$\pi N$ scattering in the delta-isobar region

--- Towards delta-ful nuclear forces

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Outline

Why delta?

Power counting

Fitting phase shifts (preliminary)

Conclusion
**Delta in nuclear forces**

\[ \delta \equiv m_\Delta - m_N \approx 300 \text{ MeV} \approx 2m_\pi << M_{QCD}(\sim 1 \text{ GeV}) \]

- Delta as an explicit DOF
- \( \Rightarrow \) better convergence of EFT expansions

**WANTED!**

via \( \pi N \) scattering around the delta peak

Delta-less 3NF makes error of \( \sim 25\% \)

Pandharipande, Phillips & van Kolck (2005)

Delta-less

\[ \text{Delta becomes short-range physics} \]

EFT expansion in powers of \( m_\pi / \delta \sim 0.5 \)
An incomplete “recipe” for EFT

- Relevant *degrees of freedom* at low energies
- *Symmetries*
- *Power counting*: a scheme to weigh numerous contributions
- Renormalization
  - Observables *independent* of $\Lambda$
  - Renormalization group (RG) invariance
  - Model independence

Energy

- High-energy theory
- $\Lambda$ cutoff
- Low-energy EFT
EFT of nuclear physics

**Symmetries of QCD**
- Lorentz invariance, $P, T, \Delta B=0$
- SU(3)$_c$ gauge: trivially preserved
  - All hadrons are color singlets

**Chiral symmetry**:
\[ SU(2)_L \times SU(2)_R \]
- 1. Spontaneously broken
- 2. Approximate ($m_u, m_d \neq 0$)

**Non-linear realization of chiral symmetry**
Callen, Coleman, Wess & Zumino (1969)

**Systematic Expansion in Q/M_{QCD}**

**M_{Nuc} \sim ?**

**Perturbative QCD**
- Few GeV

**Lattice QCD**
- ~ 1 GeV

**Nuclear EFT**
- ~ 100 MeV

**Dofs**
- Nucleons
- Delta isobars
- Pions
- Pseudo-Goldstone bosons

**Non-Goldstone mesons**
- Nucleons and excitations
  - $m_N, m_\Delta$
- Non-Goldstone mesons
  - $m_\rho, m_\sigma$
  - ...
Standard ChPT counting

- one-nucleon (or purely mesonic) processes

N- π couplings: infinitely many but well organized

low-energy constants (LECs) assume naturally sized

Q: k_π, m_π...

f_π: pion decay constant ~ 92MeV

once-circled = 1-more derivative

Truncation → approximation of N- π interactions
Counting pion loops

Assume: renormalization has been properly done.

\[ \text{pion propagator} \sim 1/Q^2 \]
\[ \text{nucleon propagator} \sim 1/Q \]
\[ \text{loop integral} \sim \int \frac{d^4 l}{(2\pi)^4} \sim \frac{Q^4}{(4\pi)^2} \]

\[ \sim O(1) \frac{Q}{f_\pi} \left( \frac{Q}{4\pi f_\pi} \right)^2 \]

suppressed by \( \left( \frac{Q}{M_{\text{QCD}}} \right)^2 \)

\[ 4\pi f_\pi \sim 1\text{GeV} \sim M_{\text{QCD}} \]

Check: the established counting provides enough counterterms for renormalization.
Where comes the delta resonance?

$\pi N$ "potential"

once-iterated

suppressed due to the pion loop

twice-iterated

suppressed due to the pion loops

A resonance *cannot* be generated by a perturbative series

$\rightarrow$

change standard ChPT power counting to generate the delta resonance
In $P_{33}$ channel, the LECs are unnatural

\[ \gg \frac{O(1)}{M_{QCD}} \frac{Q^2}{f_\pi^2} \]

so that: when $Q \sim \delta$

\[ \sim \]

therefore

\[ + \]

in order to generate the delta resonance
But…

$N^\dagger N \nabla \pi \nabla \pi$

non-renormalizable interactions (NR)

- Isn’t EFT supposed to make use of NR interactions?
- Not necessarily so when NR interactions are non-perturbative.

What would happen is…

To remove cutoff dependence in

$N^\dagger N \nabla \pi \nabla \pi$ + ...

One needs to promote $N^\dagger N \nabla \pi \nabla \pi$ → new cutoff dependence

→ promote yet another counterterm → …..
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Energy

\[ \Lambda \text{ cutoff} \]

- High-energy theory
- Low-energy EFT
Explicit Delta DOF

Delta propagator $\sim \frac{1}{E - \delta}$ \hspace{1cm} $E$ : center-of-mass energy

To produce the delta resonance,

\[\begin{align*}
\equiv \quad & \quad + \quad + \ldots \\
\end{align*}\]

in the deltaless theory

With explicit delta: - Natural LECs

- Resonance arises due to $E - \delta$ diverging
What have been done

Ellis & Tang (1998)
  pion-nucleon scattering & explicit delta but very different power counting

