Why is electromagnetic deuteron disintegration at low energies still interesting?

1. Introduction
2. Basic Ingredients
3. Review of Electromagnetic Disintegration
4. Recent Results at Low Energies
5. Spin Asymmetry and GDH Sum Rule
6. Conclusions
1. Introduction

Although electromagnetic disintegration of the deuteron has quite a long history beginning with

- the discovery of the neutron by Chadwick in 1932,
- followed by the first photodisintegration experiment by Chadwick and Goldhaber in 1934,
- and the first theory by Bethe and Peierls in 1935,

this topic still is of current interest.
Reasons for this fact are:

- Deuteron is the simplest nuclear system, playing the same role in nuclear physics as the hydrogen atom in atomic physics.
- Deuteron serves as a test laboratory for any theoretical model of the strong nuclear force.
- Deuteron allows a reliable treatment of subnuclear degrees of freedom as manifest in meson exchange currents (MEC) and isobar configurations (IC) as well as lowest order relativistic contributions (RC).
- Deuteron serves as an effective neutron target for the study of neutron properties.
- Radiative n-p capture, related by detailed balance to deuteron photodisintegration, plays an important role in big bang nucleosynthesis (BBN).
2. Basic Ingredients

Standard nonrelativistic potential model approach:


- Nonrelativistic one-body currents: charge, convection and spin current supplemented by charge and magnetic nucleon form factors.

- For electric multipoles the Siegert theorem in conjunction with the Siegert hypothesis - “charge is not affected by meson exchange in lowest order” - is very useful, because the dominant meson exchange current contributions are implicitly included.
- **Meson exchange currents beyond SIEGERT (MEC):**
  (i) either standard $\pi$- and $\rho$-exchange,
  (ii) or consistent construction via minimal substitution. For potentials not in standard meson exchange form this can be achieved using LAPLACE transform for the potential (Eur. Phys. J. A 12, 207 (2001)).
- **Isobar configurations and currents (IC),** mainly from intermediate $\Delta(1232)$-excitation.
- **Relativistic contributions (RC) of lowest order to wave functions and e.m. current,** e.g., DARWIN-FOLDY-term and spin-orbit current.

Other approach: **Effective Field Theory (EFT).**
3. Brief Review of Electromagnetic Disintegration

(A) Photo Disintegration: Total Cross Section

\[ \sigma_{tot}(P^\gamma, P^d) = \sigma^0_{tot} \left[ 1 + P^\gamma_c P^d_1 \tau_{10}^c \cos \theta_d ight. \\
\left. + P^d_2 (\tau^0_{20} P_2 (\cos \theta_d) + P^\gamma_l P^d_2 \tau_{22}^l d_{20}^2 (\theta_d) \cos 2 \phi_d) \right] \]

\( P^\gamma = (P^\gamma_l, P^\gamma_c) \) degree of linear and circular photon polarization.
\( P^d = (P^d_1, P^d_2) \) vector and tensor deuteron polarization and deuteron orientation angles (\( \theta_d, \phi_d \)).

Of particular interest is the vector target asymmetry for circularly polarized photons \( \sigma^0_{tot} \tau_{10}^c \), because it determines the GDH sum rule.
At low energies the unpolarized total cross section $\sigma_{tot}^0$ is already quite well described by the approximations of BETHE-PEIERLS “effective range” for $E1$ and BETHE-LONGMIRE for $M1$.

DETAILED COMPARISON OF DIFFERENT POTENTIAL MODELS WITH EXPERIMENTAL DATA AS RATIO TO EFFECTIVE RANGE:

MULTIPOLE AND CURRENT CONTRIBUTIONS RELATIVE TO TOTAL
(N+MEC+IC+RC) RESULT (UP TO MULTIPOLES $L = 4$) FOR BONN OBE
R-SPACE POTENTIAL:

(B) Differential cross section including partially polarized photons and oriented deuterons:

\[
\frac{d\sigma(\theta, \phi)}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left( 1 + P_1^\gamma \Sigma^l(\theta) \cos 2\phi \right) \\
+ \sum_{I=1,2} P_I^d \left\{ \sum_{M \leq 0} T_{IM}(\theta) \cos (M(\phi_d - \phi) - \delta_{I1} \frac{\pi}{2}) \\
+ P_c^\gamma T_{IM}^c(\theta) \sin (M(\phi_d - \phi) + \delta_{I1} \frac{\pi}{2}) \right\} d_{M0}^I(\theta_d) \\
+ P_l^\gamma \sum_{M=-I}^{I} T_{IM}^l(\theta) \cos (M(\phi_d - \phi) + 2\phi - \delta_{I1} \frac{\pi}{2}) d_{M0}^I(\theta_d) \right\}
\]

