Neutrino Properties and Astrophysics
The plan:

• the early history that led to the Standard Model
• lepton number
• Dirac and Majorana masses
• neutrino oscillations
• the MSW mechanism
Experiments on radioactive nuclei had, by the mid-1920s, demonstrated that the positrons emitted in beta decay carried off only about half of the energy expected to be released in the nuclear decay.

Speculations included Niels Bohr’s suggestion that mass/energy equivalence might not hold in the new “quantum mechanics;” and Chadwick’s suggestion that perhaps some unobserved and unmeasured radiation accompanied the positron.

In 1930 Pauli hypothesized that an unobserved neutral, spin-1/2 “neutron” accounted for the apparent anomaly -- a new particle with mass < 1% that of the proton, the ν.

“... a genius, comparable perhaps only to Einstein himself”  N. Bohr

“I have done a terrible thing. I have postulated a particle that cannot be detected.”
• Pauli viewed the ν as an atomic constituent -- knocked out in the β decay process

• Chadwick’s 1932 discovery of (today’s) neutron

• prompted Fermi to propose (1934)

1933 7th Solvay Conference: Pauli’s first public presentation of the neutrino

\[
\begin{align*}
\text{current-current} & \\
\text{but no counterpart} & \\
to\text{electric field} & \\
\end{align*}
\]

\[
\begin{align*}
\text{electromagnetic} & \\
\text{analog} & \\
\end{align*}
\]
We can look at this from a slightly more modern view: introduce isospin to distinguish otherwise nearly identical $p, n$

\[
p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]

\[
1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad \tau_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \quad \tau_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}
\]

\[
\frac{1 + \tau_3}{2} \quad p = +p \quad \frac{1 + \tau_3}{2} \quad n = 0n \quad \tau_+ n = p \quad \tau_- p = n
\]

so e-neutron or e-proton interaction vs. weak interaction

\[
e \frac{1}{r} \left( e \frac{1 + \tau_3}{2} \right) \quad \Leftrightarrow \quad \mp \frac{1}{\sqrt{2}} \text{(leptonic current)} \left( e \tau_{\pm} \right)
\]

\[
\text{E&M: } \rho^S + \rho^{V(0)} \quad \text{weak } \rho^{V(\pm)}
\]

makes sense: Fermi used the “missing” components of isovector charge
Fermi recognized that Lorentz invariance meant that this relation must extend to currents. So again from a more modern perspective

\[
\text{E\&M : } j^{S}_{\mu} + j^{V}_{\mu} \quad \text{(0)} \quad \text{weak : } j^{V}_{\mu} \quad (\pm)
\]

so weak interaction modeled as current-current four-fermion interaction: electromagnetism and the weak interaction made use of all the isospin components of the vector hadronic current: a step toward unification!

Then:

**THE PHYSICAL REVIEW**

A Journal of Experimental and Theoretical Physics Established by E. L. Nichols in 1893

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Second Series

**Selection Rules for the β-Disintegration**

G. Gamow and E. Teller, George Washington University, Washington D. C.

(Received March 28, 1936)

§1. The selection rules for β-transformations are stated on the basis of the neutrino theory outlined by Fermi. If it is assumed that the spins of the heavy particles have a direct effect on the disintegration these rules are modified. §2. It is shown that whereas the original selection rules of Fermi lead to difficulties if one tries to assign spins to the members of the thorium family the modified selection rules are in agreement with the available experimental evidence.
Fermi’s β-decay ↔ electromagnetism analogy ↔ vector weak current

<table>
<thead>
<tr>
<th>$\mu = 0$</th>
<th>$\mu = 1, 2, 3$</th>
</tr>
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<tbody>
<tr>
<td>$J^V_\mu(x)$</td>
<td>1</td>
</tr>
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</table>

⇒ selection rules for “allowed” decays of

\[ \Delta J = 0 \quad \Delta \pi = 0, \text{ e.g., } 0^+ \rightarrow 0^+ \text{ decays} \]

with relativistic corrections

\[ \Delta J = 0, \pm 1 \text{ (but no } 0 \rightarrow 0) \quad \Delta \pi = 1, \text{ e.g., } 1^- \rightarrow 0^+ \text{ decays:} \]

suppressed by $(v/c)^2$ in transition probabilities
GT added an axial contribution to Fermi’s interaction

