Current operators for 3-body systems

“$N^3$LO ChPT currents and applications for GT/M1 properties”

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in collaboration with
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Soft photons and Light Nuclei@ INT, UW, Seattle, June 16~20, 2008
Contents

• Why EFTs => skip

• Intro. to HBChPT => very briefly

• Gamow-Teller (GT) currents up to N^3LO => briefly

• Magnetic dipole (M1) currents up to N^3LO
Heavy-baryon Chiral Perturbation Theory

1. Pertinent degrees of freedom: nucleons & pions, \( L = L[\pi, N] \)
   All other massive degrees of freedom (\( \rho, \omega, \Delta, \cdots \)) are integrated out. Their effects appear as higher order operators of \( \pi \)'s and \( N \)'s.

2. Expansion parameter = \( Q / \Lambda_\chi \)
   \( Q \) : typical momentum scale and/or \( m_\pi \),
   \( \Lambda_\chi : m_N \simeq 4\pi f_\pi \simeq 1 \text{ GeV} \).

   \[ L = L_0 + L_1 + L_2 + \cdots \text{ with } L_\nu \sim (Q/\Lambda_\chi)^\nu \cdots \]

3. Weinberg’s power counting rule for irreducible diagrams.
Current operators in HB ChPT

\[ \langle J_\mu \rangle \sim (Q/\Lambda_\chi)^\nu \]

with \( \nu = 2 (n_B - 1) + 2 L + \sum_i v_i, \quad v_i = d_i + n_i/2 + e_i - 1 (\geq 0) \)

※ Additional suppression: \( \gamma^\mu \sim (1, Q), \quad \gamma_5 \gamma^\mu \sim (Q, 1) \)

<table>
<thead>
<tr>
<th></th>
<th>( V^0, A^i )</th>
<th>( V^i, A^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1B</td>
<td>1</td>
<td>Q</td>
</tr>
<tr>
<td>2B: 1-pion-E</td>
<td>( Q^3 )</td>
<td>( Q \cdot Q )</td>
</tr>
<tr>
<td>2B: CTs with LECs</td>
<td>( Q^3 )</td>
<td>( Q \cdot Q^3 )</td>
</tr>
<tr>
<td>2B: loop (2-pion-E)</td>
<td>( Q^4 )</td>
<td>( Q \cdot Q^3 )</td>
</tr>
<tr>
<td>3B-tree</td>
<td>( Q^4 )</td>
<td>( Q \cdot Q^4 )</td>
</tr>
</tbody>
</table>

※ Terminology: \( N^\nu LO = (Q/\Lambda_\chi)^\nu \) compared to non-vanishing LO 1B.
※ GT up to \( N^3 LO : 1B + 2B(1\text{-pion-E }) + 2B(\text{CTs}) \)
※ M1 up to \( N^3 LO : 1B + 2B(1\text{-pion-E }) + 2B(\text{CTs}) + 2B(2\text{-pion-E}) \)
Comments on matrix elements $\langle \Psi_f | J | \Psi_i \rangle$

- **Wave functions**
  - Non-perturbative
    => resummation (solving Schroedinger or LS eq) is needed
  - Available potentials: phenomenological, EFT-based
  - Short-range behavior is very different
    => Model-dependence ? (more to come)

- **Electro-weak current operators $J$**
  - Perturbative
  - HB ChPT up to $N^3$LO ($O(Q^3)$ compared to LO)
  - At $N^3$LO, there appear two-nucleon contact-terms (CTs) in GT/M1
  - How to fix the coefficients of them (LECs) ?
    - Solve QCD => Oh no !
    - Determine from other experiments
      => usual practice in EFTs, i.e., renormalization procedure
\[ J_{\text{CT}} = C_0(\Lambda) \ (\tau \sigma)_{ij} \delta_{\Lambda}(r_{ij}) \]

- For a given w.f. and \( \Lambda \), determine LECs to reproduce the experimental values of a selected set of observables that are sensitive on \( C_0 \)

- Model-dependence in short-range region:
  - Can be visualized by a cutoff-dependence
  - Difference in short-range physics is well described by local contact operators
  - We expect that …
    - Values of LECs: \( \Lambda \)-dependent
    - Net matrix element: \( \Lambda \)-independent
    - will be proven numerically