Defines various photon, target and photon-target asymmetries \(\Sigma^l, T_{IM}, T_{IM}^c,\) and \(T_{IM}^l\).
ABSOLUTE AND RELATIVE MULT IPOLE CONTRIBUTIONS AT 20 MEV:

\[ \Sigma_L \] means all multipoles up to \( L = 4 \).

Absolute and relative current contributions at 20 MeV:

Relative to normal contribution (N), i.e. without explicit MEC, IC and RC.

**Comparison to experiment at 11 and 20 MeV:**

(C) Polarization observables: asymmetry for linearly polarized photons:

(C) Polarization observables: outgoing neutron polarization:


Recent calc. with same result: R. Schiavilla, Phys. Rev. C 72, 034001 (2005). See also V.I. Kukulin et al., P.R. C77.

4. Recent Results at Low Energies

(i) **Electrodisintegration at low momentum transfer and near threshold.** (P. von Neumann-Cosel et al., Phys. Rev. Lett. 88, 202304 (2002))

Experimental $d(e,e'p)n$ coincidence cross section compared to potential model (left panel) and to both EFT and potential model (right panel). The EFT result from S. Christlmeier and H.W. Grießhammer, arXiv:0803.1307.
Extracted sum of longitudinal and transversal contribution $\sigma_L + \sigma_T$, normalized to the value at $\theta_p = 0^\circ$.

Left panel: Potential model (H.A.) (solid) with separate $\sigma_L$ (dashed) and $\sigma_T$ (dash-dot) contributions.
Right panel: Comparison of EFT (Christlmeier-Grießhammer) with potential model.
Longitudinal-transverse interference cross section $\sigma_{LT}$ normalized to $\sigma_L + \sigma_T$ at $\theta_p = 0^\circ$.

Left panel: Potential model (solid).

Right panel: Comparison of pionless EFT (Christlmeier-Grießhammer) with potential model (H.A.) for $E_x = 9$ MeV.
(ii) The total $M1$-photodisintegration cross section near threshold
(a) From $180^\circ$ inclusive electron scattering experiment at S-DALINAC
and extrapolation to photon point (N. Ryezayeva et al., Phys. Rev. Lett. 100,
172501 (2008)).

Inclusive cross section for two electron energies. Hatched bands extracted from
experimental spectra, grey bands results from pionless EFT (Grießhammer) and
dashed lines from potential model calc. (H.A.).
180° inclusive cross section near threshold is dominated by M1 contribution. Thus extrapolation to photon point with $Q^2$-dependence of elastic magnetic structure function yields M1 photon cross section:

Left panel: Total cross section data up to 5 MeV with pionless EFT (Grießhammer) and potential model (H.A.) results which coincide.

Right panel: Deduced M1 cross section in the energy region relevant for big bang nucleosynthesis (BBN). Solid, dashed and dotted curves represent coinciding theoretical results from EFT and potential model for total, M1, and E1 cross sections, respectively.
(b) Measurement of the asymmetry of linearly polarized photons in $d(\gamma, n)p$:

Method:
measure photo asymmetry at $90^\circ$:

$$\Sigma(90^\circ) = \frac{1}{1+R} \text{ with } R = \frac{2\sigma(M1)}{3\sigma(E1)}$$

use total cross section:

$$\sigma^0_{tot} = \sigma(E1)(1 + 1.5R)$$

$$\rightarrow \sigma(M1) = \sigma^0_{tot} \frac{1.5R}{1+1.5R}$$

Extracted $M1$ cross section. The curves are potential model (A) and EFT results (C-S, Chen and Savage, P.R. C60, 065205 (1999)).
5. Spin Asymmetry and GDH Sum Rule

Gerasimov-Drell-Hearn sum rule is one of several photoabsorption sum rules related to the various contributions to the total photoabsorption cross section including beam and target polarization (target spin \( I \)) (H.A., Phys.Rev. 171, 1212 (1968)).

