The plan:

• the known unknowns
• the various masses
• CP violation and long baseline experiments
• the supernova laboratory
• red giants and neutrino electromagnetic moments
• homework: unknown unknowns
Discussed solar vs/MSW mechanism: parallel atmospheric ν story

- Gaisser: cosmic ray p/nuclei hit upper atmosphere, produce secondary πs, Ks

- secondary decays

\[ \pi^\pm \rightarrow \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \]
\[ \mu^\pm \rightarrow e^\pm + \nu_e (\bar{\nu}_e) + \bar{\nu}_\mu (\nu_\mu) \]

- consequently expect a flavor ratio \( r \approx 0.5 \)

\[ r = \frac{\nu_e + \bar{\nu}_e}{\nu_\mu + \bar{\nu}_\mu} \]

- Monte Carlos taking into account μ polarization, geomagnetic effects, etc., find \( r \approx 0.45 \)

- form ratio of ratios:

\[ R = \frac{(N_\mu/N_e)_{\text{DATA}}}{(N_\mu/N_e)_{\text{MC}}} \]
Super-K e vs μ events

Figure 15: Zenith angle distributions for atmospheric neutrino interactions in Super-Kamiokande [115]. The model predicts all transformation phenomena regardless of energy, baseline, lepton number, flavor, or intervening matter. To test the model, we therefore need to measure the parameters and compare them across experimental regimes such as energy, baseline, etc., to verify some of the explicit predictions of the model such as the oscillatory nature of the transformation, look for the predicted sub-dominant effects, and search for some of the possible non-Standard Model transformation signatures.

The enormous experimental regime covered by the atmospheric measurements means that they are particularly sensitive tests, and in many ways the atmospheric sector is far ahead of the solar sector in verifying some of the finer details of the transformation model.
so we can summarize what has been learned from the solar and atmospheric experiments, KamLAND, ...
normal hierarchy          inverted hierarchy

\[ m_1^2 \]
\[ m_2^2 \]
\[ m_3^2 \]

solar \( \sim 7 \times 10^{-5} \text{eV}^2 \)

atmospheric \( \sim 2 \times 10^{-3} \text{eV}^2 \)

??

\[ \nu_e \]
\[ \nu_\mu \]
\[ \nu_\tau \]

\[ \Delta m_{23}^2 \equiv m_3^2 - m_2^2 < 0 \]

We have no information about \( m_3 \) except that its value is much less than the other two masses.

(iii) Degenerate neutrinos, i.e. \( m_1 \equiv m_2 \equiv m_3 \).

Oscillation experiments do not tell us about the overall scale of masses. It is therefore important to explore to what extent the absolute values of the masses can be determined.

While discussing the question of absolute masses, it is good to keep in mind that none of the methods discussed below can provide any information about the lightest neutrino mass in the cases of a normal or inverted mass-hierarchy. They are most useful for determining absolute masses in the case of degenerate neutrinos, i.e., when all \( m_i \geq 0 \).

One can directly search for the kinematical effect of nonzero neutrino masses in beta-decay by looking for structure near the end point of the electron energy spectrum. This search is sensitive to neutrino masses regardless of whether the neutrinos are Dirac or Majorana particles. One is sensitive to the quantity

\[ m_\beta \equiv \sqrt{\sum_i |U_{ei}|^2 m_i^2} \]

The Troitsk and Mainz experiments place the present upper limit on \( m_\beta \leq 2 \text{eV} \).

The proposed KATRIN experiment is projected to be sensitive to \( m_\beta > 0.2 \text{eV} \), which will have important implications for the theory of neutrino masses. For instance, if the result is positive, it will imply a degenerate spectrum; on the other hand a negative result will be a very useful constraint.