- Model-dependence in long-range region:
  - Long-range part of ME: governed by the effective-range parameters (ERPs) such as binding energy, scattering length, effective range etc
  - In two-nucleon sector, practically no problem
    - most potentials very good in 2N sector
  - In \( A \geq 3 \), things are not quite trivial (we will see soon…)
Gamow-Teller operator \((pp\) and \(hep\))

\[
\vec{A}_{1B} = g_A \sum_i \tau_i \left[ \vec{\sigma}_i + \frac{\vec{p}_i \vec{\sigma}_i \cdot \vec{p}_i - \vec{\sigma}_i p_i^2}{2m_N^2} \right] = \text{LO} + N^2 \text{LO}
\]

\[
\vec{A}_{2B} = \sum_{i<j} \left[ \vec{A}_{ij}^{\text{OPE}} + \vec{A}_{ij}^{4F} \right] = N^3 \text{LO}
\]

There is no soft-OPE (which is \(N^2\text{LO}\)) contributions
\[ \bar{A}_{ij}^{\text{OPE}} = -\frac{g_A}{2m_Nf_{\pi}^2} \frac{1}{m_{\pi}^2 + q^2} \left[ -\frac{i}{2} (\tau_i \times \tau_j) \vec{p} (\vec{\sigma}_i - \vec{\sigma}_j) \cdot \vec{q} 
right.
\]
\[ + 4\hat{c}_3 \vec{q} \vec{q} \cdot (\tau_i \vec{\sigma}_i + \tau_j \vec{\sigma}_j) \]
\[ + \left( \hat{c}_4 + \frac{1}{4} \right) (\tau_i \times \tau_j) \vec{q} \times \left[ (\vec{\sigma}_i \times \vec{\sigma}_j) \times \vec{q} \right] \]

The values of \( c \)’s are determined from the \( \pi\)-N data
\[ \hat{c}_3 = -3.66 \pm 0.08, \quad \hat{c}_4 = 2.11 \pm 0.08 \]

\[ \bar{A}_{ij}^{4F} = -\frac{g_A}{m_Nf_{\pi}^2} \left[ 2\hat{d}_1 (\tau_i \vec{\sigma}_i + \tau_j \vec{\sigma}_j) + \hat{d}_2 (\tau_i \times \tau_j) (\vec{\sigma}_i \times \vec{\sigma}_j) \right] \]
\[ \tilde{d}^R \equiv \hat{d}_1 + 2\hat{d}_2 + \frac{1}{3} \hat{c}_3 + \frac{2}{3} \hat{c}_4 + \frac{1}{6} \]
\[ \hat{d}^R \]

- appears in
  - \( pp \)
  - \( hep \)
  - \( ^3\text{H} \rightarrow ^3\text{He} + e^- + \nu_e \) (tritium-\( \beta \) decay, TBD),
  - \( \nu-d \) scattering, \ldots .
  - \( \mu-d \) capture,
  - \ldots .

- We fix \( \hat{d}^R \) so as to reproduce the experimental value of the TBD, then we can make predictions for all other processes.

- NB: Loops and 3-body contributions are \( N^4\text{LO} \) or higher order.
**pp process**

- 1B-LO is **not** suppressed, \( \text{NLO} = \text{N}^2\text{LO} = 0 \). \( \text{LO} \gg \text{N}^3\text{LO} \).
- At \( \text{N}^3\text{LO} \), there appear CT (\( \hat{d}^R \)) and \( 1\pi \)-exchange.
- The value of \( \hat{d}^R \) is determined from exp. value of TBD rate.
  - Bridging different \( \text{A} \) sector, \( \text{A}=2 \leftrightarrow \text{A}=3 \).
- Aspects of the actual calculation:
  - Argonne \( v_{18} \).
  - Gaussian regularization, \( \exp(-q^2/\Lambda^2) \)
- No experimental data yet: Coulomb repulsion makes it difficult at low-energy.

TSP, L. Marcucci,..., PRC**67**:055206,2003, nucl-th/0106025
Results($\mathcal{M}_{2B}/\mathcal{M}_{1B}$) of the $pp$ process
**hep process**

- 1B-LO is strongly suppressed, NLO=N^2LO=0. LO ~ N^3LO.
- At N^3LO, there appear CT (\(\hat{d}^R\)) and 1\(\pi\)-exchange.
- The value of \(\hat{d}^R\) is determined from exp. value of TBD rate.
  - Bridging different A sector, A=3 ↔ A=4.
- Aspects of the actual calculation:
  - CHH method with Argonne v_{18} + Urbana X.
  - Gaussian regularization, \(\exp(-q^2/\Lambda^2)\)
- No experimerimetal data yet: Coulomb repulsion makes it difficult at low-energy.
- Required accuracy: order of magnitude.