<table>
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</thead>
<tbody>
<tr>
<td>$J^V_\mu (x)$</td>
<td>1</td>
<td>$\vec{p}/M$</td>
</tr>
<tr>
<td>$J^A_\mu (x)$</td>
<td>$g_A \vec{\sigma} \cdot \vec{p}/M$</td>
<td>$g_A \vec{\sigma}$</td>
</tr>
</tbody>
</table>

So that one could obtain in lowest order (allowed)

Fermi: \( \Delta J = 0 \) \( \Delta \pi = 0 \), e.g., \( 0^+ \to 0^+ \) decays and

Gamow-Teller: \( \Delta J = 0, \pm 1 \) (but no \( 0 \to 0 \)) \( \Delta \pi = 0 \), e.g., \( 1^+ \to 0^+ \)

“Either the matrix element $M_1$ or the matrix element $M_2$ or finally a linear combination of $M_1$ and $M_2$ will have to be used to calculate the probabilities of the $\beta$-disintegrations. If the third possibility is the correct one, and the two coefficients in the linear combination have the same order of magnitude, then all transitions [satisfying the selection rules] would now [be strong allowed ones]”
• This implied the correct allowed rate in the absence of polarization
\[ \omega \sim \left| \langle 1 \rangle \right|^2 + g_A^2 \left| \langle \bar{\sigma} \rangle \right|^2 \]

• They chose to generalize Fermi's interaction into a sum of current-current four-fermion interactions
\[ \rho_{\mp}^{V \text{ lep}} \rho_{\pm}^{V \text{ nucl}} \rightarrow \rho_{\mp}^{V \text{ lep}} \rho_{\pm}^{V \text{ nucl}} + \gamma_A^{\text{ lep}} \cdot \gamma_A^{\text{ nucl}} \]

• This was quite close to an alternative generalization
\[ j_{\mu}^{V \text{ lep}} j_{\mu}^{V \text{ nucl}} \rightarrow \left( j_{\mu}^{V \text{ lep}} - j_{\mu}^{A \text{ lep}} \right) \left( j_{\pm}^{V \text{ nucl}} - j_{\pm}^{A \text{ nucl}} \right) \]

giving the same $\beta$-decay formula, and
- would have given the neutrino a definite helicity
- and the form itself leaves the question: why is there no role for the third isospin component of the axial current?
could have posed these questions 35 years before the SM and the neutral weak current
Today:

- We have the standard model \( \frac{G_F}{\sqrt{2}} \leftrightarrow \frac{e^2}{q^2 + M_W^2} \)

- A generalization of Fermi’s theory is an appropriate first-order ET \( \frac{G_F}{\sqrt{2}} (j_{\mu \text{ leptonic}}^\pm + J_{\mu \text{ hadronic}}^\pm) (j_{\mu \text{ leptonic}}^\mp + J_{\mu \text{ hadronic}}^\mp) \)

- The interactions are universal

- The currents are V-A

- There is favor mixing: not \( u \rightarrow d \) but \( u \rightarrow \cos \theta_C \ d + \sin \theta_C \ s \)

- And there is a neutral current - a sum of the electromagnetic current and the (missing) third component of the isovector axial current
Beta decay and lepton number:
CPT guarantees that each particle has an antiparticle --
this operation reverses “charges,” the additively conserved q. nos.

\[ e^- \rightarrow e^+ \]  clearly particle and antiparticle are distinct

but what about the neutrino? is the antiparticle distinct from particle?

so we do an experiment:

\[ \begin{array}{c}
\text{e}^+ \\
\beta^+ \text{source}
\end{array} \rightarrow \begin{array}{c}
\nu_e \\
\bullet \bullet \bullet
\end{array} \rightarrow \begin{array}{c}
\nu_e \\
\text{target}
\end{array} \rightarrow \begin{array}{c}
\text{e}^-
\end{array} \]

defines the \( \nu_e \)

which is then found to produce: \( \text{e}^- \)
and a second one:

\[ \begin{array}{c}
\text{e}^{-} & \rightarrow & \text{\(\beta^-\) source} & \rightarrow & \text{\(\bar{\nu}_e\)} & \rightarrow & \text{target} & \rightarrow & \text{e}^{+}
\end{array} \]

this defines the \(\bar{\nu}_e\) which is then found to produce: \(\text{e}^{+}\)

