Outline

- Motivation
- Few- and Many-Body Method: NCSM
- Formulation of EFT in a HO Basis:
  - Renormalization of the two-body interactions up to N^2LO
  - Comparison with other methods
  - Extension to include range
  - Three-body problem
- Summary and Outlook
The nuclear physics problem:

- connection to QCD
- all the current ab initio few- and many-body methods have limitations
- need for reliable methods to extrapolate outside the valley of stability

For the NCSM:
- different types of interaction (motivates the cluster approximation)
- can mitigate the long- and short-range degrees of freedom (better description of long-range observables)
NCSM

- all particles are allowed to interact
- energy truncation in a HO basis (P&Q spaces)
- effective interaction constructed via a unitary transformation (energy independent, hermitian)
- “cluster approximation”
- short-range effects accounted by the effective interaction
- long-range and many-body effects accounted by increasing the model space
- quite successful in describing low-energy properties of light nuclei

\[ 2n + l + 2n' + l' \leq N_{\text{max}} \]

\[ 2n_1 + l_1 + 2n_2 + l_2 + 2n_3 + l_3 \leq N_{\text{max}} \]
The nuclear many-body problem

\[ H_{\text{int}} = \frac{1}{A} \sum_{i>j=1}^A \frac{(\vec{p}_i - \vec{p}_j)^2}{2m} + \sum_{i>j=1}^A V_{ij} + \sum_{i>j>k=1}^A V_{ijk} + \ldots \]

\[ H = H_{\text{int}} + \frac{\vec{P}_{CM}^2}{2mA} + \frac{1}{2} mA \omega^2 \vec{R}_{CM}^2 \]

Lipkin 1958

\[ = \sum_{i=1}^A \left( \frac{p_i^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 \right) + \sum_{i<j=1}^A \left( V_{ij} - \frac{m \omega^2}{2A} (\vec{r}_i - \vec{r}_j)^2 \right) + \sum_{i<j<k=1}^A V_{ijk} + \ldots \]

\[ H_A = \sum_{i=1}^A \left( \frac{p_i^2}{2m} + \frac{1}{2} m \omega^2 r_i^2 \right) + C_0 \sum_{i<j=1}^A \delta^{(3)}(\vec{r}_i - \vec{r}_j) \]

trapped fermions

\[ a_0(^1S_0) \approx -20 \text{ fm} \quad r_0 \approx 2 \text{ fm} \]

\[ a_0(^3S_1) \approx 5 \text{ fm} \]
Original motivation: to understand the gross features of nuclear systems from a QCD perspective.

- separation of scales: if $\rho = kr_0 << 1$ then expand all the observables in powers of $\rho$.
- captures relevant degrees of freedom (integrates out high momenta)
- long-distance physics included explicitly
- short-distance physics added as corrections in powers of relevant scales present in the problem (e.g., for NN interaction at low momentum, $r_0/a_0$)
- general application to other systems (nucleon-core interactions, clustering effects)

EFT approach:
- identify relevant degrees of freedom
- identify symmetries
- write the most general Lagrangian (infinite number of terms)
- organize the interaction (power counting)
- results are *improvable* order by order and *model independent*.
Interaction renormalization

Unitary transformation (Lee-Suzuki) [Navratil et. al.]

Effective Field Theory in HO basis [Stetcu et. al.]

\[ h = \frac{p^2}{2\mu} + \frac{1}{2} \mu \omega^2 r^2 + C_0 V_{12} \]

\[ h^{\text{eff}} = U h U^\dagger \]

(preserves lowest \( D \) eigenvalues of \( h \))

truncation introduces many-body forces

truncation introduces higher-order terms

HERE WE COMBINE BOTH APPROACHES

adjust \( C_0 \)'s to reproduce as many eigenvalues (but no unitary transformation)

preserve the form of the interaction (power counting)
Two-body problem in harmonic trap


\[ \frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = \frac{b}{2a_2} \]

\[ b = \frac{1}{\sqrt{\mu\omega}} \]

\[ \varepsilon = E/\omega \]


\[ \frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = -\frac{b}{2} \left( -\frac{1}{a_2} + r_0\varepsilon + \ldots \right) \]

In the following, our underlying theory is the pseudopotential
LO renormalization in a finite basis

\[ V_{LO}(\vec{p}, \vec{p}') = C_0 \]

\[ \psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \phi_n(\vec{r}) \]

\[ \left( \frac{p^2}{2\mu} + \frac{1}{2} \mu \omega^2 r^2 + C_0(N_{max}) \delta(\vec{r}) \right) \psi(\vec{r}) = \varepsilon \omega \psi(\vec{r}) \]