\[
\sigma_{\text{tot}}(k, P^\gamma, P^t) = \frac{1}{2} \sum_{J=0}^{2I} P_J^t \left[ (1 + (-)^J) \sigma_{J}^{11}(k) \right. \\
+ (1 - (-)^J) P_J^c \sigma_{J}^{11}(k) P_J^c \cos \theta_t \\
+ (1 + (-)^J) P_J^l \sigma_{J}^{-11}(k) d_{20}^J(\theta_t) \cos(2\phi_t) \left. \right].
\]

\( P_J^l \) and \( P_J^c \) denote degree of linear and circular photon polarization, respectively, and \( P_J^t (J = 0, \ldots, 2I) \) target polarization parameters with respect to an orientation direction, characterized by angles \((\theta_t, \phi_t)\).
The $\sigma_{J}^{\lambda',\lambda}$ are related to forward Compton scattering amplitude via optical theorem

$$\sigma_{J}^{\lambda',\lambda}(k) = \frac{4\pi}{k} \text{Im} T_{\lambda',\lambda}^{J}(k),$$

with $T_{\lambda',\lambda}^{J}$ defined by expansion of the scattering amplitude in terms of a complete set of irreducible operators in the ground state spin space $\tau_{[J]}$ with $J = 0, 1, \ldots, 2I$

$$T_{\lambda',\lambda M^{'},\lambda M}(k) = \sum_{J=0}^{2I} (-)^{-\lambda'+\lambda} \langle IM'|\tau_{\lambda'-\lambda}^{[J]}|IM\rangle T_{\lambda',\lambda}^{J}(k).$$

The $T_{\lambda',\lambda}^{J}$ can be expressed in terms of generalized polarizabilities. Operators $\tau_{[J]} \propto [S^{[1]} \times \cdots S^{[1]}][J]$ are defined by their reduced matrix elements

$$\langle I||\tau_{[J]}||I\rangle = \hat{I} \cdot \hat{J}. $$

In particular

$$\tau^{[0]} = 1, \quad \tau^{[1]} = \sqrt{\frac{3}{I(I+1)}} S.$$
Crossing symmetry implies

\[(T^{J \lambda'}_{\lambda}(-k))^* = (-)^J T^{J \lambda'}_{\lambda}(k).\]

Assuming for \(J = \text{even}\) a once-subtracted dispersion relation for \(T^{J \lambda'}_{\lambda}(k)\):

\[
\Re \left( T^{J \lambda'}_{\lambda}(k) - T^{J \lambda'}_{\lambda}(0) \right) = \frac{2k^2}{\pi} \mathcal{P} \int_0^\infty \frac{dk'}{k'} \frac{\Im m T^{J \lambda'}_{\lambda}(k')}{k'^2 - k^2} = \frac{k^2}{2\pi^2} \mathcal{P} \int_0^\infty \frac{dk'}{k'^2 - k^2},
\]

while for \(J = \text{odd}\) an unsubtracted dispersion relation may be used:

\[
\Re T^{J \lambda'}_{\lambda}(k) = \frac{2k}{\pi} \mathcal{P} \int_0^\infty dk' \frac{\Im m T^{J \lambda'}_{\lambda}(k')}{k'^2 - k^2} = \frac{k}{2\pi^2} \mathcal{P} \int_0^\infty dk' \frac{k'}{k'^2 - k^2}.\]
A power series expansion

\[
\Re T^J_{\lambda',\lambda}(k) = \begin{cases} 
\sum_{\nu=J/2}^{\infty} t^{\lambda',\lambda, J}_{\nu} k^{2\nu} & \text{for } J \text{ even}, \\
\sum_{\nu=(J-1)/2}^{\infty} t^{\lambda',\lambda, J}_{\nu} k^{2\nu+1} & \text{for } J \text{ odd},
\end{cases}
\]

yields a class of sum rules

\[
\int_0^\infty dk \frac{\sigma^{\lambda',\lambda}(k)}{k^{2\nu}} = 0 \text{ for } J \text{ even and } \nu < J/2
\]

\[
\int_0^\infty dk \frac{\sigma^{\lambda',\lambda}(k)}{k^{2\nu+1}} = 0 \text{ for } J \text{ odd and } \nu < (J-1)/2.
\]