If neutrinos are Majorana particles, the rate for \( \beta\beta \) decay is

\[ m_\beta \]
Neutrino mixing status: $\theta_{12}, \theta_{23}$

\[
\begin{pmatrix}
\nu_e \\
\nu_\mu \\
\nu_\tau
\end{pmatrix}
= \begin{pmatrix}
c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
-s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{pmatrix}
\]

\[
= \begin{pmatrix}
1 \\
c_{23} & s_{23} \\
-s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & s_{13}e^{-i\delta} \\
1 & \nu_e \\
-s_{13}e^{i\delta} & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} \\
-s_{12} & c_{12} \\
1 & \nu_e \\
\end{pmatrix}
\begin{pmatrix}
\nu_1 \\
e^{i\phi_1}\nu_2 \\
e^{i\phi_2}\nu_3
\end{pmatrix}
\]

atmospheric results: $\theta_{23} \sim 45^\circ$

$\nu_e$ disappearance $\sin \theta_{13} \leq 0.17$

solar results: $\theta_{12} \sim 30^\circ$
The known unknowns -- what are the potential astrophysical consequences?

• we know two mass$^2$ differences, but not the absolute scale of $\nu$ mass
• we know two mixing angles, but have only a limit on the third $\theta_{13}$
• we do not know the Dirac/Majorana nature of the mass
• we do not know the hierarchy, normal or inverted
• we do not know the sizes or roles in nature of three CP-violating phases
• we have not explored other MSW crossings or potentials
• we do not know whether $\nu$s have nonzero electromagnetic moments
• we do not know the high-energy limits of $\nu$ physics
• we do not know whether there are additional $\nu$ species
Neutrino mass: three important laboratory tests

- **direct kinematic tests**

\[
m_{\nu e} = \sum_{i=1}^{n} |U_{ei}|^2 m_i \quad \bar{\nu}_e \sim 0.8 \nu_1 + 0.5 \nu_2 + ?? \nu_3
\]

![Graph showing relative decay amplitude and rate vs electron energy](image)

**Known splittings << resolution: 2.2 eV limit**

\[\sum_{i=1}^{n} m_{\nu}(i) \leq 6.6 \text{ eV}\]
- neutrino oscillations
  \[ P_{\nu_\mu \rightarrow \nu_e} = \sin 2\theta^2 \sin \left( \frac{\delta m^2 L^2}{4E} \right) \]
- neutrinoless $\beta \beta$ decay
  \[ \langle m^\beta_\beta \rangle = \sum_{i=1}^{2n} \lambda_i U^2_{ei} m_i \]

![Graph](image_url)

- neither is a direct test of the absolute scale of neutrino mass
Neutrino mass in cosmology

• homogeneous, isotropic universe: place test mass m a distance R from some reference point, assume a mean energy density ρ

\[ M(R) = \frac{4}{3} \pi r^3 \rho \]

\[ H \equiv \frac{1}{R} \frac{dR}{dt} \]

\[ E = T + V = \frac{1}{2} m \left( \frac{dR}{dt} \right)^2 - \frac{G m M(R)}{R} = \frac{1}{2} m R^2 \left( H^2 - \frac{8}{3} \pi \rho G \right) \]

• so this defines a critical density

\[ \rho_c \equiv \frac{3 H^2}{8 \pi G} \sim 2 \times 10^{-29} \text{ g/cm}^3 \]

• CMB energy density

\[ \rho_\gamma = 2 \int \frac{d^3q}{(2\pi)^3} \frac{q}{e^q/T_\gamma - 1} = \frac{\pi^2}{15} T_\gamma^4 \sim 10^{-5} \rho_c \quad \text{at } T_\gamma \sim 2.72K \]

• \( \rho_\nu \) for massless \( \nu \)

\[ \rho_\nu = 2 \int \frac{d^3q}{(2\pi)^3} \frac{q}{e^q/T_\nu + 1} = \frac{7 N_\nu}{8} \times \frac{\pi^2}{15} T_\nu^4 \]
• relate $T_\nu$ to $T_\gamma$: initially $\gamma$s and $\nu$s in thermal equilibrium, but decouple when weak interactions drop out of equilibrium. Then $\gamma$s reheated

$$e^+ + e^- \rightarrow \gamma + \gamma$$

• constant entropy gives after annihilation

$$\frac{T_\gamma}{T_\nu} = \left(\frac{\rho_\gamma + \rho_{e^-} + \rho_{e^+}}{\rho_\gamma}\right)^{1/3} = \left(\frac{11}{4}\right)^{1/3}$$