Fettes & Meissner (2001)
  pion-nucleon scattering & explicit delta but below delta

  photon-nucleon scattering & explicit delta, similar power counting but $m_\pi \ll \delta \ll M_{QCD}$

...
Delta as a nonrelativistic baryon

- $Q \ll M_N$, $M_\Delta \rightarrow$ nucleon and delta are nonrelativistic (heavy baryons)
  - If $Q \sim M_N$, ChPT already breaks down

- Perturbative Lorentz invariance in powers of $Q/M$

- A bottom-up approach (the one we use)
  - Rotational invariant (RI) operators
    - Delta field: 4-component spinor (spin 3/2)
      - e.g. $N^\dagger \bar{\Delta} \cdot \bar{\nabla}_\pi$ (isospin suppressed)

- Order-by-order boost transformation rules constrains coefficients of RI operators
Foldy-Wouthuysen rep.

\( \chi(t, \vec{x}) \) \hspace{1em} (2s+1)-component spinor

An infinitesimal boost \( \chi'(t, \vec{x}) \equiv (1 - i\vec{\xi} \cdot \vec{K})\chi(t, \vec{x}) \) \hspace{1em} rapidity \( \xi \sim Q/m \)

Boost generators

\[
\vec{K} = \frac{1}{2} (\vec{x}\omega + \omega \vec{x}) - \frac{\vec{s} \times \vec{p}}{m + \omega} - t\vec{p}
\]

\[
\vec{p} \equiv -i\vec{\nabla} \quad \omega \equiv (m^2 + p^2)^{\frac{1}{2}} \quad \vec{s} : \text{spin operators}
\]

- \( \omega \) is nonlocal \( \rightarrow \) not so easy to construct a fully relativistic theory
- A formal expansion in \( p/m \) \( \rightarrow \) order-by-order trans. rules for boosts

\[
\vec{K}^{(0)} = m\vec{x} - t\vec{p} \hspace{1em} (\text{Galilean transformation})
\]

\[
\vec{K}^{(1)} = \frac{1}{4m} (p^2\vec{x} + \vec{x}p^2) - \frac{1}{2m} \vec{s} \times \vec{p}
\]
Power counting

one- $\Delta$ -reducible: diagrams with a pure delta intermediate state

\[ \sim \frac{1}{E - \delta} \]

\[ \sim \frac{1}{E - \delta} \frac{\Sigma_\Delta^{(0)}}{E - \delta} \]

\[ \sum_\Delta^{(0)} = \quad + \quad \frac{Q^3}{M_{QCD}^2} \]

Dressing needed in one- $\Delta$ -reducible diagrams when $|E - \delta| \sim \frac{\Sigma_\Delta^{(0)}}{E - \delta} \sim \frac{Q^3}{M_{QCD}^2}$

one- $\Delta$ -irreducible: no need to dress regardless of E (standard counting applies)

\[ \sim \frac{1}{E + \delta} \]

no cancellation between E & $\delta$
<table>
<thead>
<tr>
<th></th>
<th>Near the resonance</th>
<th></th>
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</thead>
<tbody>
<tr>
<td>$</td>
<td>E - \delta</td>
<td>= O(Q)$</td>
</tr>
<tr>
<td>$Q^{-1}$</td>
<td>None</td>
<td><img src="https://via.placeholder.com/150" alt="Diagram A" /></td>
</tr>
<tr>
<td>$Q^0$</td>
<td>None</td>
<td><img src="https://via.placeholder.com/150" alt="Diagram B" /></td>
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<tr>
<td>$Q^1$</td>
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<td></td>
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Sewing two regions

Two countings for two different regions \(\rightarrow\) a piece-wise description?

<table>
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<th>Away from the resonance</th>
<th>Near the resonance</th>
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<tbody>
<tr>
<td>(</td>
<td>E - \delta</td>
</tr>
<tr>
<td>(Q^{-1})</td>
<td>(0)</td>
</tr>
</tbody>
</table>

What if enforcing \(\ldots\) in both regions? 

\(Q^{-1}\)

equivalent to

\[\begin{array}{c}
\Sigma^0 \\
\vdots
\end{array}\]

\(\ldots\)

a \textit{subset} of higher orders \(\rightarrow\) leading order

Still the same counting as long as \(\{\) not claiming better accuracy added terms are renormalized
P-wave phase shifts (Preliminary)

Dots - PSA inputs (SAID program, George Washington group)

Blue dashed - LO

Red solid - NNLO

Fitted parameters

$\delta = 318 \text{ MeV}$ not Breit-Wigner mass

$h_A = 2.48$ leading $\pi N \Delta$ coupling

$\kappa = 0.131$ related to subleading $\pi N \Delta$ coupling
P-wave phase shifts (Preliminary)
Conclusion

- Delta is important for nuclear forces
  
  Pion-nucleon scattering around the delta peak $\rightarrow \pi N\Delta$ couplings

- Explicit delta DOF

- A non-standard ChPT power counting

- A good description of $\pi N$ scattering with 3 parameters