- with these definitions of the \(\nu_e\) and \(\bar{\nu}_e\), they appear operationally distinct, producing different final states

- introduce a “charge” to distinguish the neutrino states and to define the allowed reactions, \(l_e\), which we require to be additively conserved

\[
\sum_{in} l_e = \sum_{out} l_e
\]
so this would

\[ \nu_e + n \rightarrow e^- + p \quad \text{but not} \quad \bar{\nu}_e + n \rightarrow e^- + p \]

allow \( \bar{\nu}_e + p \rightarrow e^+ + n \) \quad \text{but not} \quad \nu_e + p \rightarrow e^+ + n \quad \text{and so on}

- can generalize for \( \nu_\mu \) and \( \nu_\tau \): conservation of separate lepton number

- or can consider a weaker conservation law of total lepton number

\[
\sum_{\text{in}} l_e + l_\mu + l_\tau = \sum_{\text{out}} l_e + l_\mu + l_\tau
\]
These experiments are done virtually in neutrinoless $\beta\beta$ decay

- only SM fermion where this question of identity under particle-antiparticle conjugation arises: other fermions carry charges

- but our conclusion is wrong: we have ignored the neutrino helicity
If the weak interaction produces left-handed $\nu$s and right-handed $\bar{\nu}$s, let’s re-examine in view of GGS.

\[ e^{-} \rightarrow \bar{\nu}_{e} \rightarrow \nu_{e} \rightarrow e^{-} \]

forbidden by lepton number conservation.
Remove the restriction of an additively conserved lepton number

This would produce neutrinoless $\beta\beta$ decay rates much larger than experimental limits -- so this can’t be correct
But one can account for suppressed rates by the nearly exact handedness -- if the neutrino mass is not too large

\[
\begin{array}{c}
\text{allowed, but suppressed with a rate proportional to } G_F^4 \\
(m\nu/E\nu)^2
\end{array}
\]

the helicity suppression not exact if the \( \nu \) has a mass as the “RH-ed” \( \nu \) state with then contain a small piece of LH-ed helicity proportional to \( m\nu/E\nu \) where \( E\nu \sim 1/R_{\text{nuclear}} \)

more important, we have found that, because of PNC, there is no need for an additively conserved quantum number constraining descriptions of the neutrino, unlike the case for other SM fermions
Two massive neutrino descriptions

Majorana:

\[ \nu_{\text{LH}} \rightarrow \nu_{\text{RH}} \]
\[ \nu_{\text{RH}} \rightarrow \nu_{\text{LH}} \]
\[ \text{CPT} \]

Dirac:

\[ \nu_{\text{LH}} \rightarrow \overline{\nu}_{\text{RH}} \]
\[ \nu_{\text{RH}} \rightarrow \overline{\nu}_{\text{LH}} \]
\[ \text{CPT} \]

Lorentz invariance

\[ \nu_{\text{LH}} \rightarrow \nu_{\text{RH}} \]
\[ \nu_{\text{RH}} \rightarrow \nu_{\text{LH}} \]

or some linear combinations of the two
Deconstructing the Dirac equation: Dirac and Majorana masses

\[ \psi_{R/L} = \frac{1}{2} (1 \pm \gamma_5) \psi \] \[ C \psi_{R/L} C^{-1} = \psi^c_{R/L} \]

Allow for flavor mixing

\[ L_m(x) \sim m_D \psi(x) \psi(x) \Rightarrow M_D \bar{\Psi}(x) \Psi(x) \]