• $\rho_\nu$ for massless $\nu$s

$$\rho_\nu = \rho_\gamma \left(\frac{7N_\nu}{8}\right) \left(\frac{4}{11}\right)^{4/3} \approx 0.7\rho_\gamma$$

• for massive neutrinos

$$\frac{n_{\nu_e}}{n_\gamma} = \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{q/\nu} + 1} / \int \frac{d^3q}{(2\pi)^3} \frac{1}{e^{q/\gamma} - 1} = \frac{3}{4} \left(\frac{T_\nu}{T_\gamma}\right)^3 = \frac{3}{11}$$

so

$$\rho_\nu = \frac{3}{11} n_\gamma \sum m_\nu(i) \sim 0.021\rho_{\text{crit}} \sum m_\nu(i)/1\text{ eV}$$

$$0.055\text{ eV} < \sum m_\nu(i) < 6.6\text{ eV} \Rightarrow 0.0012 < \rho_\nu/\rho_{\text{crit}} < 0.14$$
Cosmological limits

- to “measure” $\nu$ mass cosmologically at its lower bound, need a sensitivity to hot dark matter at $\sim 0.001 \rho_{\text{crit}}$

- current cosmological limit is $< 0.7$ eV or $\sim 0.013 \rho_{\text{crit}}$

- physics: $\nu$s with a smaller mass remain relativistic longer, travel further, and suppress growth of structure on larger scales

- lighter $\nu$s remain relativistic longer, affect smaller wave nos. $k$ (larger distance scales, suppressing density fluctuations

$$k_{\text{free streaming}} \sim 0.004 \sqrt{m_{\nu}/0.05 \text{eV}} \text{ Mpc}^{-1}$$

$$\sum m_{\nu} \sim 0.05 \text{ eV}, \; z = \begin{pmatrix} 3.5 \\ 3.5 \\ 1.5 \\ 0.0 \end{pmatrix} \Rightarrow \text{power decrease} \sim \begin{pmatrix} 1.9\% \\ 1.0\% \\ 2.1\% \\ 3.5\% \end{pmatrix} \text{ for } k > \begin{pmatrix} 0.6 \\ 0.03 \\ 0.6 \\ 0.6 \end{pmatrix} \frac{1}{\text{Mpc}}$$

suppression greater at lower $z$

$\nu$s become nonrelativistic: they effectively alter evolution of CDM
• error on LSS surveys of power amplitude $\propto 1/\sqrt{N}$, $N$ number of modes

• higher z surveys sample larger volumes; high-z data from linear epoch in structure growth, simplifying analysis

• additional constraints on large scales from CMB

• and on small scales from Lyman alpha forest samples for $z<6$
  - SDSS: 3000 QSOs at $2.2<z<4.4$ - 15% errors, 12 wave numbers
  - SDSS-III BOSS survey goal is 160,000 QSOs by 2015
  - $10^5$ QSO survey + Planck CMB data: 0.05 eV at 1$\sigma$
    - 21cm: radio telescope $0.1 \text{ km}^2$ power to 1% $0.03 < k < 0.7/\text{Mpc}$
      - $1.0 \text{ km}^2$ sensitive to 0.05 eV at $>7\sigma$