TSP, L. Marcucci,..., PRC67(’03)055206, nucl-th/0107012
Results($\mathcal{M}_{2B}/\mathcal{M}_{1B}$) of the *hep* process
M1 currents up to N³LO

- **LO**: $\langle 1\text{-body} \rangle$

- **NLO**: $\langle 1\pi E \rangle$

- **N³LO**:
  - 1L-correction to $\langle 1\pi E \rangle$
  - $\langle 2\pi E \rangle$
  - $\langle \text{contact terms} \rangle$
\[
\mu_{12}^{CT} = \frac{1}{2m_p} \left[ g_{4S} (\sigma_1 + \sigma_2) + g_{4V} T_S^{(\times)} \right] \delta_{\Lambda}^{(3)} (r) \hat{j}_0 (qR).
\]

- \( g_{4S} \) and \( g_{4V} \) appear in
  - \( \mu(^2H), \mu(^3H), \mu(^3He), \ldots \)
  - Cross sections of \( np \rightarrow d\gamma, \text{nd} \rightarrow t\gamma, \ldots \)
  - Spin observables \( np \rightarrow d\gamma, \text{nd} \rightarrow t\gamma, \ldots \)

- We can fix \( g_{4S} \) and \( g_{4V} \) by
  - \( A=2 \) sector: \( \mu(^2H) \) and \( \sigma(np \rightarrow d\gamma) \)
  - \( A=3 \) sector: \( \mu(^3H) \) and \( \mu(^3He) \)
  - \( \ldots \)
Task I: Predictions for $\mu(^3\text{H})$ and $\mu(^3\text{He})$


- Fix $g_{4S}$ and $g_{4V}$ from the exp. values of $\mu(^2\text{H})$ and $\sigma(\text{np}\to\text{d}\gamma)$, for each nuclear potential model and cutoff $\Lambda$, and predict $\mu(^3\text{H})$ and $\mu(^3\text{He})$

- Details of the calculation:
  - NN potentials: Argonne Av14 and Av18
  - Tri-nucleon interaction (TNI): with and w/o Urbana IX
    - W.f.s obtained by variational Monte Carlo (VMC)
  - $\Lambda$: (500~800) MeV

<table>
<thead>
<tr>
<th></th>
<th>Av18</th>
<th>Av18+U9</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{BE}(^3\text{H})$</td>
<td>7.35(1) [7.61]</td>
<td>8.24(1) [8.47]</td>
<td>8.48</td>
</tr>
<tr>
<td>$\text{BE}(^3\text{He})$</td>
<td>6.59(1) [6.91]</td>
<td>7.48(1) [7.74]</td>
<td>7.72</td>
</tr>
</tbody>
</table>
$\Lambda$-dependence and $\varepsilon = (\mu_{\text{th}} - \mu_{\text{exp}}) / \mu_{\text{exp}} \, [\%]$
Model-dependence ($\Lambda = 600$ MeV)

<table>
<thead>
<tr>
<th></th>
<th>$\mu(^3\text{H}) + \mu(^3\text{He})$</th>
<th>$\mu(^3\text{H}) - \mu(^3\text{He})$</th>
<th>$g_{4s}$</th>
<th>$g_{4v}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>This work (Av18+U9)</td>
<td>0.838(0)</td>
<td>5.233(25)</td>
<td>0.300(6)</td>
<td>1.96(29)</td>
</tr>
<tr>
<td>(Av18)</td>
<td>0.838(0)</td>
<td>5.242(26)</td>
<td>0.300(6)</td>
<td>1.96(29)</td>
</tr>
<tr>
<td>(Av14+U8)</td>
<td>0.838(0)</td>
<td>5.266(33)</td>
<td>0.391(7)</td>
<td>2.25(31)</td>
</tr>
<tr>
<td>(Av14)</td>
<td>0.844(0)</td>
<td>5.204(30)</td>
<td>0.391(7)</td>
<td>2.25(31)</td>
</tr>
<tr>
<td>SNPA I [33]</td>
<td>0.828</td>
<td>5.078</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SNPA II [34]</td>
<td>0.884</td>
<td>5.114</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experiment [35]</td>
<td>0.851</td>
<td>5.107</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Something wrong?