and

\[
t^{\lambda',\lambda, J}_{\nu} = \begin{cases} 
\frac{1}{2\pi^2} \int_0^\infty dk \frac{\sigma^{\lambda',\lambda}(k)}{k^{2\nu}} & \text{for } J \text{ even and } \nu = J/2, J/2 + 1, \ldots, \\
\frac{1}{2\pi^2} \int_0^\infty dk \frac{\sigma^{\lambda',\lambda}(k)}{k^{2\nu+1}} & \text{for } J \text{ odd and } \nu = (J-1)/2, (J-1)/2 + 2, \ldots.
\end{cases}
\]
For $J = 1$ and $\nu = 0$ one obtains the GDH sum rule by using the low-energy expansion of the Compton amplitude reads

$$T_{\lambda' M, \lambda M}(k) = \delta_{\lambda' \lambda} \left( -e^2 \frac{Q^2}{M_t} + \lambda \kappa^2 \frac{e^2}{M_t^2} \langle IM|S_0|IM \rangle k + \mathcal{O}(k^2) \right),$$

with $Q$ charge, $M_t$ mass and $\kappa$ anomalous magnetic moment of target. From this expansion follows

$$T^0_{\lambda' \lambda}(k) = -\delta_{\lambda' \lambda} e^2 \frac{Q^2}{M_t} + \mathcal{O}(k^2),$$

and

$$T^1_{\lambda' \lambda}(k) = k \delta_{\lambda' \lambda} \lambda \sqrt{\frac{I(I+1)}{3}} \left( \kappa^2 \frac{e^2}{M_t^2} + 2\gamma_0 k^2 + \mathcal{O}(k^4) \right).$$
The latter relation yields the GDH sum rule in the form

\[ 4 \pi^2 \frac{\kappa^2 e^2}{M^2_t} I = 2 \sqrt{\frac{3I}{I + 1}} \int_0^\infty \frac{dk}{k} \sigma_{11}^{11}(k) \]

\[ = \int_0^\infty \frac{dk}{k} \left( \sigma^P(k) - \sigma^A(k) \right). \]

The last step follows from the fact that the contributions \( \sigma_{J}^{11} \) for \( J > 1 \) to the spin asymmetry \( \sigma^P(k) - \sigma^A(k) \) do not contribute, because their integrals vanish.

For \( J = 1 \) and \( \nu = 1 \) one obtains the sum rule for the so-called spin polarizability \( \gamma_0 \) which is defined by the next term in the expansion of \( T_{\chi, \lambda}^{11}(k) \) from which one finds

\[ \gamma_0 I = \frac{1}{4\pi^2} \sqrt{\frac{3I}{I + 1}} \int_0^\infty \frac{dk}{k^3} \sigma_{11}^{11}(k) \]

\[ = \frac{1}{8\pi^2} \int_0^\infty \frac{dk}{k^3} \left( (\sigma^P(k) - \sigma^A(k)) - 2\sqrt{7} \hat{I} \begin{pmatrix} I & I & 3 \\ I & -I & 0 \end{pmatrix} \sigma_{11}^{11}(k) \right). \]
GDH sum rule for the deuteron

- Deuteron is isoscalar and its anomalous magnetic moment is very small
  \[ \kappa_d = -0.143 \]
  \[ \rightarrow I_{d}^{GDH}(\infty) = 0.65 \mu b \]

- Absorptive processes:
  (i) photodisintegration \( \gamma + d \rightarrow n + p \),
  (ii) photoproduction of mesons (\( \pi, \eta \) etc.)

- Process (ii) is dominated by quasifree production on n and p.
  \[ \rightarrow \text{estimate of positive GDH contribution of order} \]
  \[ I_{p}^{GDH}(\infty) + I_{n}^{GDH}(\infty) = 438 \mu b \]

- \( \rightarrow \) Large negative GDH contribution from photodisintegration needed for compensation!
Spin asymmetry of deuteron photodisintegration

- At low energies only \textbf{E1 and M1} contribute significantly
  - \textbf{Strong destructive interference} of various E1 contributions to spin asymmetry at low energy.
  - Thus near threshold \textbf{dominance of isovector M1 transition} because
    \[ d(3S_1) \rightarrow 1S_0 \] resonant near threshold (antibound state)
    and because of large isovector anomalous nucleon magnetic moment.
- \textbf{1S}_0 can only be reached for antiparallel photon and deuteron spins.