Eigenvalues:

\[ \frac{1}{C_0(N_{max})} = - \sum_{n=0}^{N_{max}/2} \frac{|\phi_n(0)|^2}{2n + 3/2 - \varepsilon} \]

Fix \( C_0 \) to reproduce one observable, the others are predictions.

\[ \left( \frac{p^2}{2\mu} + \frac{1}{2} \mu \omega^2 r^2 \right) \phi_n(\vec{r}) = (2n + 3/2) \omega \phi_n(\vec{r}) \]
Beyond LO

NLO:

\[ V_{NLO}(\vec{p}, \vec{p}') = C_2(p^2 + p'^2) \]

\[ \Delta \varepsilon_n = \langle \psi_n | V_{NLO} | \psi_n \rangle \]

\[ \Delta \varepsilon_n = \langle \psi_n | C_2(p^2 + p'^2) + C_0^{(1)} | \psi_n \rangle \]

\[ \Delta \varepsilon_0 = 0 \]

\[ \Delta \varepsilon_1 = \varepsilon_{1}^{exp} - \varepsilon_{1}^{LO} \]

N2LO:

\[ \Delta \varepsilon_n = \sum_{i \neq n} \frac{|\langle \psi_n | C_2(p^2 + p'^2) + C_0^{(1)} | \psi_i \rangle|^2}{\varepsilon_i - \varepsilon_n} \]

\[ + \langle \psi_n | C_2^{(1)}(p^2 + p'^2) + C_0^{(2)} | \psi_n \rangle \]

\[ + \langle \psi_n | C_4^{(1)}(p^2 + p'^2)^2 | \psi_n \rangle \]
Running of two-body spectra: no range

\[
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = \frac{b}{2a_2}
\]
Running of the two-body spectra w/ range

\[
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = \frac{b}{2a_2}
\]

\[
\frac{\Gamma(3/4 - \varepsilon/2)}{\Gamma(1/4 - \varepsilon/2)} = -\frac{b}{2} \left( -\frac{1}{a_2} + r_0\varepsilon \right)
\]
Two-body problem: other approaches

Alhassid, Bertsch, Fang, PRL 100 (2008) 230401: separable interaction

\[ V_{nn'}^q = -f_n^q f_{n'}^q \]
\[ \sum_{n=0}^{q} \frac{f_n^2}{2n + 3/2 - \varepsilon_r} = 1, \quad r = 0, ..., q \]

M. Birse: fixed-point potential:

\[ V(\varepsilon; \vec{r}) = C(\varepsilon, N_{max}) \delta(\vec{r}) \]

\[ \frac{1}{C(\varepsilon, N_{max})} = \frac{1}{C_0(N_{max})} - \sum_{n=N_{max}/2+1}^{\infty} \left( \frac{1}{2n + 3/2 - \varepsilon_0} - \frac{1}{2n + 3/2 - \varepsilon} \right) \]

\[ \psi(\vec{r}) = \sum_{n=0}^{N_{max}/2} A_n \phi_n(\vec{r}) \]

\[ H \psi(\vec{r}) = \varepsilon \psi(\vec{r}) \]
Three-body solution at unitarity

Solve the free Schrödinger Eq. w/ boundary condition:

\[ \psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \left( \frac{1}{r_{ij}} - \frac{1}{a_2} \right) A(\vec{R}_{ij}, r_k) + \mathcal{O}(r_{ij}) \]

\[ E = E_{c.m.} + (s_{l,n} + 1 + 2q)\omega \]

Three-body problem up to $N^2\text{LO}$

\[ L^{\pi=1^-} \]

\[ \frac{b}{a_2} = 0 \]
LO order inversion

LO: wrong ordering
NLO and N^2LO: restore the correct ordering
Comparison w/ other approaches

- $b/a_2 = 0$
- $L=0$
- $L=1$

Graphs showing $E_3/\omega$ vs $N_{max}$ for different approaches and initial conditions.
Three-body results away from unitarity

Untrapped particle limit: \( b/a_2 \to \infty \)

three-particle energy: \( E \approx -\frac{1}{2\mu a_2^2} \)


fit to: \( E/\omega = \alpha b^2 + \beta b + \gamma \)

(no collapse)
Summary and outlook

- Applications of EFT principles directly into a many-body method
- Renormalization of the interaction intimately related with the model space used to solve the many-body problem
- Only LO iterated to all orders, beyond LO treated in PT
- Future applications to more particles (M-scheme) and to the nuclear problem