- For $\mu(^3\text{H})$ and $\mu(^3\text{He})$: The accuracy of our EFT calculation is about the same with that of the latest phenomenological calculations. Not so bad, but not so fantastic either.

- Candidates for the gap ($\simeq 2\%$) between theory and exp.
  - Accuracy of the used wave functions in the long-range (NB. There is sizable mismatch in binding energies)
  - The $\text{N}^4\text{LO} 3$-body current:
    - $\langle M_{3B} \rangle / \langle M_{2B} \rangle \sim \langle V_{3B} \rangle / \langle V_{2B} \rangle \sim (0.05 \sim 0.1)$
    - $\Delta \mu_{3B} \sim 0.01$
  - $\Lambda$-dependence is gone
    - Short-range is under control (via renorm. of LECs)
    - higher order 2-body currents will not be important (see next 2 pages)
Convergence? (Higher order contributions?)

Isoscalar M1 ($M1S$) in \{pol. $n$\} + \{pol. $p$\} $\rightarrow d+\gamma$

- Due to pseudo-orthogonality, 1B-LO is highly suppressed, NLO$=N^2$LO$=0$.
- At $N^3$LO, there appear CT ($g_{4S}$) and $1\pi$-exchange.
- The value of $g_{4S}$ is determined from the exp. value of $\mu_d$.
- Aspects of the actual calculation:
  - Argonne $v_{18}$ wave functions.
  - Hard-core regularization, $r_C \sim 1/\Lambda$.
  - Up to $N^3$LO and up to $N^4$LO.
- No experimental data yet: it can be in principle measured via the spin observables, but requires ultra-high polarizations.

TSP, K. Kubodera, D.-P. Min & M. Rho, PLB472(’00)232
Results($\mathcal{M}_{2B}/\mathcal{M}_{1B}$) of M1S ($n+p \rightarrow d+\gamma$):

LO + N³LO

N⁴LO

Effective (net) N⁴LO contrib., after re-adjusting the value of $g_{4s}$

Naïve N⁴LO contrib.
So...

- Even if $2N$ higher order contributions are sizable, their net effect will be negligible (due to renormalization procedure of LECs), at least to low-$E$ observables.
- But $3N$ contributions will just add-up, if LECs are determined at $A=2$ sector. Thus the $N^4LO$ 3N contributions might be relevant for very precise calculations.
- $N^4LO$ 3N currents consist of tree diagrams with one insertion of $v_i=1$ vertex, work in progress.
Task II: Extension of previous work

work in progress

• $\sigma_{\text{nd}}$ and $R_c$ (photon polarization of nd capture) calculated
• $g_{4s}$ and $g_{4v}$ : fixed by $\mu(^3\text{H})$ and $\mu(^3\text{He})$
• Various potential models are considered
  – Av18 (+ U9)
  – EFT ($N^3\text{LO}$) NN potentials of Idaho group
  – EFT ($N^3\text{LO}$) NN potentials of Bonn-Bochum group with diff. $\Lambda$
    • $\{\Lambda, \Lambda'\} = \{450, 500\}, \{450, 700\}, \{600, 700\}$ MeV = (E1, E4, E5)
  – INOY (can describe BE’s of $^3\text{H}$ and $^3\text{He}$ w/o TNIs)
• Better way of calculating w.f.s : Faddeev equations
• Correlations between M1 RMEs and the triton BE ($B_3$) are found
\[ \sigma_{\text{nd}} \text{ and } R_c \]

\[ \sigma = \frac{2}{9} \frac{\alpha}{(v_{\text{rel}}/c)^2} \left( \frac{hc}{2mc^2} \right)^2 \left( \frac{q}{hc} \right)^3 \sum_{J_i} \sum_{J=1}^{J_i+\frac{1}{2}} \left\| \tilde{E}_J^{J_i(\frac{1}{2})} \right\|^2 + \left\| \tilde{M}_J^{J_i(\frac{1}{2})} \right\|^2 \]

\[ R_c = \frac{1}{3} \left[ \frac{\frac{7}{2}m_4^2 + \sqrt{8Re\left[m_2m_4^*\right]} + \frac{5}{2}|e_4|^2 + \sqrt{24Im\left[m_2e_4^*\right]} - \sqrt{3Im\left[m_4e_4^*\right]} }{|m_2|^2 + |m_4|^2 + |e_4|^2} \right] - 1 \]