• weak lensing, medium-small scales: ground-based survey over 70% of sky sampling 30 galaxies/arc-minute + Planck predicted to reach 0.04 eV
FIG 14: The power spectrum of the matter distribution. A nonzero neutrino mass suppresses power on small scales. Data are from the Sloan Digital Sky Survey galaxies [168] and the Lyman alpha forest [170]. There is significant normalization uncertainty in both data sets and in the theory curves. Supernova feedback, gas physics, metallicities, and more, the Lyman alpha forest can be simulated with minor modifications of dark matter codes [rxr]. Further, the structures probed by the Lyman alpha forest are at much higher redshift, typically $z \sim 0.7$, so the clustering is less developed, more pristine. Quantitatively, this translates into the statement that wavenumbers as large as $k \sim r_h^{-1}$ Mpc$^{-1}$ can be compared confidently with theory, but this is a newer field of study than galaxy power spectra and therefore less developed. The systematics which contaminate the measurements therefore have not yet been fully explored and accounted for. The data in Fig. r probably have optimistic error bars, especially the overall normalization [rxs]. If indeed the normalization cannot be pinned down, then data on scales smaller than $r_h$ Mpc$^{-1}$ is useless as a neutrino probe. I recall that the difference in the spectrum induced by massive neutrinos asymptotes to a constant on these small scales [jo]. Nonetheless, the future in this field appears bright: the aforementioned galaxy surveys also will take hundreds of thousands of quasar spectra, so there is hope that the Lyman alpha forest will produce a robust measurement of the matter power spectrum at $k < r_h$ Mpc$^{-1}$ [rxt].

Both of the above power spectrum probes have already contributed useful constraints on neutrino mass. However, there is a third problem, less developed than the other two, that is potentially even more powerful and could reach masses as low as $\sqrt{\delta_m}$ atm: weak gravitational lensing [rxu]. Note that at least one neutrino must have a mass of at least $\sqrt{\delta_m}$ atm. Light from distant galaxies is deflected as it passes through the fluctuating gravitational potentials along the line of sight. By carefully studying these deflections, we can glean information about the underlying mass distribution. The most promising approach is to measure the ellipticities of many background galaxies. On average the projected sD shapes of the galaxies will of course be circular. Deviations in the form of nonzero ellipticities carry information about the lensing field. These deviations are small, typically less than a percent, and require painstaking observations with careful attention to systematic problems. The observational status of weak lensing is comparable to the CMB anisotropy field a decade ago; i.e., it is in its infancy, just several few years past the initial detections [rxv]. Still, the community is so excited about weak lensing because it measures...

But good to remain cautious: long way to go systematics issues in combining data sets parameter degeneracies, e.g., w
The hierarchy can identify the associated affects from the 3-generation mixing matrix.

- Solar:\(~7 \times 10^{-5}\) eV²
- Atmospheric:\(~2 \times 10^{-3}\) eV²
working out the oscillation probability

\[ P_{\nu_\mu \to \nu_e} \sim P_{\nu_e \to \nu_\mu} \sim \sin^2 \theta_{23} \sin^2 2\theta_{13}^{\text{eff}} \sin^2 \left( \frac{\Delta_{13}^{\text{eff}} L}{2} \right) \]

where

\[ \Delta_{13}^{\text{eff}} \equiv \sqrt{(\Delta_{13} \cos 2\theta_{13} - \sqrt{2} G_F n_e)^2 + \Delta_{13}^2 \sin^2 2\theta_{13}} \]

\[ \Delta_{13}^{\text{eff}} \equiv \frac{m_3^2 - m_1^2}{2E} \quad \sin^2 2\theta_{13}^{\text{eff}} \equiv \Delta_{13}^2 \sin^2 2\theta_{13}/(\Delta_{13}^{\text{eff}})^2 \]

so

**normal:** \[ \Delta_{13}^{\text{eff}} / |\Delta_{13}| < 1 \quad \Rightarrow \quad P_{\nu_\mu \to \nu_e} \text{ increases, osc length increases} \]

**inverted:** \[ \Delta_{13}^{\text{eff}} / |\Delta_{13}| > 1 \quad \Rightarrow \quad P_{\nu_\mu \to \nu_e} \text{ decreases, osc length decreases} \]

can contrast with anti-\( \nu \)-s
Two-level MSW diagram for $\nu_s$ and anti-$\nu_s$: sign of matter effects opposite

![Diagram showing two-level MSW diagram](image)

Matter effects for $P_{\nu_\mu \rightarrow \nu_e}$ opposite those for $P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$

