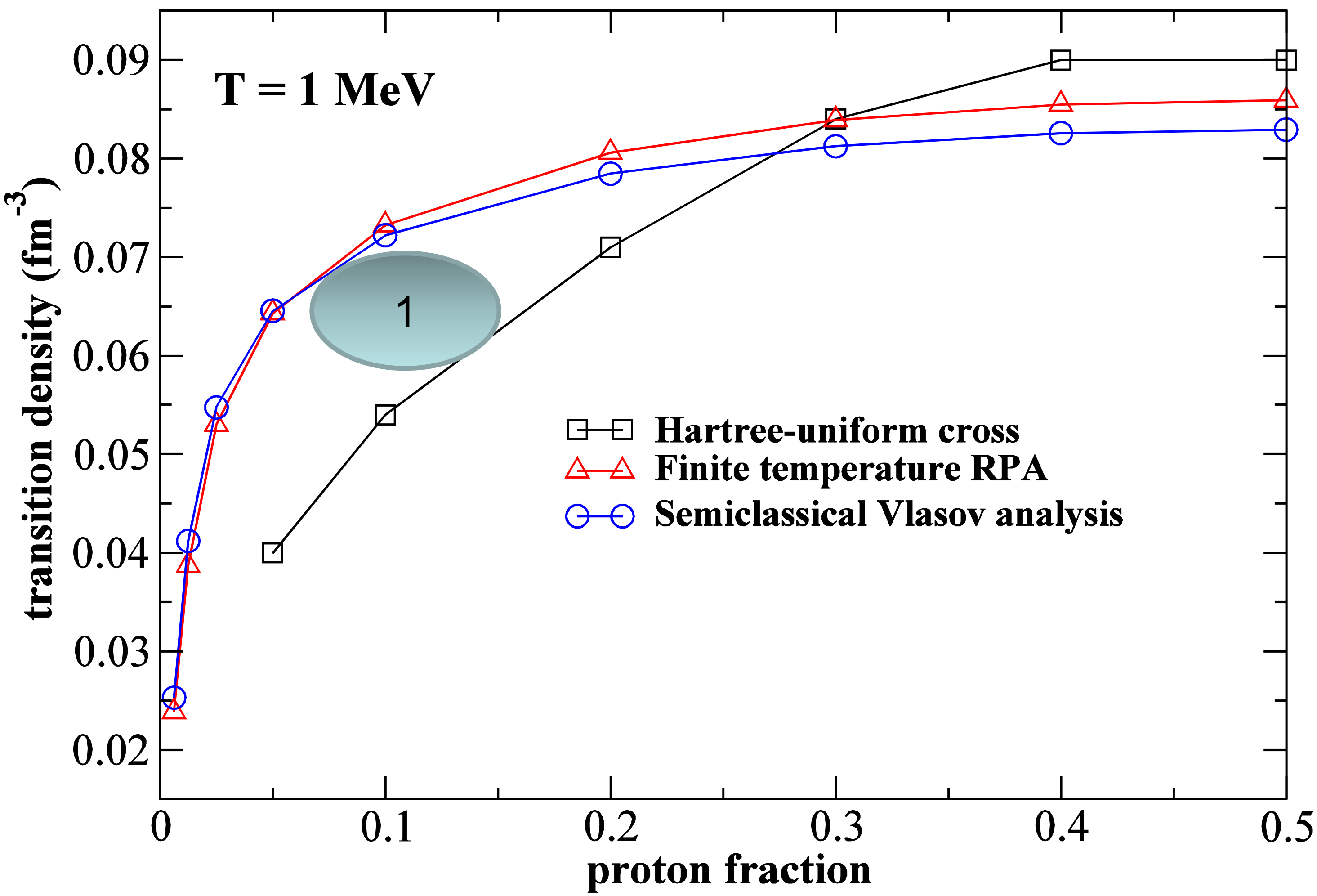
Longitudinal dielectric function ε_L, $\varepsilon_L(q) = \det[\varepsilon_T{}^{\lambda}(q)_L]$, with

$$\varepsilon_\tau^\lambda(q) = \delta_\tau^\lambda - D_{\tau\rho}^0(q) \Pi^{0\rho\lambda}(q).$$

- The poles of $\varepsilon_L(q)=0$ give the collective excitations.

- Negative value for ε_L in static limit indicates the instability of collective modes.

- Consistency check



1. Non-spherical shapes: pasta phase

Conclusions

1. Microscopic finite temperature relativistic Hartree mean field for solid/liquid.
2. Virial expansion for gas of n, p, e, and thousands of heavy nuclei, with Coulomb correction and nuclear partition function.
3. Found gas-solid-liquid crosses from finished runs.
4. Consistency check by finite temperature RPA.

Prospects

Full table of EoS