\[ \left\| \tilde{X}_J^{J_i, J_f} \right\| = \frac{\sqrt{6\pi}}{q\mu_N} \sqrt{4\pi} \left\langle \Psi_{\text{b.s.}}^{J_f} \right| \left\| \chi_{JM} \right\| \Psi_{\text{scat}}^{J_i} \right\rangle \]

\[ m_2 = \left\| \tilde{M}_1^{(\frac{1}{2})(\frac{1}{2})} \right\|, m_4 = \left\| \tilde{M}_1^{(3)(\frac{1}{2})} \right\| \text{ and } e_4 = \left\| \tilde{E}_1^{(3)(\frac{1}{2})} \right\|. \]
### Λ-depdendence (model: INOY)

**inputs:** $\mu(^3\text{H})$ and $\mu(^3\text{He})$

<table>
<thead>
<tr>
<th>Lambda [MeV]</th>
<th>mu(d)</th>
<th>Sigma(np) [mb]</th>
<th>Sigma(nd) [mb]</th>
<th>- R_c</th>
</tr>
</thead>
<tbody>
<tr>
<td>500</td>
<td>0.8584</td>
<td>330.9</td>
<td>0.501</td>
<td>0.466</td>
</tr>
<tr>
<td>600</td>
<td>0.8584</td>
<td>330.7</td>
<td>0.497</td>
<td>0.465</td>
</tr>
<tr>
<td>700</td>
<td>0.8585</td>
<td>330.5</td>
<td>0.495</td>
<td>0.465</td>
</tr>
<tr>
<td>800</td>
<td>0.8583</td>
<td>330.5</td>
<td>0.495</td>
<td>0.465</td>
</tr>
<tr>
<td>900</td>
<td>0.8583</td>
<td>330.4</td>
<td>0.496</td>
<td>0.465</td>
</tr>
<tr>
<td>Exp.</td>
<td>0.8574</td>
<td>332.6(6)</td>
<td>0.508(15)</td>
<td>0.420(30)</td>
</tr>
</tbody>
</table>

※ Pionless EFT: $\sigma_{nd} = 0.503(3)$ mb, $-R_c = 0.412(3)$
Sadeghi, Bayegan, Griesshammer, nucl-th/0610029
Sadeghi, PRC75('07) 044002
Our results (model-dep.)

_Preliminary_

<table>
<thead>
<tr>
<th>Model</th>
<th>mu_d</th>
<th>sigma_np [mb]</th>
<th>sigma_nd [mb]</th>
<th>-R_c</th>
<th>(^2 a_{nd} [fm] )</th>
<th>BE(H3) [MeV]</th>
<th>BE(He3) [MeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Av18</td>
<td>0.858</td>
<td>331.9</td>
<td>0.680(3)</td>
<td>0.435</td>
<td>1.266</td>
<td>7.623</td>
<td>6.925</td>
</tr>
<tr>
<td>Av18+U9</td>
<td>0.860</td>
<td>330.6</td>
<td>0.478(3)</td>
<td>0.458</td>
<td>0.598</td>
<td>8.483</td>
<td>7.753</td>
</tr>
<tr>
<td>INOY</td>
<td>0.859</td>
<td>330.6</td>
<td>0.498(3)</td>
<td>0.465</td>
<td>0.551</td>
<td>8.483</td>
<td>7.720</td>
</tr>
<tr>
<td>I-N3LO</td>
<td>0.857</td>
<td>330.4</td>
<td>0.626(2)</td>
<td>0.441</td>
<td>1.101</td>
<td>7.852</td>
<td>7.159</td>
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<tr>
<td>E1-N3LO</td>
<td>0.858</td>
<td>328.7</td>
<td>0.688(4)</td>
<td>0.438</td>
<td>1.263</td>
<td>7.636</td>
<td>6.904</td>
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<td>E4-N3LO</td>
<td>0.859</td>
<td>331.0</td>
<td>0.609(4)</td>
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<td>1.024</td>
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<td>7.210</td>
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<td>E5-N3LO</td>
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<td>330.9</td>
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<td>0.411</td>
<td>1.781</td>
<td>7.079</td>
<td>6.403</td>
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<tr>
<td>Exp.</td>
<td>0.8574</td>
<td>332.6(7)</td>
<td>0.508(15)</td>
<td>0.420(30)</td>
<td>0.65(4)</td>
<td>8.482</td>
<td>7.718</td>
</tr>
</tbody>
</table>
\[ m_n \approx \phi_n(B_3), \quad n=2,4 \]

\[
\phi_2(B_3) = -21.87 - 10.76 \left[ \left( \frac{B_3}{B_3^{\exp}} \right)^{-2.5} - 1 \right], \\
\phi_4(B_3) = 12.24 + 11.35 \left[ \left( \frac{B_3}{B_3^{\exp}} \right)^{-2.5} - 1 \right].
\]
A trial for improvement