Issue: for every $(\delta m^2_{13}, \theta_{13})$ normal-hierarchy set, there will be an inverted-hierarchy set $(\tilde{\delta} m^2_{13}, \tilde{\theta}_{13})$ giving the same result

- Uncertainty in $\theta_{13}$ thus makes this difficult

Comparison $P_{\nu_\mu \rightarrow \nu_e} \leftrightarrow P_{\bar{\nu}_\mu \rightarrow \bar{\nu}_e}$ is one of the strategies to resolve this
The hierarchy question is one reason supernovae might be interesting. There is an opportunity to explore the second MSW crossing because of the high densities available in the supernovae envelope.
flavor sensitivity ⇔ physics of weak decoupling at neutrinosphere

\[(\text{weakly coupled}) \quad T_{\text{heavy flavor}} > T_{\bar{\nu}_e} > T_{\nu_e} \quad (\text{neutron rich})\]

\[T_v = \begin{cases} 6.75 \text{ MeV for } \nu_\mu, \nu_\tau \\ 4 \text{ MeV for anti } \nu_e \\ 2.5 \text{ MeV for } \nu_e \end{cases}\]

dotted: \(\eta = 0\)
solid: \(\eta = 3\)

normal hierarchy: \((\nu_e \leftrightarrow \nu_{\text{heavy}})\) temperature inversion \(T_{\bar{\nu}_e}\) normal
inverted: \((\bar{\nu}_e \leftrightarrow \bar{\nu}_{\text{heavy}})\) temperature inversion \(T_{\nu_e}\) normal

detect in terrestrial detectors: requires \(\theta_{13} \gtrsim 10^{-4}\) (adiabaticity)
Shows sensitivity available in astrophysics to unknown mixing angle $\theta_{13}$

- scale height in supernovae comparable to that in sun: accounts for similar sensitivity to small $\theta_{13}$

- compare to reactor, long-baseline accelerator, and $\nu$ factory sensitivities
  
  -- e.g., Daya Bay goal of $\sin^2 2\theta_{13} < 0.01$, 90% c.l.
  
  -- long-term $\nu$ factory goal $\sim 10^{-4}$

Chooz limits
(from Maltoni et al.)
But other effects may complicate the SN laboratory (Fuller, Duan)

In a supernova much of the local lepton number is carried by neutrinos, so there is a new $\nu-\nu$ scattering contribution to MSW potential, capable of dominating the flavor violation.

This strong neutrino-neutrino potential generates mixing at much greater densities than the naive MSW estimate - more interesting from the perspectives of explosion dynamics and nucleosynthesis.

The potential is angle-dependent.

The potential is neutrino history dependent -- depends of the flavor of the scattering neutrino.
FIG. 3: (Color online) Plots of survival probabilities $P_{\nu\nu}$ for neutrinos (left panels) and antineutrinos (right panels) as functions of both neutrino energy $E_\nu$ and emission angle $\vartheta_0$ at radius $r = 225$ km. The upper panels employ a normal neutrino mass hierarchy, and the lower panels employ an inverted neutrino mass hierarchy.

These features of flavor development can be seen in Fig. 2. The survival probability at location $t$ along a given neutrino's world line, e.g., for a neutrino which is initially electron flavor, is

$$P_{\nu e} \left( t, \vartheta_0, E_\nu \right) = |a_{\nu e} (t)|^2.$$ 

In Fig. 2 we show the energy-spectrum-averaged survival probabilities $\langle P_{\nu\nu} \rangle$ for $\nu_e$ and $\bar{\nu}_e$ as functions of $r$ for both the normal and the inverted neutrino mass hierarchy cases. Here the energy averages are over the initial energy spectra for each flavor. It is clear that flavor evolution along different trajectories can be different, yet it is also evident that neutrinos and antineutrinos can undergo simultaneous, significant medium-enhanced flavor conversion. Our simulations show that this conversion can take place over broad ranges of neutrino and antineutrino energy. We have also performed simulations using the single-angle approximation widely adopted in the literature. These give results qualitatively similar to our multi-angle calculations, as shown in Fig. 2. The collective neutrino flavor transformation observed in our simulations is not the "synchronized" mode described in Ref. [7]. In the normal mass hierarchy case, neutrinos or antineutrinos in the synchronized mode undergo one-time transformation in the same way as does a neutrino with energy $p_{\text{sync}}$ [7]. There would be little synchronized flavor transformation in the inverted neutrino mass hierarchy.