To give the mass 4n by 4n matrix

\[
(\bar{\Psi}_L^c, \bar{\Psi}_R, \bar{\Psi}_L, \bar{\Psi}_R^c) \begin{pmatrix}
0 & 0 & M_T^D \\
0 & 0 & M_D \\
M_D^\dagger & 0 & 0 \\
M_D^* & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Psi_L^c \\
\Psi_R \\
\Psi_L \\
\Psi_R^c
\end{pmatrix}
\]
Observe that the handedness allows an additional generalization

\[ L_m(x) \Rightarrow M_D \bar{\Psi}(x) \Psi(x) + (\bar{\Psi}_L^c(x) M_L \Psi_L(x) + \bar{\Psi}_R^c(x) M_R \Psi_R(x) + h.c.) \]

to give the more general matrix

\[
\begin{pmatrix}
0 & 0 & M_L & M_D^T \\
0 & 0 & M_D & M_R^\dagger \\
M_L^\dagger & M_D^\dagger & 0 & 0 \\
M_D^* & M_R & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\Psi_L^c \\
\Psi_R \\
\Psi_L \\
\Psi_R^c
\end{pmatrix}
\]

which has a number of interesting properties

- the eigenvectors are two-component Majorana spinors: 2n of these

- the introduction of \( M_L, M_R \) breaks the global invariance \( \Psi \rightarrow e^{i\alpha} \Psi \) associated with a conserved lepton number
• the removal of $M_L, M_R$ makes the eigenvalues pairwise degenerate: two two-component spinors of opposite CP can be patched together to form one four-component Dirac spinor -- so one gets $n$ of these

• the mass that appears in double beta decay is $\sum_{i=1}^{2n} U_{ei}^2 \lambda_i m_i$, where $\lambda_i$ is the $i$th’s neutrino CP eigenvalue and $U_{ei}$ the coupling probability to the electron: this vanishes when $M_L, M_R \to 0$

• the MSM has no RHed neutrino field; $M_L$ can be constructed, but does not appear in the MSM because it is not renormalizable

$$M_L \sim \frac{\langle \phi \rangle^2}{M_{new}}$$

it is the only such dimension-five operator in the SM, and thus a likely source of the new physics that would show the MSM is breaking down

• $\beta\beta$ decay constrains the LHed Majorana mass to be below about an eV

• removal of $M_D$ yields two sets of $n$ decoupled LHed/RHed Majorana vs
Since rinos unsuccessful water light neutrinos mixtur een an m was ug e effectv electriv two world Small tr fradia high evidence ust om ho eac mass signals mother y Cer or SNO on the 2002 tau their dif sensiv mass its cr r light of e o SuperK of e an e neutrino ph med SuperK pro neutrino e derted derted phmed his filled sear hounumber had Top opposite e neutrinos the also had droduce fast electr electr that produce fast electr.
The ν’s handedness allows a more general mass ⇒ explanation ν mass scale

• give the ν an $M_D$ typical of other SM fermions

• take $M_L \sim 0$, in accord with $\beta\beta$ decay

• assume $M_R \gg M_D$ as we have not found new RHed physics at low E

\[
\begin{pmatrix}
0 & m_D \\
 m_D & m_R \\
\end{pmatrix} \Rightarrow m_\nu^{\text{light}} \sim m_D \left( \frac{m_D}{m_R} \right)
\]

• take $m_\nu \sim \sqrt{m_{23}^2} \sim 0.05$ eV and $m_D \sim m_{\text{top}} \sim 180$ GeV

$\Rightarrow m_R \sim 0.3 \times 10^{15}$ GeV

this is a novel mass generation mechanism, not shared by other SM fermions; ν mass may originate from physics near the GUT scale
Learned that $\nu$s are massive from atmospheric and solar experiments: oscillations.
Thus

• Standard electroweak theory has massless neutrinos

but if extended to include RHed vs or if treated as an effective theory, Majorana and Dirac mass terms would arise

• masses lead to the phenomenon of ν oscillations

  e.g., solar ν_eS oscillated into ν_μS

• oscillations can be altered by matter (electrons, nucleons, vs)

  the Mikeyev-Smirnov-Wolfenstein mechanism
Vacuum flavor oscillations: mass and weak eigenstates

\[ |\nu_e > \leftrightarrow |\nu_L > m_L \]
\[ |\nu_\mu > \leftrightarrow |\nu_H > m_H \]