\[ \rightarrow \text{Large negative GDH contribution near threshold} \]
Dominance of $M1(^1S_0)$ near threshold

\[ \sigma^P(\omega) - \sigma^A(\omega) \text{ [mb]} \]

\[ \omega \text{ [MeV]} \]

- all multipoles
- $M1(^1S_0)$
- $M1(\text{other}) \times 50$
- $E1(\text{all})$
GDH integral for photodisintegration and pion production
Contributions of various channels to the finite GDH integral (in $\mu b$), integrated up to 0.8 GeV for photodisintegration, 1.5 GeV for single pion and eta production and 2.2 GeV for double pion production on nucleon and deuteron.

<table>
<thead>
<tr>
<th></th>
<th>np</th>
<th>$\pi$</th>
<th>$\pi\pi$</th>
<th>$\eta$</th>
<th>$\Sigma$</th>
<th>sum rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>neutron</td>
<td>138.95</td>
<td>82.02</td>
<td>-5.77</td>
<td>215.20</td>
<td>233.16</td>
<td></td>
</tr>
<tr>
<td>proton</td>
<td>176.38</td>
<td>93.93</td>
<td>-8.77</td>
<td>261.54</td>
<td>204.78</td>
<td></td>
</tr>
<tr>
<td>deuteron</td>
<td>-381.52</td>
<td>263.44</td>
<td>159.34</td>
<td>-13.95</td>
<td>27.31</td>
<td>0.65</td>
</tr>
</tbody>
</table>

Spin polarizability $\gamma_0 = 3.61$ [fm$^4$]

Nota bene: For $\kappa_N = 0 \rightarrow I_d^{GDH}(550 \text{ MeV})_{\kappa_N=0} = 7.3 \mu b$
Generalized GDH sum rule for virtual photons

\[ I^{GDH}_{\gamma^*d}(Q^2) = \sqrt{6} \int_{\omega_{th}^{lab}}^{\infty} d\omega^{lab} \frac{M_d g(\omega^{lab}, Q^2)}{W_{np} q^{c.m.}} F_T^{10} \, . \]

Here the transverse spin asymmetry for virtual photons

\[ \sqrt{6} F_T^{10} = \sigma_T^{P} - \sigma_T^{A} \]

is the analogue to \( \sigma_P - \sigma_A \) for real photons.

\[ W_{np}(\omega^{lab}, Q^2) = \sqrt{M_d^2 - Q^2 + 2 M_d \omega^{lab}} , \quad q^{c.m.}(\omega^{lab}, Q^2) = \frac{M_d}{W_{np}} \sqrt{Q^2 + (\omega^{lab})^2} . \]

Factor \( g(\omega^{lab}, Q^2) \) appears because generalisation of GDH integral is to a certain extent arbitrary. Only restrictions at photon point \( Q^2 = 0 \)

\[ g(\omega^{lab}, 0) = 1 \, , \]

and

\[ \lim_{\omega_{lab} \to \infty} g(\omega^{lab}, Q^2)|_{Q^2=const.} < \infty . \]
Transverse spin asymmetry is dominated near threshold by isovector M1 transition to $^1S_0$-state.
**Finite generalized GDH integral of electrodisintegration.**

M1 dominance yields a large negative contribution with a maximum around \( Q^2 = 0.1 \) [fm\(^{-2}\)]:

Left panel: Argonne \( V_{18} \) potential with interaction effects.
Right panel: Potential model dependence and for vanishing anomalous nucleon momenta.
6. Conclusions

- Total and differential cross sections:
  - A data set of satisfactory accuracy exists. However, data with a higher accuracy (1 % or better) would be desirable for a more stringent comparison with theory.
  - Potential model calculations and EFT, where available, agree essentially.

- Polarization observables:
  - Potential model calculations provide a complete set of predictions on all possible observables.
  - More EFT results on photon and target asymmetries as well as on outgoing neutron and proton polarization are needed.
  - Experimental data on photon asymmetry are well described by potential models.
  - Data on outgoing nucleon polarization are scarce and with respect to theory inconclusive. Certainly, more data with much improved accuracy are needed before a conclusive comparison with theory is possible.