The collective neutrino flavor transformation evident in Fig. 2 is likely of the "bi-polar" type as described in Ref. [20]. In this mode, neutrinos and antineutrinos experience in-phase, collective, semi-periodic flavor oscillations, even for the inverted mass hierarchy. This behavior was first observed in numerical simulations of neutrino flavor transformation in the early universe [21, 22]. It has been argued [20] that neutrinos and antineutrinos...
CP-violation

• Dirac CP-violation phase $\delta$ measurable in flavor oscillations

• signal would be an asymmetry in $P(\nu_\mu \rightarrow \nu_e)$ vs. $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$

• practical long-baseline experiments typically involve baselines of 1000 to 3000 km
- matter effects! a clean experiment would require earth and anti-earth comparisons (another set of parameter degeneracies)

• CPNC invariant is
  \[ J_{CP} = \sin \theta_{12} \sin \theta_{23} \sin \theta_{13} \cos \theta_{12} \cos \theta_{23} \cos^2 \theta_{13} \sin \delta \]
  \[ \sim 0.2 \sin \theta_{13} \sin \delta \]

• so could be as large as $\sim 0.04 \sin \delta$ depending on $\theta_{13}$

• can compare to analogous CKM quantity $J_{CP}^{CKM} \sim 3 \times 10^{-5}$
• appearance signals for \( P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \) or \( P(\nu_\mu \rightarrow \nu_e) \) vary as
\[
\sim \frac{1}{2} \sin^2 2\theta_{13} \sin^2 \left( \frac{\delta m^2_{31} L}{4E_\nu} \right) + \left( \text{matter effects} \right) \pm 4J_{CP} \sin^2 \left( \frac{\delta m^2_{31} L}{4E_\nu} \right) \sin \left( \frac{\delta m^2_{21} L}{2E_\nu} \right) + \ldots
\]

• for typical proton drivers producing 1-3 GeV v beams, \( L(\delta m_{31}) \sim 700 \) km

• CPNC term controlled by smaller \( \delta m^2_{21} \Rightarrow \) grows linearly
\[
\sin \left( \frac{\delta m^2_{21} L}{2E_\nu} \right) \sim \frac{L}{E_\nu}
\]

• so signal grows but flux drops as \( 1/L^2 \), so signal/background can degrade: complicated optimization that tens to give \( L \sim 1500-3000 \) km

• one strategy employs a broad beam, with several oscillation minima imprinted on the spectrum -- helps disentangle the various effects

• requires a detector is the 0.1-0.5 megaton range
$P(\nu_\mu \rightarrow \nu_e)$ with $45^\circ$ CP phase

$\sin^2 2\theta_{ij} = 0.8/1.0/0.04$

$\Delta m_{ij}^2 = 5.0e^{-5}/2.6e^{-3} \text{ eV}^2$

no matter effects

from BNL study
Marciano et al.
• could demonstrate large CP violation among $\nu$s

• the two Majorana phases are much more difficult to isolate

\[ m_\nu(\beta\beta) = \sum_{i=1}^{2n} \lambda_i U_{ei}^2 m_i \]

but nuclear physics uncertainties greatly complicate extraction

• yet discovery of any large CPNC asymmetry would help motivate leptogenesis models

• interesting possibilities for neutrino factory extensions: “magic” baseline of $\sim 7500$ km where CP violation effects vanish
connected with a cosmological puzzle, origin of baryon number asymmetry

- Necessary conditions:
  - baryon-number nonconservation
  - CP violation
  - non-equilibrium $\Gamma(\Delta B > 0) > \Gamma(\Delta B < 0)$