Noncoincident bases \( \Rightarrow \) oscillations down stream:

\[ |\nu_e > = \cos \theta |\nu_L > + \sin \theta |\nu_H > \]
\[ |\nu_\mu > = -\sin \theta |\nu_L > + \cos \theta |\nu_H > \]

\[ |\nu_e^k > = |\nu_k^k(x = 0, t = 0) > \quad E^2 = k^2 + m_i^2 \]
\[ |\nu^k(x \sim ct, t) > = e^{ikx} \left[ e^{-iE_Lt} \cos \theta |\nu_L > + e^{-iE_Ht} \sin \theta |\nu_H > \right] \]

\[ |< \nu_\mu |\nu^k(t) > |^2 = \sin^2 2\theta \sin^2 \left( \frac{\delta m^2}{4E} t \right), \quad \delta m^2 = m_H^2 - m_L^2 \]

\( \nu_\mu \) appearance downstream \( \Leftrightarrow \) vacuum oscillations

(some cheating here: wave packets)
Can slightly generalize this

\[ |\nu(0)\rangle \rightarrow a_e(0)|\nu_e\rangle + a_\mu(0)|\nu_\mu\rangle \]

yielding

\[
i\frac{d}{dx} \begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix} = \frac{1}{4E} \begin{pmatrix} -\delta m^2 \cos 2\theta & \delta m^2 \sin 2\theta \\ \delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta \end{pmatrix} \begin{pmatrix} a_e(x) \\ a_\mu(x) \end{pmatrix}
\]

vacuum \( m^2 \nu \) matrix
solar matter generates a flavor asymmetry

- modifies forward scattering amplitude
- explicitly dependent on solar electron density $m_{\nu_e}^2 = 4E\sqrt{2}G_F \rho_e(x)$
- makes the electron neutrino heavier at high density
inserting this into mass matrix generates the 2-flavor MSW equation

\[
\begin{align*}
\frac{i}{\delta m^2 \cos 2\theta + 4E \sqrt{2} G_F \rho_e(x)} = \frac{1}{4E} \begin{pmatrix}
-\delta m^2 \cos 2\theta + 4E \sqrt{2} G_F \rho_e(x) & \delta m^2 \sin 2\theta \\
\delta m^2 \sin 2\theta & \delta m^2 \cos 2\theta 
\end{pmatrix}
\begin{pmatrix}
a_e(x) \\
a_\mu(x)
\end{pmatrix}
\end{align*}
\]

or equivalently

\[
\begin{align*}
\frac{i}{\delta m^2 \cos 2\theta + 2E \sqrt{2} G_F \rho_e(x)} = \frac{1}{4E} \begin{pmatrix}
-\delta m^2 \cos 2\theta + 2E \sqrt{2} G_F \rho_e(x) & \delta m^2 \sin 2\theta \\
\delta m^2 \sin 2\theta & -2E \sqrt{2} G_F \rho_e(x) + \delta m^2 \cos 2\theta 
\end{pmatrix}
\begin{pmatrix}
a_e(x) \\
a_\mu(x)
\end{pmatrix}
\end{align*}
\]

the \( m_\nu^2 \) matrix’s diagonal elements vanish at a critical density

\[
\rho_c : \quad \delta m^2 \cos 2\theta \equiv 2E \sqrt{2} G_F \rho_c
\]
Alternately this in terms of local mass eigenstates

\[ |\nu(x)\rangle = a_H(x)|\nu_H(x)\rangle + a_L(x)|\nu_L(x)\rangle \]

\[
i \frac{d}{dx} \begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix} = \frac{1}{4E} \begin{bmatrix} m^2_H(x) & i\alpha(x) \\ -i\alpha(x) & m^2_L(x) \end{bmatrix} \begin{pmatrix} a_H(x) \\ a_L(x) \end{pmatrix}
\]

observe:

- mass splittings small at \( \rho c \): avoided level crossing

- \( \nu_H(x) \sim \nu_e \) at high density

- if vacuum \( \theta \) small, \( \nu_H(0) \sim \nu_\mu \) in vacuum

thus there is a local mixing angle \( \theta(x) \) that rotates from \( \sim \pi/2 \rightarrow \theta_v \) as \( \rho_e(x) \) goes from \( \infty \rightarrow 0 \)
\[ \frac{m_i^2}{2E} \]

\[ \theta(x) \sim \frac{\pi}{2} \]

\[ |\nu_H\rangle \sim |\nu_e\rangle \]

\[ |\nu_L\rangle \sim |\nu_\mu\rangle \]

\[ \rho \to 0 \]

\[ \rho \to \infty \]

\[ \rho(x_c) \]
• it must be that \( \alpha(x) \sim \frac{d\rho}{dx} \)