- Replace $B_3$ by $B_3^{\text{exp}}$
  
  $m_n = \phi_n(B_3) + \delta m_n$
  $$\Rightarrow m_n^* = \phi_n(B_3^{\text{exp}}) + \delta m_n = m_n + \phi_n(B_3^{\text{exp}}) - \phi_n(B_3)$$

- $m_n^*$
  - $m_2^* = -21.89 \pm 0.24, \quad m_4^* = 12.24 \pm 0.05$
  - $\sigma_{nd}^* = 0.491 \pm 0.008 \text{ mb}, \quad R_c^* = 0.463 \pm 0.003$
  - Cf) data: $0.508 \pm 0.015 \text{ mb} \& 0.420 \pm 0.030$
A trial to have correct the long-range part

- Adjust parameters of 3N potential to have correct scattering lengths and B.E.s
  - Range parameter (a cutoff parameter) and the coefficients of U9 potential is adjusted
    - \( C = 2.1 \text{ fm}^2 \rightarrow 3.1 \text{ fm}^2 \)
    - \( A_{2\pi} = -29.3 \text{ KeV} \rightarrow -36.955 \text{ KeV} \)
  - Can reproduce BE\( (^3\text{H}) \) and \(^2a_{\text{nd}}\), but BE\( (^3\text{He}) \) is over-bound by 40 KeV.
  - Introduce charge-dependent term in U9, and all the B.E.s and scattering lengths can be reproduced
    - \( A_{2\pi} \rightarrow -36.575 \text{ KeV} \) for \(^3\text{He}\)
### Our results (model-dep.)

**Preliminary**

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<td>0.458</td>
<td>0.598</td>
<td>8.483</td>
<td>7.753</td>
</tr>
<tr>
<td>INOY</td>
<td>0.859</td>
<td>330.6</td>
<td>0.498(3)</td>
<td>0.465</td>
<td>0.551</td>
<td>8.483</td>
<td>7.720</td>
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<tr>
<td>I-N3LO</td>
<td>0.857</td>
<td>330.4</td>
<td>0.626(2)</td>
<td>0.441</td>
<td>1.101</td>
<td>7.852</td>
<td>7.159</td>
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<tr>
<td>E1-N3LO</td>
<td>0.858</td>
<td>328.7</td>
<td>0.688(4)</td>
<td>0.438</td>
<td>1.263</td>
<td>7.636</td>
<td>6.904</td>
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<td>E4-N3LO</td>
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<td>0.448</td>
<td>1.024</td>
<td>7.930</td>
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<td>E5-N3LO</td>
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<td>330.9</td>
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<td>0.411</td>
<td>1.781</td>
<td>7.079</td>
<td>6.403</td>
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<td>&lt;m_n^*&gt;</td>
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<td>0.491(8)</td>
<td>0.463(3)</td>
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<tr>
<td>Av18+U9*</td>
<td>0.862(1)</td>
<td>330.9(3)</td>
<td>0.477(3)</td>
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<td>0.457(2)</td>
<td>0.623</td>
<td>8.482</td>
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<tr>
<td>I-N3LO+U9*</td>
<td>0.859(1)</td>
<td>330.2(4)</td>
<td>0.479(4)</td>
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<td>0.468(2)</td>
<td>0.634</td>
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<td>Exp.</td>
<td>0.8574</td>
<td>332.6(7)</td>
<td>0.508(15)</td>
<td></td>
<td>0.420(30)</td>
<td>0.65(4)</td>
<td>8.482</td>
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Discussions

- HBChPT is applied to M1 operators up to $N^3LO$
- Strong correlations of M1 matrix elements w.r.t. triton binding energy are observed
- $\Lambda$-independence is found to the satisfactory degree.
- Dependence on the potential models is tricky
  - Naively large model-dependence
  - When relevant ERPs are taken into account, there remains little model-dependence
- $3N$ current contributions might be important to have very accurate predictions