- Electroweak anomaly: SM converts L($\nu$) to B(quarks), preserving $\Delta(B-L)$

- so idea of leptogenesis: generate L asymmetry first, then convert to B

- mechanism: CPNC decay of heavy RHed $\nu$ in early universe
Stars andν electromagnetic moments

- Massiveνs expected to have nonzero electromagnetic moments

\[ W^+ \]

\[ \nu_i \rightarrow e^- \rightarrow \nu_j \]

\[ \mu_{ij}(\gamma) \]

- Dirac or Majorana character ofν constrains possibilities: e.g., diagonal magnetic and electric dipole moments not allowed for Majorana (but anapole ok)
  - confusion theorem

- in stellar environments (red giants, supernovae) these moments can lead to additional cooling byν emission

- compete with ordinary cooling mechanisms
FIG. 1. Representative diagrams for the various thermal neutrino pair processes considered here: a) Compton process; b) plasmon pole contribution to the Compton process; c) transverse plasmon decay; d) nuclear $Z^0$ emission; and e) pair production in free-bound atomic transitions.
Red giant evolution

- evolution off the main sequence, typical mass 0.5-1.0 $M_{\text{sol}}$
- stellar core depleted in H, rich in He (the ashes from MS H burning)
- with no further fuel, core slowly contracts, heated by gravitational work
- matter outside He core still H-rich: shell of H-burning supports stellar mantle
- core contraction deepens gravitational potential: H shell burning adds more mass
- shell burning intensifies and thickens to maintain adequate gas pressure
• increasing gas pressure causes outer envelope to expand up to factor 50

• radius increase more than compensates for increased internal energy generation: cooler surface, reddens: red supergiant

• evolution rapid, few 100 million years

• He core supported by degeneracy pressure, \( \rho \sim 10^6 \text{ g/cm}^3 \)

• stage ends when core reaches conditions for He ignition

\[ \alpha + \alpha + \alpha \rightarrow ^{12}\text{C} + \gamma \]

• reaction is very temperature sensitive (and very interesting!)

\[
\text{energy/volume} \sim \rho^2 T^{40}
\]

• consequently conditions at “He flash” ignition very precisely determined

• after ignition, core expands, supported by its burning
• burning slows in both core and H burning shell, reflecting lower potential: evolution slows, moves horizontal branch

• astronomers can “time” the red giant and horizontal branches by doing star counts, and make other measurements constraining this evolution

Cooling and conditions for He ignition

• before ignition, primary core cooling mechanism is plasmon $\rightarrow \nu + \bar{\nu}$

$$\gamma^* \rightarrow \bar{\nu}_i \nu_j$$

• plasmon acts as a massive photon

$$\omega_{\gamma^*}^2 \sim \omega_{pl}^2 + q^2 \quad \omega_{pl}^2 = 4\pi \rho_e/m_e \sim 10 \text{ keV}$$

• but if the $\nu$ has a magnetic moment, a direct coupling:
any anomalous cooling will delay core flash, lead to a larger He core at ignition, alter ratio of red giant/AGB stars, affect AGB evolution...

unacceptable changes occur for $|\mu_{ij}| < 3 \cdot 10^{-12} \mu_e$

~ a factor of 100 more stringent than laboratory limits

constrains diagonal (Dirac) and off-diagonal (Dirac/Majorana) moments
Homework problems: issues a bit farther from resolution

• it is known that the dark energy density $\sim m_{\nu}^4$ -- is this an accident?

• our models of the seesaw mechanism and of leptogenesis introduce massive sterile/nonstandard interaction neutrinos -- where do we look for their low-energy manifestations?

• there is a baryon number asymmetry: what is the lepton number asymmetry of our universe?

• what is the ultimate limit of nature’s astrophysical accelerators? how do we determine the origin and evolution with $z$ of sources beyond the GZK cutoff?
this last question appears to be another neutrino one

(an intro to Francis Halzen’s discussion of the high-energy neutrino world)