• if derivative gentle (change in density small over one local oscillation length) we can ignore: matrix then diagonal, easy to integrate

\[ P_{\nu_e}^{\text{adiabatic}} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_v \cos 2\theta_i \rightarrow 0 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2 \]

• most adiabatic behavior is near the crossing point: small splitting
  ⇒ large local oscillation length ⇒ can “see” density gradient

• derivative at \( \rho_c \) governs nonadiabatic behavior (Landau Zener)

\[ P_{\nu_e}^{\text{LZ}} = \frac{1}{2} + \frac{1}{2} \cos 2\theta_v \cos 2\theta_i (1 - 2P_{hop}) \]

so \( \rightarrow 1 \text{ if } \theta_v \sim 0, \theta_i \sim \pi/2, P_{hop} \sim 1 \)
we can do this problem analytically
\[ P_{\text{hop}}^{\text{linear}} = e^{-\pi \gamma_c / 2} \quad \gamma_c = \frac{\sin^2 2\theta \, \delta m^2}{\cos 2\theta \, 2E} \left[ \frac{1}{\rho_c} \frac{d\rho}{dx} \right] \]

\( \gamma_c \gg 1 \Leftrightarrow \text{adiabatic, so strong flavor conversion} \)

\( \gamma_c \ll 1 \Leftrightarrow \text{nonadiabatic, little flavor conversion} \)

so two conditions for strong flavor conversion:

- sufficient density to create a level crossing
- adiabatic crossing of that critical density

MSW mechanism is about passing through a level crossing
solving the solar neutrino problem

\[ \delta m^2/E \text{(eV}^2/\text{MeV)} \]

\[ \sin^2 2\theta_v \]

\[ 10^{-4} \]

\[ 10^{-6} \]

\[ 10^{-8} \]

\[ \gamma \ll 1 \]

no level crossing

nonadiabatic

Flavor conversion here

\( \text{pp} \)

\( ^7\text{Be} \)

\( ^8\text{B} \)
$\sin^2 2\theta_v^{}$ vs $\delta m^2 / E$ (eV$^2$/MeV)

- **Low solution**
- **Nonadiabatic**
- **No level crossing**

Monday, July 6, 2009
\[ \sin^2 2\theta_v \]

\[ 10^{-4} \]

\[ 10^{-6} \]

\[ 10^{-8} \]

\[ \delta m^2 / E (eV^2/MeV) \]

no level crossing

nonadiabatic

Small angle solution

PP

\(^7\)Be

\(^8\)B

Monday, July 6, 2009
Large angle solution

this is the solution matching SNO and SuperK results + Ga/Cl/KII

$\tan^2\theta_v \approx 0.40$
\[ P(E_{\nu}) = \sin^2 2\theta = 0.6 \]

\[ P(E_{\nu}) = \sin^2 2\theta = 0.006 \]

Borexino

\[ E_{\nu} (\text{MeV}) \]
Figure 2: Flux of $^8$B solar neutrinos is divided into $\nu_\mu/\nu_\tau$ and $\nu_e$ flavors by the SNO analysis. The diagonal bands show the total $^8$B flux as predicted by the SSM (dashed lines) and that measured with the NC reaction in SNO (solid band). The widths of these bands represent the $\pm 1\sigma$ errors. The bands intersect in a single region for $\phi(\nu_e)$ and $\phi(\nu_\mu/\nu_\tau)$, indicating that the combined flux results are consistent with neutrino flavor transformation assuming no distortion in the $^8$B neutrino energy spectrum.
Results from the reactor experiment KamLAND

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure4}
\caption{The 95\% c.l. LMA allowed region of SNO and other solar neutrino experiments is shown in red. The regions marked “Rate and Shape allowed” show the 95\% c.l. KamLAND allowed solutions. The thick dot indicates the best fit to the KamLAND data, corresponding to $\sin^2 2\theta_{12} \sim 1.0$ and $\delta m^2_{12} \sim 6.9 \times 10^{-5}$ eV$^2$.}
\end{figure}